

TRDNOST (VSŠ) - 2. KOLOKVIJ (23. 01. 2004)

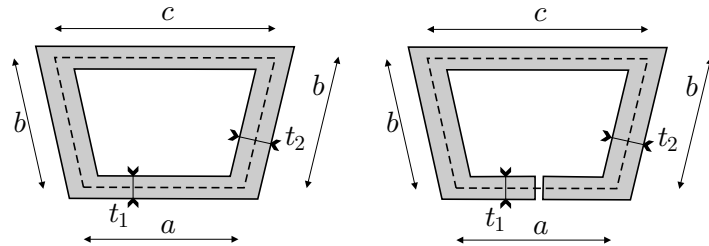
Pazljivo preberite besedilo vsake naloge!

Naloge imajo različno težo (20%, 35% in 45%), ob tem je 3. naloga še obvezna!

Pišite čitljivo! Uspešno reševanje!

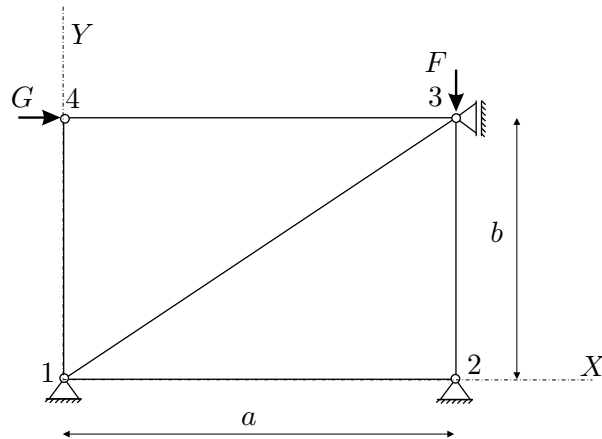
1. Odprti oziroma zaprti tankostenski prerez (enakokraki trapez) na sliki je obtežen s torzijskim momentom $M_x = 12 \text{ kNm}$. Izračunajte za oba prereza ekstremne vrednosti strižnih napetosti! (20%)

Podatki: $t_1 = 1 \text{ cm}$, $t_2 = 2 \text{ cm}$,
 $a = 25 \text{ cm}$, $b = 12 \text{ cm}$, $c = 35 \text{ cm}$.



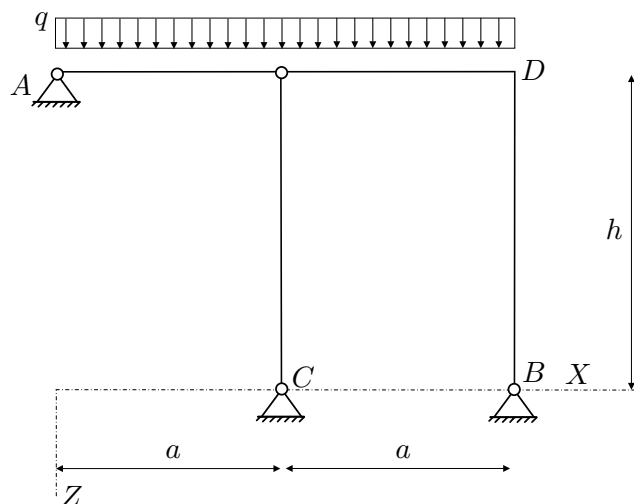
2. Za paličje na sliki določite pomike vozlišč, reakcije in osno silo v palici 23! (35%)

Podatki: $a = 3 \text{ m}$, $b = 2 \text{ m}$,
 $F = 10 \text{ MN}$, $G = 10 \text{ MN}$,
 $E = 2 \cdot 10^5 \text{ MPa}$, $A = 0.01 \text{ m}^2$.



3. Za konstrukcijo na sliki izračunajte notranje statične količine in zasuk v točki D! (OBVEZNA NALOGA! 45%)

Podatki: $a = 3 \text{ m}$, $h = 4 \text{ m}$, $q = 10 \text{ kN/m}$
 $E = 21000 \text{ kN/cm}^2$, $J_y = 90000 \text{ cm}^4$,
 $A_x = 1200 \text{ cm}^2$.



