Zveza med koordinatami v stari in novi ortonormirani bazi

Uvedemo vektorje
$\vec{f}_1 = \vec{e}_\xi, \vec{f}_2 = \vec{e}_\eta, \vec{f}_3 = \vec{e}_\zeta, \quad \vec{e}_1 = \vec{e}_x, \vec{e}_2 = \vec{e}_y, \vec{e}_3 = \vec{e}_z$
in pripadajoče koordinate
$y_1 = \xi, y_2 = \eta, y_3 = \zeta, \quad x_1 = x, x_2 = y, x_3 = z.$
Vektor

$$\vec{r} = \sum_{i=1}^{3} x_i \, \vec{e}_i = \sum_{j=1}^{3} y_j \, \vec{f}_j$$

pomnožimo po vrsti skalarno z $\vec{f}_1$, $\vec{f}_2$ in $\vec{f}_3$. Dobimo

$$y_j = \sum_{i=1}^{3} x_i \, \vec{e}_i \cdot \vec{f}_j = \sum_{i=1}^{3} x_i \, Q_{ij} = \sum_{i=1}^{3} x_i \, Q^T_{ji} = \sum_{i=1}^{3} Q^T_{ji} \, x_i.$$

Od tu neposredno izpeljemo

$$[Q] = \begin{bmatrix} \vec{e}_1 \cdot \vec{f}_1 & \vec{e}_1 \cdot \vec{f}_2 & \vec{e}_1 \cdot \vec{f}_3 \\ \vec{e}_2 \cdot \vec{f}_1 & \vec{e}_2 \cdot \vec{f}_2 & \vec{e}_2 \cdot \vec{f}_3 \\ \vec{e}_3 \cdot \vec{f}_1 & \vec{e}_3 \cdot \vec{f}_2 & \vec{e}_3 \cdot \vec{f}_3 \end{bmatrix}, \qquad \begin{bmatrix} \xi \\ \eta \\ \zeta \end{bmatrix} = [Q]^T \begin{bmatrix} x \\ y \\ z \end{bmatrix}.$$