

Geometrijske karakteristike ravninskih likov

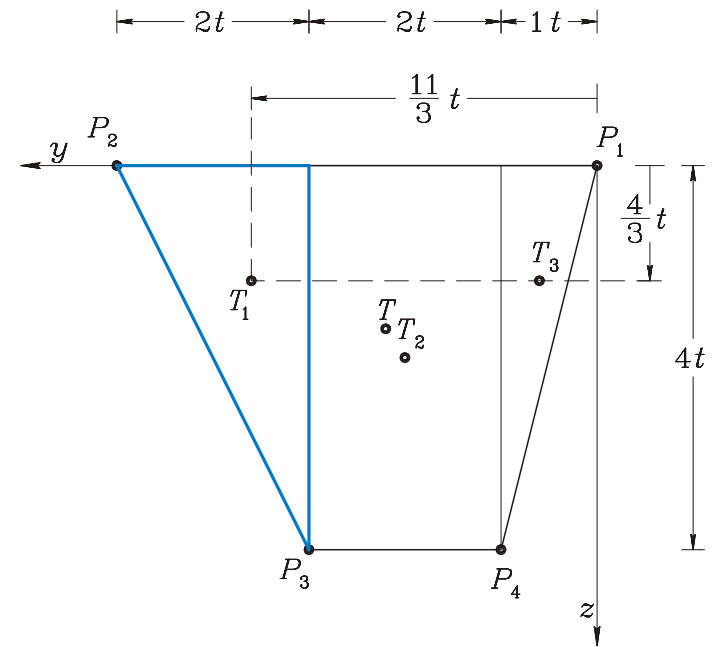
Rado Flajs¹

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Izračun geometrijskih karakteristik - 1.del