REŠITVE

1. naloga

odprti prerez: $\sigma_{\xi\zeta, \text{ext}} = \pm 28.6 \text{ kN/cm}^2$

zaprti prerez: $\sigma_{\xi\zeta, \text{min}} = 0.9 \text{ kN/cm}^2$, $\sigma_{\xi\zeta, \text{max}} = 1.8 \text{ kN/cm}^2$

TABELA DOLŽIN, KOSINUSOV IN OSNIH TOGOSTI ZA PODANO PALICJE

```
=====
palica  vozell  vozell2  dolzina  cos(a_ij)  cos(b_ij)  k_ij
=====
  1      1      2      3.000    1.000     0.000    666.667
-----
  2      1      3      3.606    0.832     0.555    554.700
-----
  3      1      4      2.000    0.000     1.000   1000.000
-----
  4      2      3      2.000    0.000     1.000   1000.000
-----
  5      3      4      3.000   -1.000     0.000    666.667
-----
```

TOGOSTNA MATRIKA PALICJA

```
=====
-1050.690  -256.015   666.667    0.000   384.023   256.015    0.000    0.000
-256.015  -1170.677    0.000    0.000   256.015   170.677    0.000   1000.000
 666.667    0.000  -666.667    0.000    0.000    0.000    0.000    0.000
 0.000    0.000    0.000  -1000.000    0.000  1000.000    0.000    0.000
 384.023   256.015    0.000    0.000 -1050.690  -256.015   666.667    0.000
 256.015   170.677    0.000  1000.000  -256.015 -1170.677    0.000    0.000
 0.000    0.000    0.000    0.000   666.667    0.000  -666.667    0.000
 0.000  1000.000    0.000    0.000    0.000    0.000    0.000  -1000.000
=====
```

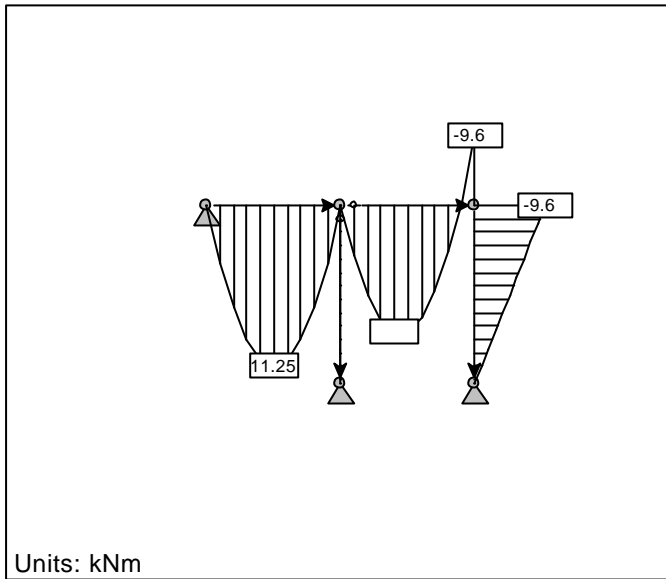
POMIKI IN REAKCIJE VOZLIŠC DANEGA PALICJA

```
=====
vozell  u_x      u_y      R_x      R_y
=====
  1      0.00000  0.00000   2.187   1.458
-----
  2      0.00000  0.00000   0.000   8.542
-----
  3      0.00000 -0.00854  -12.187
-----
  4      0.01500  0.00000
-----
```

