

Vpisna številka: 261 \_ \_ \_ \_ \_

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## TRDNOST (VSŠ) - 2. KOLOKVIJ (20. 12. 2013)

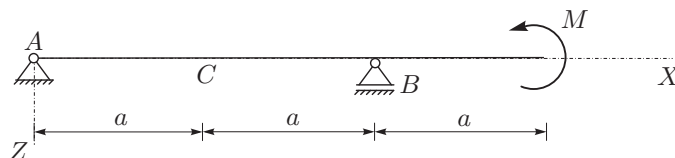
Pazljivo preberite besedilo vsake naloge! Pišite čitljivo! Uspešno reševanje!

1. Kakšno je napetostno stanje v ravnem nosilcu pri upogibu z osno silo? Na kratko opišite in pojasnite. Pojasnite tudi pojma *rezultantna osna sila* in *rezultantni upogibni moment* prečnega prereza nosilca. (15%)

2. Opišite robne pogoje pri upogibu nosilca z osno silo. Kako jih delimo? Kako so povezani z osnovnimi neznankami - pomiki osi nosilca? Navedite tudi nekaj primerov! (15%)

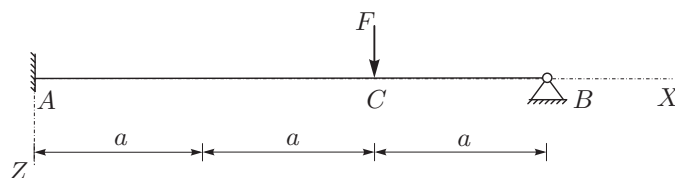
3. Za konstrukcijo na sliki izrazite prečne pomike in določite navpična pomika v točki  $C$  in na prostem koncu! (15%)

Podatki:  $a = 1.5 \text{ m}$ ,  $M = 10 \text{ kNm}$ ,  
 $E = 20000 \text{ kN/cm}^2$ ,  
 $A = 100 \text{ cm}^2$ ,  $I_y = 10000 \text{ cm}^4$ .



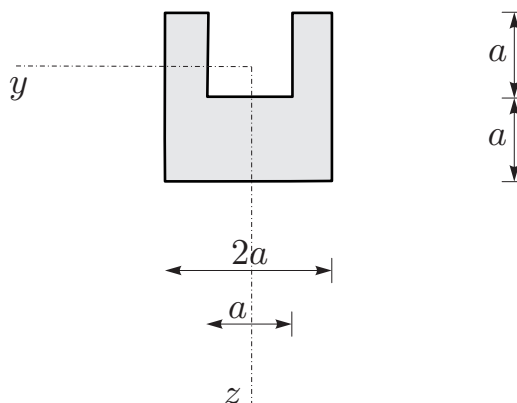
4. Za statično nedoločeno konstrukcijo na sliki izrazite upogibnico, notranje sile in vertikalni pomik v prijemališču sile! Rezultate notranjih statičnih količin prikažite z diagrami! (30%)

Podatki:  $a = 1 \text{ m}$ ,  $F = 15 \text{ kN}$ ,  
 $E = 21000 \text{ kN/cm}^2$ ,  
 $A = 100 \text{ cm}^2$ ,  $I_y = 1800 \text{ cm}^4$ .



5. Prerez na sliki je obremenjen s prečno silo  $N_z = 25 \text{ kN}$  in upogibnim momentom  $M_y = 10 \text{ kNm}$ . Določite (i) geometrijske karakteristike prereza ( $A$ ,  $y_T$ ,  $z_T$ ,  $I_y^T$ ,  $I_z^T$ ,  $I_{yz}^T$ ), (ii) potek normalnih napetosti  $\sigma_{xx}$  in (iii) potek strižnih napetosti  $\sigma_{xz}$  po prerezu! (25%)

Podatki:  $a = 10 \text{ cm}$ .