$$A_x = 14t^2,$$

$$S_y = \frac{2t \cdot 4t}{2} \cdot \frac{4}{3}t$$

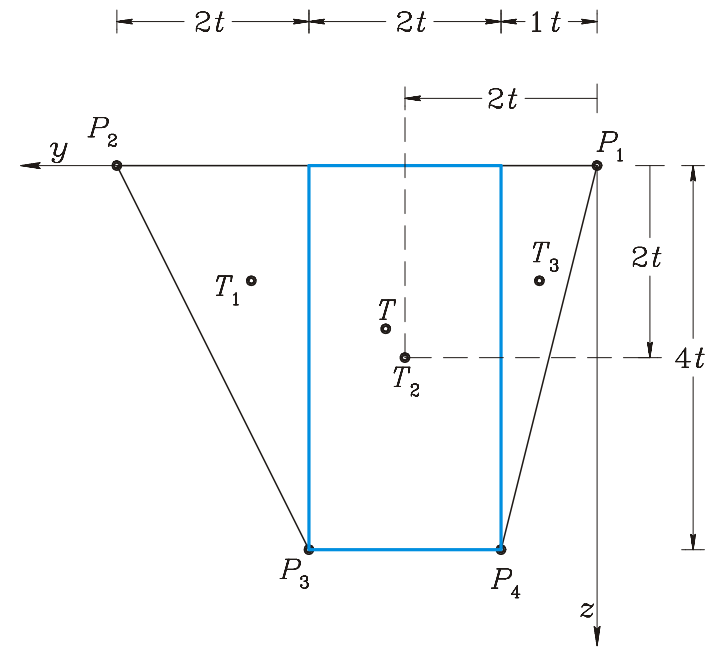


$$S_{y1} = A_{x1} \cdot z_{T1}$$

Izračun geometrijskih karakteristik - 1.del

$$A_x = 14t^2,$$

$$S_y = \frac{2t \cdot 4t}{2} \cdot \frac{4}{3}t + 2t \cdot 4t \cdot 2t$$

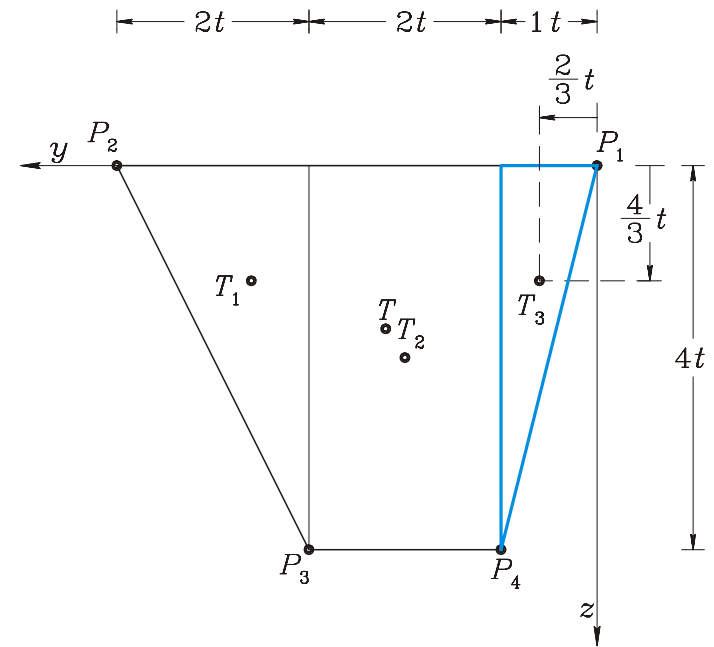


$$S_{y2} = A_{x2} \cdot z_{T2}$$

Izračun geometrijskih karakteristik - 1.del

$$A_x = 14t^2,$$

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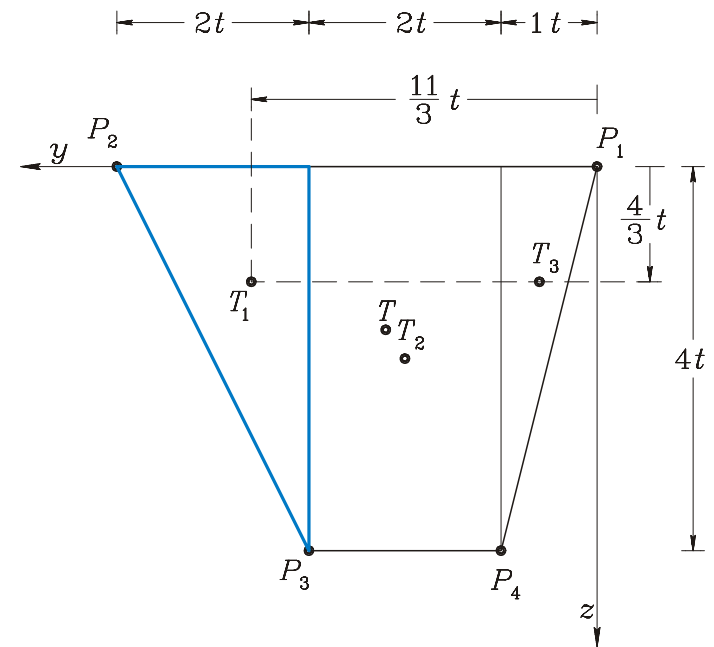
$$S_{y3} = A_{x3} \cdot z_{T3}$$

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$$S_z = \frac{2t \cdot 4t}{2} \cdot \frac{11}{3}t$$