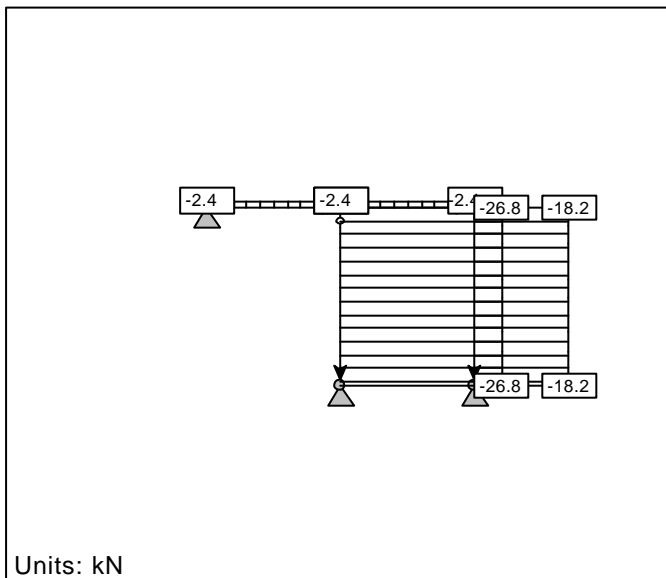
TABELA OSNIH SIL ZA PODANO PALICJE

```
=====
palica  vozell  vozell2  N_ij
=====
  1      1      2      0.000
-----
  2      1      3     -2.628
-----
  3      1      4      0.000
-----
  4      2      3     -8.542
-----
```

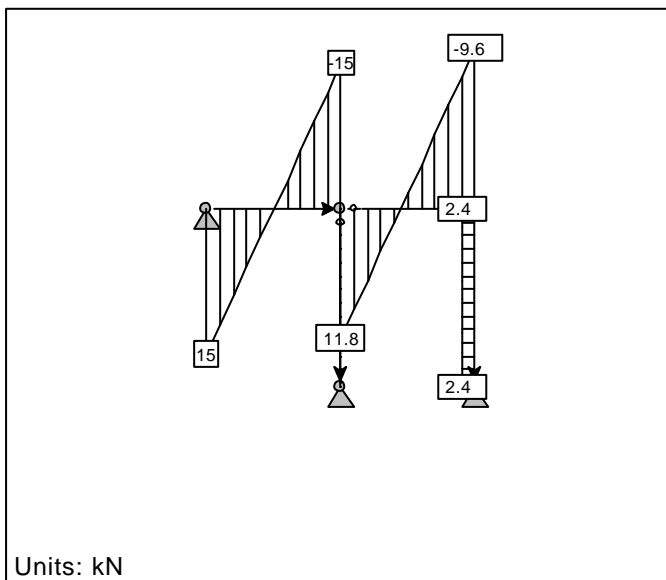
Upogibni momenti My



Osne sile Fx



Strzne sile Fz



TRDNOST (VŠŠ) - 2. KOLOKVIJ (20. 01. 2005)

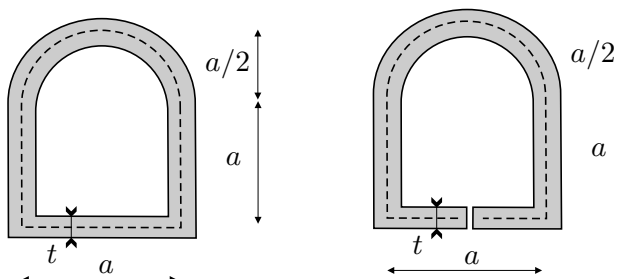
Pazljivo preberite besedilo vsake naloge!

Naloge imajo različno težo (15%, 20%, 25% in 40%), ob tem je 4. naloga še obvezna!

Pišite čitljivo! Uspešno reševanje!

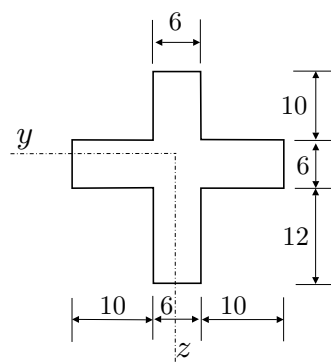
1. Odprti oziroma zaprti tankosten-ski prerez na sliki je obtežen s torzijskim momentom $M_x = 10 \text{ kNm}$. Izračunajte za oba prereza ekstremne vrednosti strižnih napetosti! (15%)

Podatki: $t = 1 \text{ cm}$, $a = 20 \text{ cm}$.