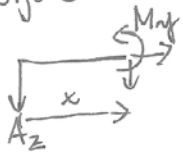


### 3. NALOGA



$$\begin{aligned} A_x &= 0 \\ A_z + B_z &= 0 \\ -B_z \cdot 3 + M &= 0 \end{aligned} \quad \begin{aligned} A_z &= -\frac{10}{3} \text{ kN} \\ B_z &= \frac{10}{3} \text{ kN} \end{aligned}$$

polje ①



$$M_y + A_z \cdot x = 0$$

$$M_y = \frac{10}{3}x$$

$$\frac{d^2 w^{\text{①}}}{dx^2} = -\frac{M_y}{EI_y} = \frac{1}{EI_y} \left(-\frac{10}{3}x\right)$$

$$\frac{dw^{\text{①}}}{dx} = \frac{1}{EI_y} \left(-\frac{5}{3}x^2 + C_1\right)$$

$$w^{\text{①}}(x) = \frac{1}{EI_y} \left(-\frac{5}{9}x^3 + C_1x + C_2\right)$$

polje ②



$$M_y = M = 10 \text{ kNm}$$

$$\frac{d^2 w^{\text{②}}}{dx^2} = -\frac{M_y}{EI_y} = \frac{1}{EI_y} (-10)$$

$$\frac{dw^{\text{②}}}{dx} = \frac{1}{EI_y} (-10x + D_1)$$

$$w^{\text{②}}(x) = \frac{1}{EI_y} (-5x^2 + D_1x + D_2)$$

ROBNI POGOJI

$$w^{\text{①}}(0) = 0 \Rightarrow C_2 = 0$$

$$w^{\text{②}}(0) = 0 \Rightarrow D_2 = 0$$

$$w^{\text{①}}(3) = w^{\text{②}}(0) = 0 \Rightarrow -\frac{5}{9} \cdot 3^3 + C_1 \cdot 3 = 0$$

$$\boxed{C_1 = 5}$$

$$\varphi^{\text{①}}(3) = \varphi^{\text{②}}(0) \Rightarrow -\frac{5}{3} \cdot 3^2 + C_1 = D_1$$

$$\boxed{D_1 = -10}$$

REŠITEV

$$w^{\text{①}}(x) = \frac{1}{EI_y} \left(-\frac{5}{9}x^3 + 5x\right)$$

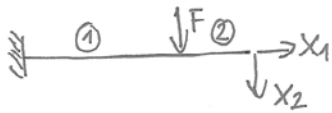
$$w^{\text{②}}(x) = \frac{1}{EI_y} (-5x^2 - 10x)$$

$$w_C = w^{\text{①}}(1.5) = 0.028 \text{ cm}$$

$$w_D = w^{\text{②}}(1.5) = -0.131 \text{ cm}$$

## 4. NALOŽBA

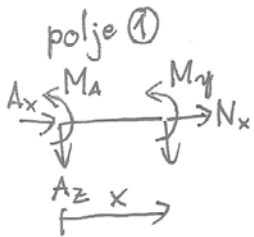
2x nedoločena konstrukcija:



$$A_x = -X_1$$

$$A_z = -15 - X_2$$

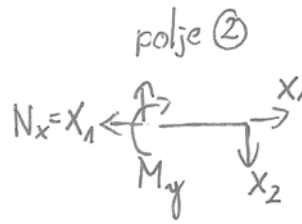
$$M_A = 30 + 3X_2$$



$$N_x = X_1$$

$$M_{yy} = -M_A - A_z \cdot x$$

$$M_{yy} = -30 - 3X_2 + 15x + X_2x$$



$$\bar{x} = 1 - x$$

$$N_x = X_1$$

$$M_{yy} = -X_2 \bar{x}$$

$$M_{yy} = X_2x - X_2$$

osni pomiki

$$\frac{du^{\text{I}}}{dx} = \frac{X_1}{EA_x}$$

$$u^{\text{I}}(x) = \frac{1}{EA_x}(X_1x + C)$$

robni pogoji

$$u^{\text{I}}(0) = 0 \Rightarrow C = 0$$

$$\frac{du^{\text{II}}}{dx} = \frac{X_1}{EA_x}$$

$$u^{\text{II}}(x) = \frac{1}{EA_x}(X_1x + D)$$

$$u^{\text{II}}(1) = 0 \quad X_1 + D = 0 \Rightarrow 3X_1 = 0$$

$$u^{\text{I}}(2) = u^{\text{II}}(0) \Rightarrow X_1 \cdot 2 = D$$

$$\Downarrow$$

$$X_1 = 0$$

$$\Downarrow$$

$$D = 0$$

$$u^{\text{I}}(x) = u^{\text{II}}(x) = 0$$

$$X_1 = 0$$

prečni pomiki

$$\frac{d^2w^{\text{I}}}{dx^2} = \frac{1}{EI_y}(30 + 3X_2 - 15x - X_2x)$$

$$\frac{dw^{\text{I}}}{dx} = \frac{1}{EI_y}\left(30x + 3X_2x - \frac{15}{2}x^2 - X_2\frac{x^2}{2} + C_1\right)$$

$$w^{\text{I}}(x) = \frac{1}{EI_y}\left(15x^2 + \frac{3}{2}X_2x^2 - \frac{15}{6}x^3 - X_2\frac{x^3}{6} + C_1x + C_2\right)$$

robni pogoji

$$w^{\text{I}}(0) = 0 \Rightarrow C_2 = 0$$

$$\varphi^{\text{I}}(0) = 0 \Rightarrow \frac{dw^{\text{I}}}{dx}\Big|_{x=0} = 0 \Rightarrow C_1 = 0$$

$$\frac{d^2w^{\text{II}}}{dx^2} = \frac{1}{EI_y}(X_2 - X_2x)$$

$$\frac{dw^{\text{II}}}{dx} = \frac{1}{EI_y}\left(X_2x - X_2\frac{x^2}{2} + D_1\right)$$

$$w^{\text{II}}(x) = \frac{1}{EI_y}\left(X_2\frac{x^2}{2} - X_2\frac{x^3}{6} + D_1x + D_2\right)$$

$$w^{\text{II}}(1) = 0 \Rightarrow X_2\frac{1}{2} - X_2\frac{1}{6} + D_1 + D_2 = 0$$

$$w^{\text{I}}(2) = w^{\text{II}}(0) \Rightarrow 60 + 6X_2 - 20 - \frac{4}{3}X_2 = D_2 \Rightarrow D_2 = 40 + \frac{14}{3}X_2$$

$$\varphi^{\text{I}}(2) = \varphi^{\text{II}}(0) \Rightarrow \frac{dw^{\text{I}}}{dx}\Big|_{x=2} = \frac{dw^{\text{II}}}{dx}\Big|_{x=0} \Rightarrow 60 + 6X_2 - 30 - 2X_2 = D_1 \Rightarrow D_1 = 30 + 4X_2$$