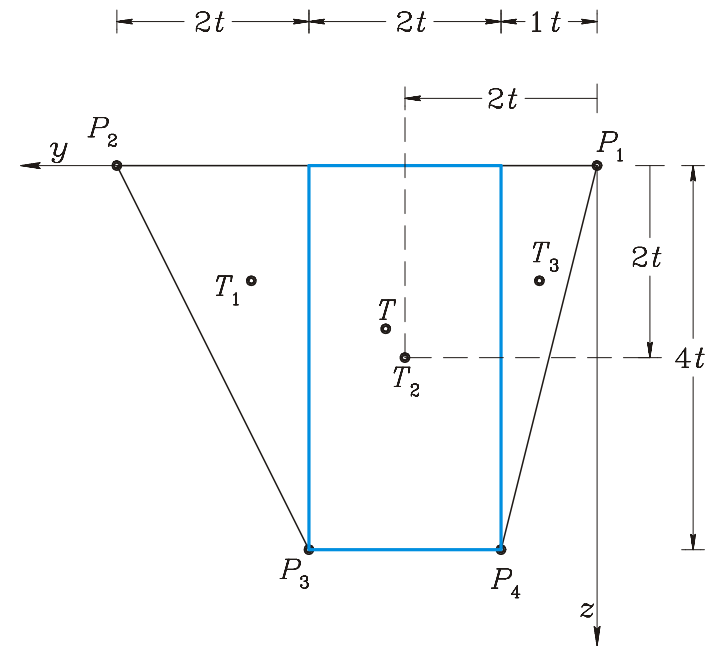
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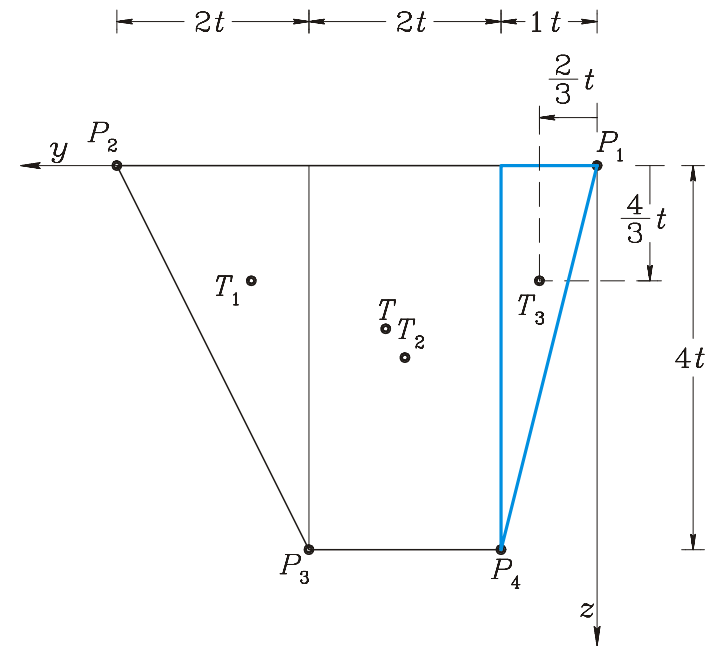
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$$S_z = \frac{2t \cdot 4t}{2} \cdot \frac{11}{3}t + 2t \cdot 4t \cdot 2t + \frac{t \cdot 4t}{2} \cdot \frac{2}{3}t = 32t^3,$$



$$S_{z3} = A_{x3} \cdot y_{T3}$$

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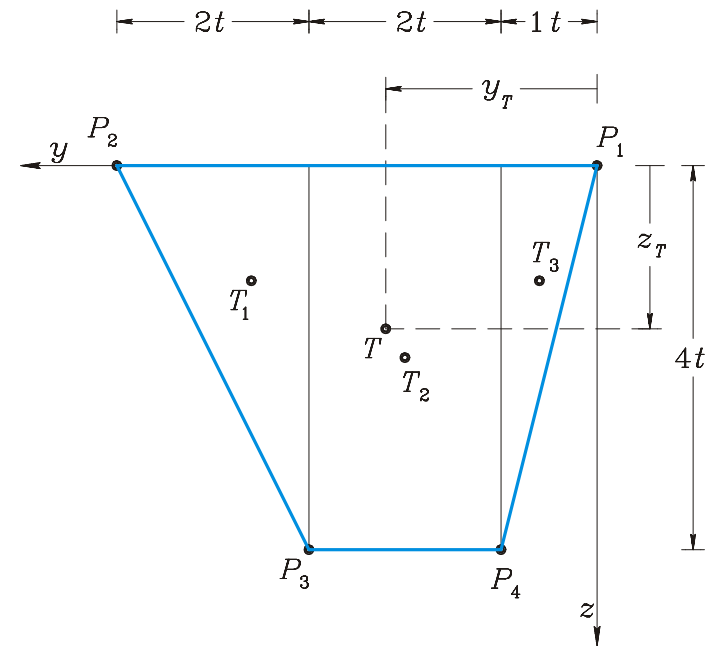
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$$y_T = \frac{S_z}{A_x} = 2.2857t,$$