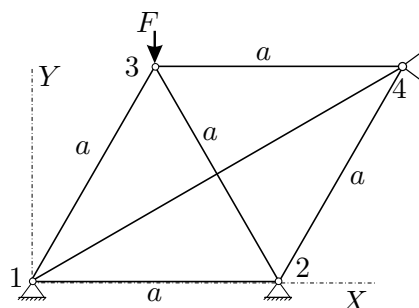
2. Prerez na sliki je obremenjen s prečno silo $N_z = 15 \text{ kN}$. Določite nekaj značilnih vrednosti in skicirajte diagram strižne napetosti σ_{xy} v tem prerezu! (20%)

Podatki za prerez so v centimetrih.



3. Za paličje na sliki določite pomike vozlišč po metodi pomikov! Nasvet: določiti je treba zgolj pomike, pri čemer je veliko prostostnih stopenj podprtih. (25%)

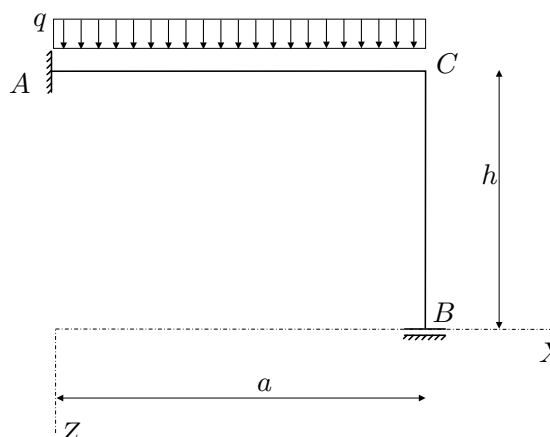
Podatki: $a = 3 \text{ m}$, $F = 20 \text{ MN}$,
 $E = 2 \cdot 10^5 \text{ MPa}$, $A = 0.02 \text{ m}^2$.



4. Za konstrukcijo na sliki izračunajte notranje statične količine in zasuk v točki C!

(OBVEZNA NALOGA! 40%)

Podatki: $a = 5 \text{ m}$, $h = 2 \text{ m}$, $q = 10 \text{ kN/m}$
 $E = 3500 \text{ kN/cm}^2$, $J_y = 90000 \text{ cm}^4$,
 $A_x = 1200 \text{ cm}^2$.



1. naloga:

odprti prerez: $\sigma_{\xi\xi} = 32.8 \text{ kN/cm}^2$

zaprti prerez: $\sigma_{\xi\xi} = 0.898 \text{ kN/cm}^2$

2. naloga:

y^*	h^*	S_y^*	$\sigma_{xy} [\text{N/cm}^2]$
-13	6	0	0
-3	6	-35	7.7
-3	28	-35	1.6
0	28	0	0
3	28	35	-1.6
3	6	35	-7.7
13	6	0	0

TABELA DOLŽIN, KOSINUSOV IN OSNIH TOGOSTI ZA PODANO PALIČJE

palica	vozel1	vozel2	dolzina	cos(a_ij)	cos(b_ij)	k_ij
1	1	2	3.000	1.000	0.000	1333.333
2	1	3	3.000	0.500	0.866	1333.333
3	1	4	5.196	0.866	0.500	769.800
4	2	3	3.000	-0.500	0.866	1333.333
5	2	4	3.000	0.500	0.866	1333.333
6	3	4	3.000	1.000	0.000	1333.333

TOGOSTNA MATRIKA PALIČJA

-2244.017	-910.684	1333.333	0.000	333.333	577.350	577.350	333.333
-910.684	-1192.450	0.000	0.000	577.350	1000.000	333.333	192.450
1333.333	0.000	-2000.000	0.000	333.333	-577.350	333.333	577.350
0.000	0.000	0.000	-2000.000	-577.350	1000.000	577.350	1000.000
333.333	577.350	333.333	-577.350	-2000.000	0.000	1333.333	0.000
577.350	1000.000	-577.350	1000.000	0.000	-2000.000	0.000	0.000
577.350	333.333	333.333	577.350	1333.333	0.000	-2244.017	-910.684
333.333	192.450	577.350	1000.000	0.000	0.000	-910.684	-1192.450