$$\frac{1}{3}X_2 + 30 + 4X_2 + 40 + \frac{14}{3}X_2 = 0$$

$$\frac{24}{3}X_2 = -70$$

$$X_2 = -\frac{210}{24} = -7.8 \text{ kN}$$

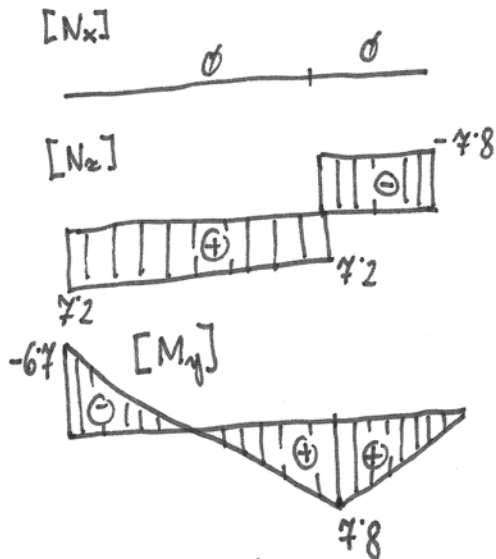
4. NALOGA, dalje

notranje sile nedoločene konstrukcije

polje ①

$$M_y^{①} = -6 \cdot 7 + 7 \cdot 2x \quad [\text{kNm}]$$

$$N_z^{①} = \frac{dM_y^{①}}{dx} = 7 \cdot 2 \text{ kN}$$



polje ②

$$M_y^{②} = 7 \cdot 8 - 7 \cdot 8x \quad \text{kNm}$$

$$N_z^{②} = \frac{dM_y^{②}}{dx} = -7 \cdot 8 \text{ kN}$$

pomik v prijemašću sile

$$\begin{aligned} w_F &= w^{②}(0) = \frac{1}{EI_y} D_2 = \frac{1}{EI_y} \left( 40 - \frac{14}{3} \cdot 7 \cdot 8 \right) = \frac{3 \cdot 7 \text{ kNm}^3}{21 \cdot 10^3 \cdot 18 \cdot 10^2 \text{ kN cm}^2 \text{ m}^2} \\ &= 0.00098 \text{ m} = 0.98 \text{ cm} \end{aligned}$$

$$w_F = 0.98 \text{ mm}$$

## 5. NALOGA

a.) geom. karakteristike

$$A_x = 300 \text{ cm}^2$$

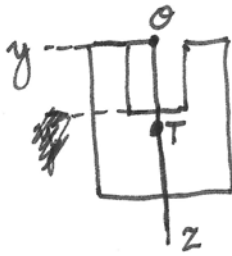
$$y_T = 0 \text{ cm}$$

$$z_T = 11.7 \text{ cm}$$

$$I_{yy} = 50000 \text{ cm}^4$$

$$I_{zz} = 12500 \text{ cm}^4$$

$$I_{yz} = 0 \text{ cm}^4$$



$$I_{yy}^T = 9167 \text{ cm}^4$$

$$I_{zz}^T = 12500 \text{ cm}^4$$

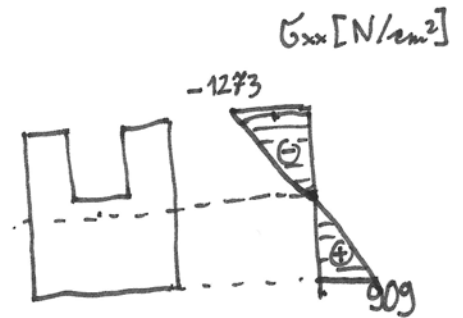
$$I_{yz}^T = 0 \text{ cm}^4$$

b.) normalne napetosti

$$\sigma_{xx} = \frac{M_y}{I_{yy}} x$$

$$\sigma_{xx}(-11.7 \text{ cm}) = -1273 \text{ N/cm}^2$$

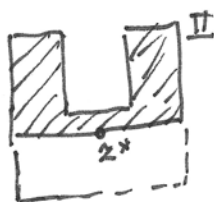
$$\sigma_{xx}(8.3 \text{ cm}) = 909 \text{ N/cm}^2$$



c.) strižne napetosti



$$S_{yy}^{*I}(z^*) = 5(z^{*2} - 136)$$



$$S_{yy}^{*II} = 10(z^{*2} - 69)$$

$z^*$	$b^*$ [cm]	$S_{yy}^*$ [cm²]	$\tau_{xz} = -\frac{N_z}{I_{yy}} \frac{S_{yy}^*}{b^*}$ [N/cm²]
-11.7	10	0	0
-1.7	10	-66.7	182
-1.7	20	-66.7	91
0	20	-69.4	95
8.3	20	0	0