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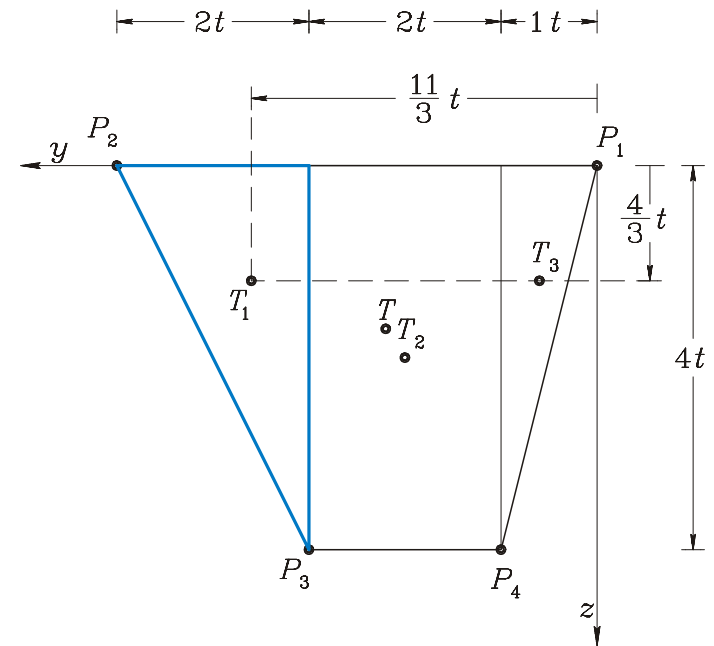
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$$I_{y1} = I_{y1}^{T1} + A_{x1} \cdot z_{T1}^2 \text{ Steinerjevo pravilo}$$

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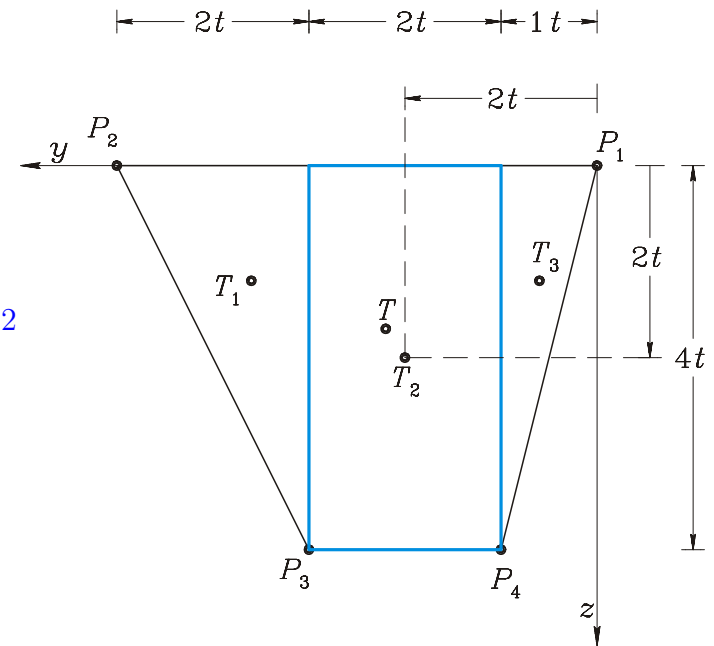
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$$I_{y2} = I_{y2}^{T_2} + A_{x2} \cdot z_{T_2}^2 \text{ Steinerjevo pravilo}$$

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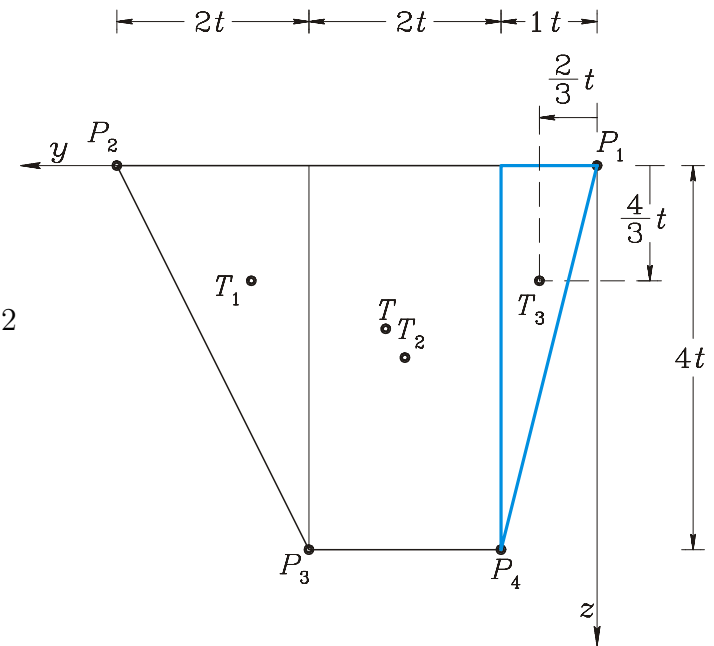
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$$I_{y3} = I_{y3}^{T3} + A_{x3} \cdot z_{T3}^2 \text{ Steinerjevo pravilo}$$