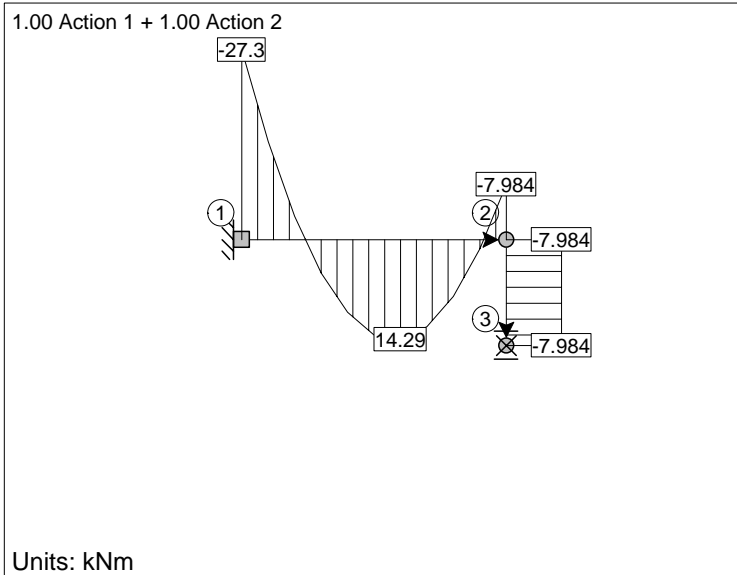
POMIKI IN REAKCIJE VOZLIŠČ DANEGA PALIČJA

vozel	u_x	u_y	R_x	R_y
1	0.00000	0.00000	5.774	10.000
2	0.00000	0.00000	-5.774	10.000
3	0.00000	-0.01000		
4	0.00000	0.00000	0.000	

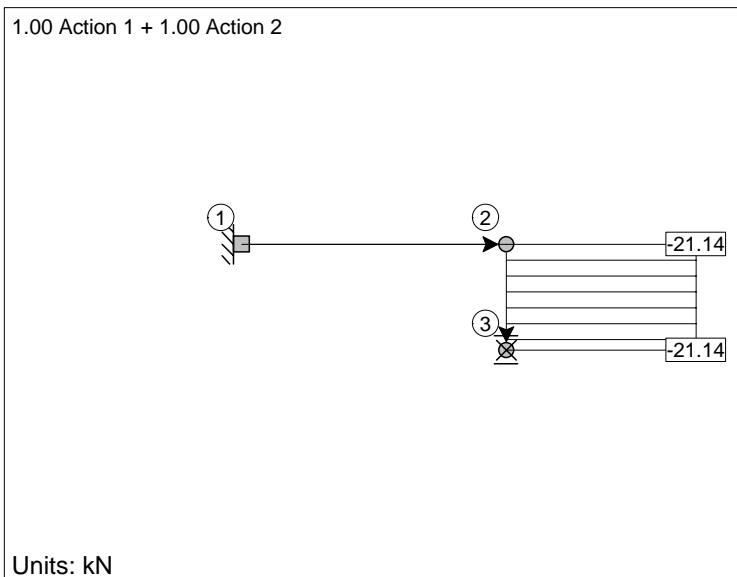
TABELA OSNIH SIL ZA PODANO PALIČJE

palica	vozel1	vozel2	N_ij
1	1	2	0.000
2	1	3	-11.547
3	1	4	0.000
4	2	3	-11.547
5	2	4	0.000
6	3	4	0.000

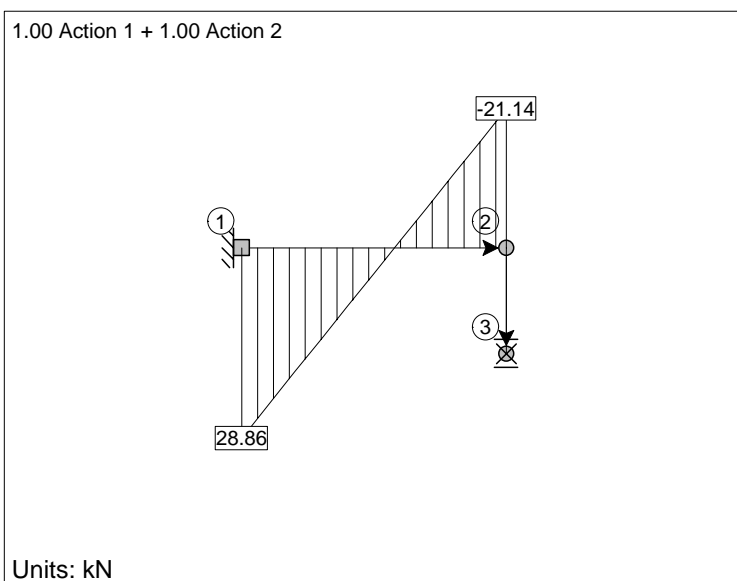
LC2: Load case 2: Bending Moments My



LC2: Load case 2: Axial Forces Fx



LC2: Load case 2: Shear Forces Fz



LC2: Load case 2: Nodal Displacements

Node	ux [cm]	uz [cm]	fiy [deg]
2:	0.00000	0.0010065	0.029043
3:	0.050689	-	-

TRDNOST (VŠŠ) - 2. KOLOKVIJ (18. 01. 2006)

Pazljivo preberite besedilo vsake naloge!

Naloge imajo različno težo (30%, 40% in 30%), ob tem je 2. naloga še obvezna!

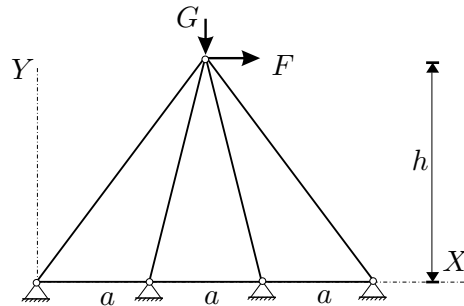
Pišite čitljivo! Uspešno reševanje!

1. Za paličje na sliki določite pomike vozlišč in notranje sile v palicah po metodi pomikov! Namig: veliko prostostnih stopenj je podprtih. (30%)

Podatki: $a = 2 \text{ m}$, $h = 4 \text{ m}$,

$F = 30 \text{ MN}$, $G = 10 \text{ MN}$,

$E = 2 \cdot 10^5 \text{ MPa}$, $A = 0.01 \text{ m}^2$.

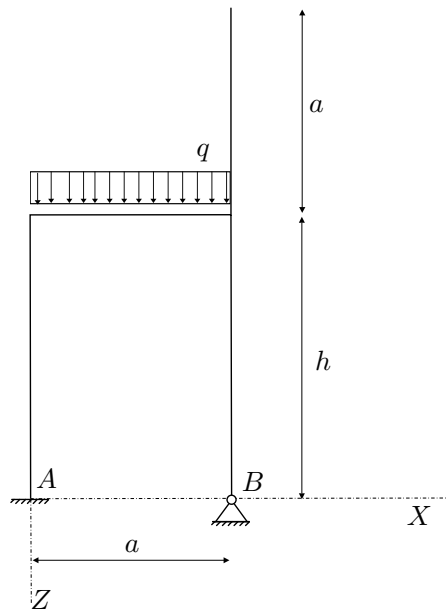


2. Za konstrukcijo na sliki izračunajte notranje statične količine!

(OBVEZNA NALOGA! 40%)

Podatki: $a = 3 \text{ m}$, $h = 5 \text{ m}$, $q = 15 \text{ kN/m}$

$E = 3500 \text{ kN/cm}^2$, $J_y = 90000 \text{ cm}^4$.



3. Za konstrukcijo na sliki izračunajte horizontalni pomik v točki C in zasuk v točki D! (30%)

Podatki: $a = 3 \text{ m}$, $q = 5 \text{ kN/m}$

$E = 21000 \text{ kN/cm}^2$, $J_y = 7908 \text{ cm}^4$.