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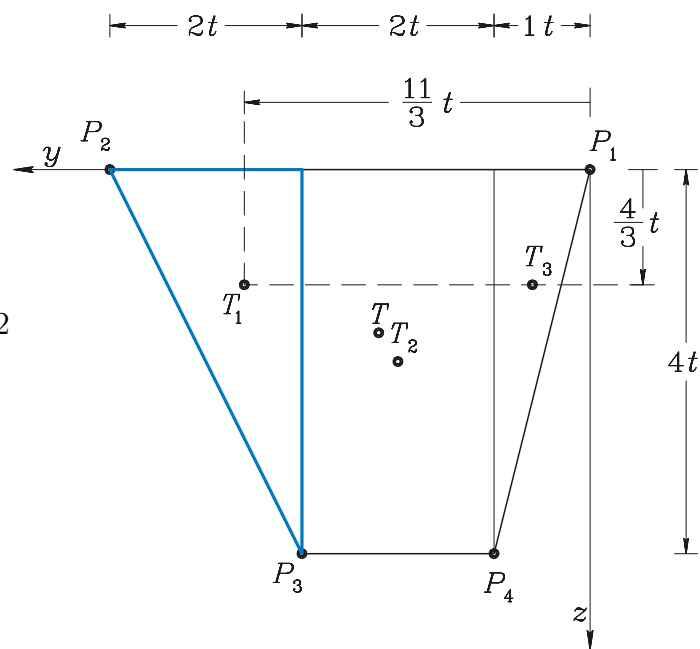
$$y_T = \frac{S_z}{A_x} = 2.2857t,$$

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$$+ \frac{1t \cdot (4t)^3}{36} + \frac{1t \cdot 4t}{2} \cdot \left(\frac{4t}{3}\right)^2 = 58.6667t^4$$

$$I_z = \frac{(2t)^3 \cdot 4t}{36} + \frac{2t \cdot 4t}{2} \cdot \left(\frac{11t}{3}\right)^2$$



$$I_{z1} = I_{z1}^{T1} + A_{x1} \cdot y_{T1}^2 \text{ Steinerjevo pravilo}$$

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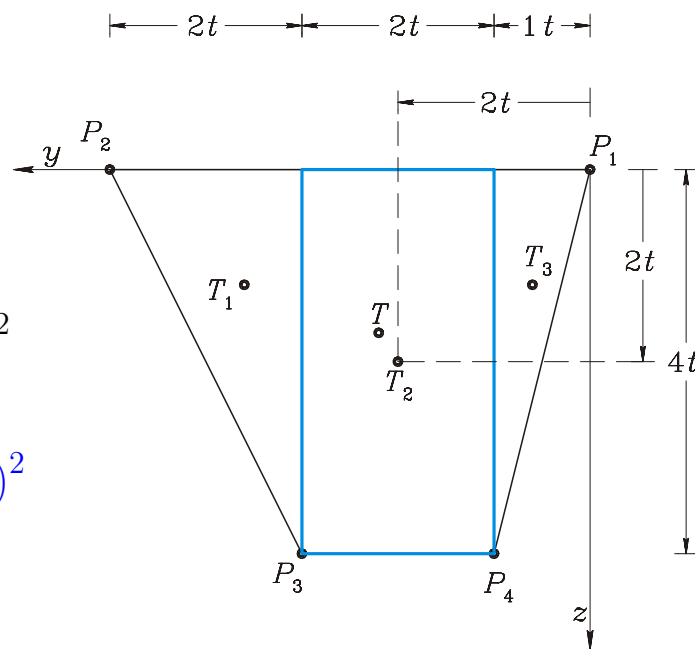
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$$I_z = \frac{(2t)^3 \cdot 4t}{36} + \frac{2t \cdot 4t}{2} \cdot \left(\frac{11t}{3}\right)^2 + \frac{(2t)^3 \cdot 4t}{12} + 2t \cdot 4t \cdot (2t)^2$$