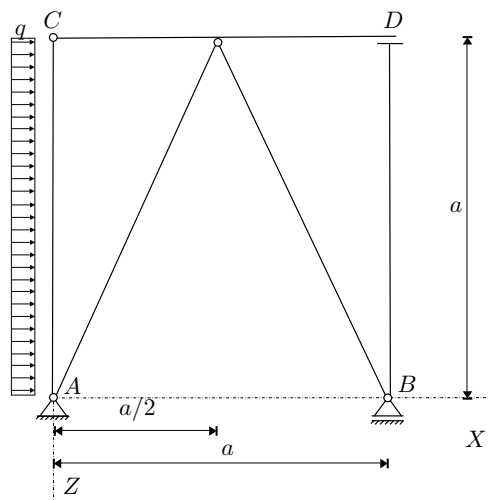


TABELA DOLŽIN, KOSINUSOV IN OSNIH TOGOSTI ZA PODANO PALIČJE

```
=====
palica  vozell  vozell2  dolzina  cos(a_ij)  cos(b_ij)  k_ij
=====
```

palica	vozell	vozell2	dolzina	cos(a_ij)	cos(b_ij)	k_ij
1	1	2	5.000	-0.600	-0.800	400.000
2	1	3	4.123	-0.243	-0.970	485.071
3	1	4	4.123	0.243	-0.970	485.071
4	1	5	5.000	0.600	-0.800	400.000
5	2	3	2.000	1.000	0.000	1000.000
6	3	4	2.000	1.000	0.000	1000.000
7	4	5	2.000	1.000	0.000	1000.000

```
-----
```

TOGOSTNA MATRIKA PALIČJA

```
=====
```

-345.067	0.000	144.000	192.000	28.534	114.134	28.534	-114.134	✓
144.000	-192.000							
0.000	-1425.075	192.000	256.000	114.134	456.538	-114.134	456.538	✓
-192.000	256.000							
144.000	192.000	-1144.000	-192.000	1000.000	0.000	0.000	0.000	✓
0.000	0.000							
192.000	256.000	-192.000	-256.000	0.000	0.000	0.000	0.000	✓
0.000	0.000							
28.534	114.134	1000.000	0.000	-2028.534	-114.134	1000.000	0.000	✓
0.000	0.000							
114.134	456.538	0.000	0.000	-114.134	-456.538	0.000	0.000	✓
0.000	0.000							
28.534	-114.134	0.000	0.000	1000.000	0.000	-2028.534	114.134	✓
1000.000	0.000							
-114.134	456.538	0.000	0.000	0.000	0.000	114.134	-456.538	✓
0.000	0.000							
144.000	-192.000	0.000	0.000	0.000	0.000	1000.000	0.000	✓
-1144.000	192.000							
-192.000	256.000	0.000	0.000	0.000	0.000	0.000	0.000	✓
192.000	-256.000							

POMIKI IN REAKCIJE VOZLIŠČ DANEGA PALIČJA

```
=====
```

vozel	u_x	u_y	R_x	R_y
1	0.08694	-0.00702		
2	0.00000	0.00000	-11.172	-14.896
3	0.00000	0.00000	-1.680	-6.719
4	0.00000	0.00000	-3.282	13.126
5	0.00000	0.00000	-13.867	18.489

```
-----
```

TABELA OSNIH SIL ZA PODANO PALIČJE

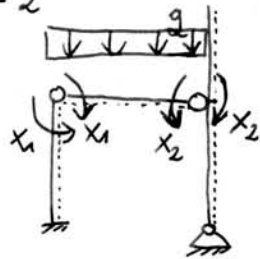
```
=====
```

```
palica    vozell    vozell2    N_ij
=====
  1         1         2         18.620
-----
  2         1         3          6.926
-----
  3         1         4        -13.530
-----
  4         1         5        -23.111
-----
  5         2         3          0.000
-----
  6         3         4          0.000
-----
  7         4         5          0.000
-----
```