$$I_{z2} = I_{z2}^{T2} + A_{x2} \cdot y_{T2}^2 \text{ Steinerjevo pravilo}$$

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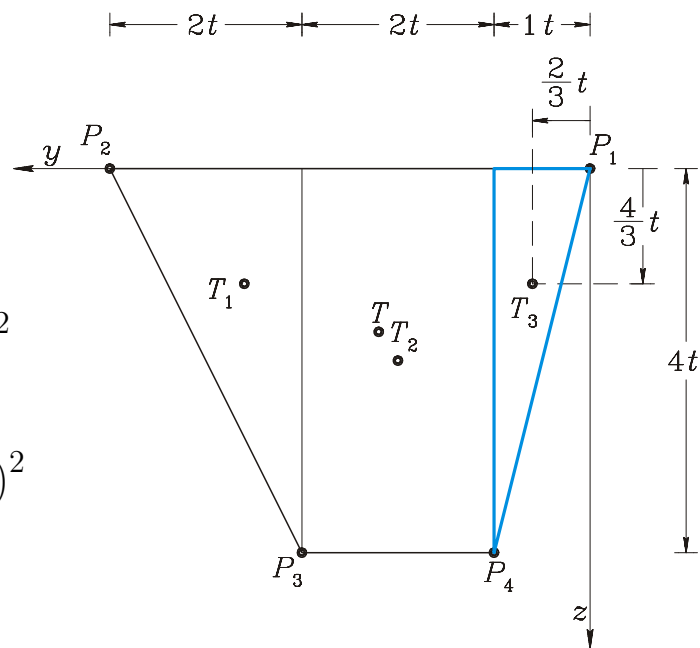
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$$I_{z3} = I_{z3}^{T_3} + A_{x3} \cdot y_{T_3}^2 \quad \text{Steinerjevo pravilo}$$

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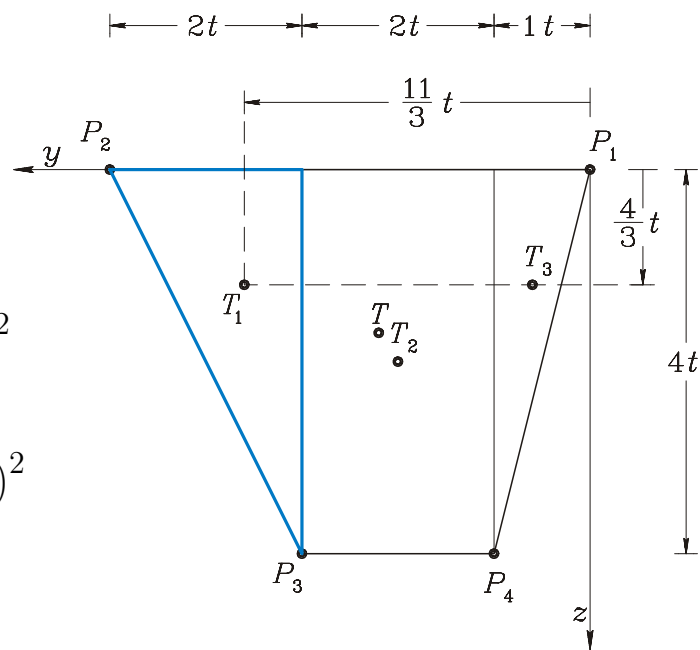
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$$I_{yz} = \frac{(2t)^2 \cdot (4t)^2}{72} - \frac{2t \cdot 4t}{2} \cdot \frac{11t}{3} \cdot \frac{4t}{3}$$



$$I_{yz1} = I_{yz1}^{T_1} - A_{x1} \cdot y_{T_1} \cdot z_{T_1} \text{ Steinerjevo pravilo}$$

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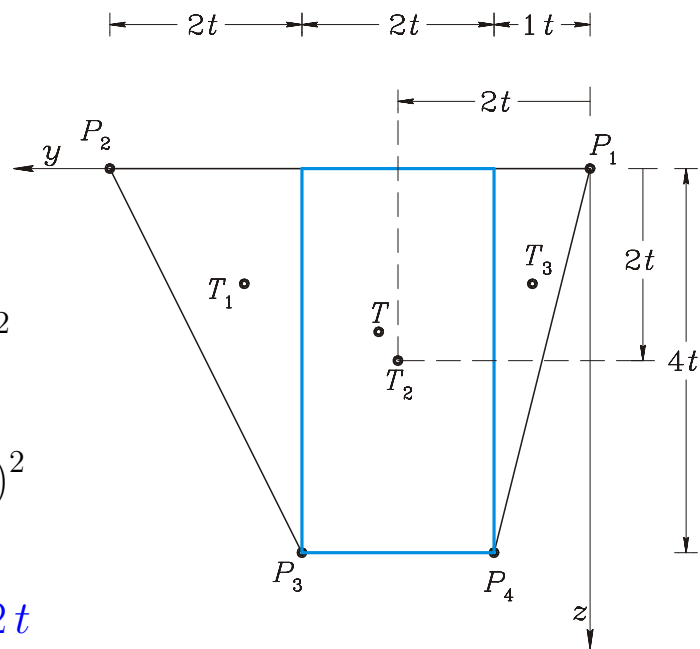
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$$I_{yz} = \frac{(2t)^2 \cdot (4t)^2}{72} - \frac{2t \cdot 4t}{2} \cdot \frac{11t}{3} \cdot \frac{4t}{3} + 0 - 2t \cdot 4t \cdot 2t \cdot 2t$$



$$I_{yz2} = I_{yz2}^{T_2} - A_{x2} \cdot y_{T_2} \cdot z_{T_2} \text{ Steinerjevo pravilo}$$

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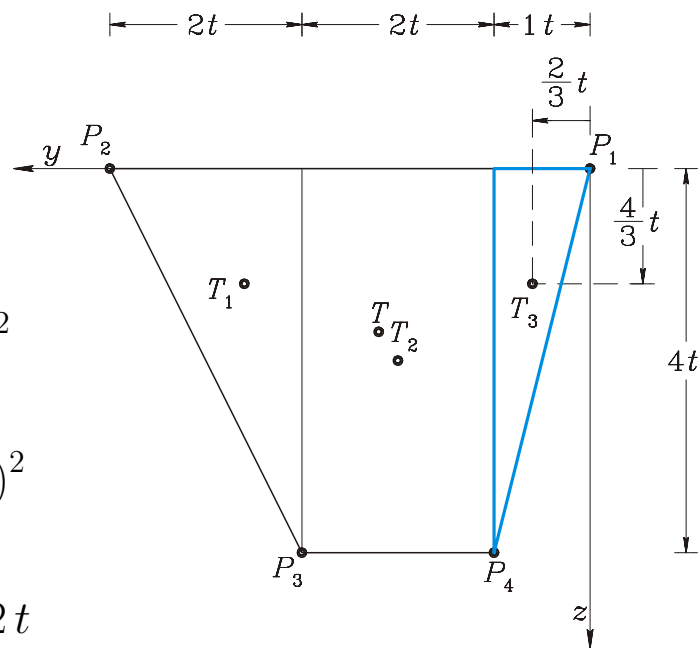
$$+ \frac{1t \cdot (4t)^3}{36} + \frac{1t \cdot 4t}{2} \cdot \left(\frac{4t}{3}\right)^2 = 58.6667t^4$$

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$$- \frac{(1t)^2 \cdot (4t)^2}{72} - \frac{1t \cdot 4t}{2} \cdot \frac{2t}{3} \cdot \frac{4t}{3} = -52.6667t^4$$



$$I_{yz3} = I_{yz3}^{T_3} - A_{x3} \cdot y_{T_3} \cdot z_{T_3} \text{ Steinerjevo pravilo}$$

Negativni predznak v prvem členu se pojavi zato, ker smo trikotnik prezrcalili okrog osi z (ali os y zamenjali z osjo $-y$) glede na lik v tabeli.

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$$I_{yz} = \frac{(2t)^2 \cdot (4t)^2}{72} - \frac{2t \cdot 4t}{2} \cdot \frac{11t}{3} \cdot \frac{4t}{3} + 0 - 2t \cdot 4t \cdot 2t \cdot 2t - \frac{(1t)^2 \cdot (4t)^2}{72} - \frac{1t \cdot 4t}{2} \cdot \frac{2t}{3} \cdot \frac{4t}{3} = -52.6667t^4$$

$$I_y^T = I_y - z_T^2 A_x = 17.5238t^4$$

$$I_z^T = I_z - y_T^2 A_x = 17.1905t^4$$

$$I_{yz}^T = I_{yz} + y_T z_T A_x = 2.1905t^4.$$