2. NALOGA

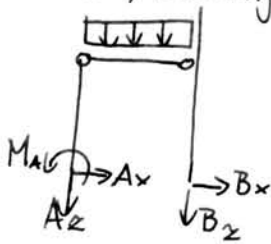
a.) $n = 3 + 2 - 3 = 2$

b.) sprostitev:



c.) notranje sile

c1.) zunanja obtežba

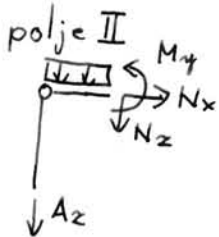


$$\begin{aligned} A_x + B_x &= 0 \\ B_x = 0 &\Rightarrow A_x = 0 \\ \Rightarrow M_A &= 0 \\ A_z + B_z &= -g \cdot a \\ -B_z \cdot a - g \frac{a^2}{2} &= 0 \end{aligned}$$

$$\begin{aligned} A_z &= -\frac{g \cdot a}{2} \\ B_z &= -\frac{g \cdot a}{2} \end{aligned}$$

polje I:

$$\begin{aligned} \uparrow N_x \\ \downarrow A_z \end{aligned} \quad \boxed{N_x = A_z = -22.5 \text{ kN}}$$

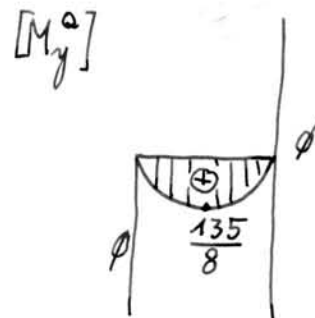
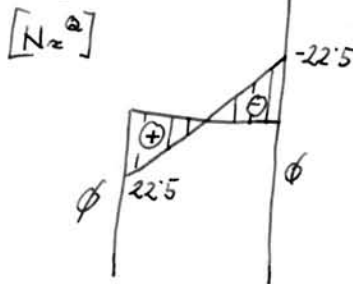
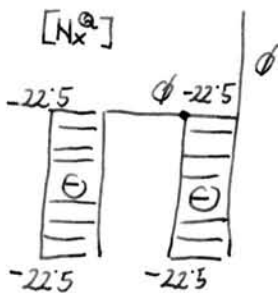


$$\begin{aligned} N_x &= 0 \\ N_z &= -A_z - g \cdot x \\ N_z &= 22.5 - 15x \\ M_y &= 22.5x - 7.5x^2 \\ M_y(1.5) &= 16.875 \text{ kNm} \\ &= \frac{135}{8} \end{aligned}$$

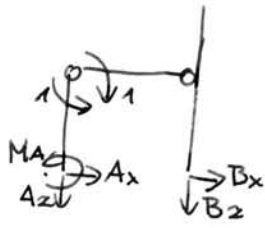
polje III

$$\begin{aligned} \uparrow N_x \\ \downarrow B_z \end{aligned} \quad N_x = B_z = -22.5 \text{ kN}$$

DIAGRAMI:



c2.) $X_1 = 1$



$$\begin{aligned} B_x &= 0 \\ A_x &= 0 \\ M_A + 1 &= 0 \Rightarrow M_A = -1 \text{ kNm} \\ A_z + B_z &= 0 \\ M_A - B_z \cdot a &= 0 \end{aligned}$$

$$\begin{aligned} B_z &= -\frac{1}{3} \text{ kN} \\ A_z &= \frac{1}{3} \text{ kN} \end{aligned}$$

polje I:

$$\begin{aligned} \uparrow N_x \\ \rightarrow N_z \\ \uparrow M_y \\ \downarrow A_z \end{aligned} \quad \begin{aligned} N_x &= \frac{1}{3} \text{ kN} \\ N_z &= 0 \\ M_y &= 1 \text{ kNm} \end{aligned}$$

polje II

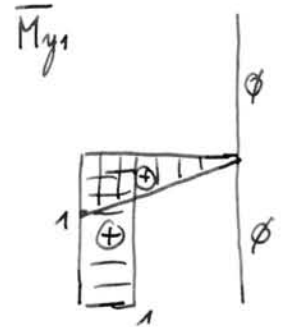
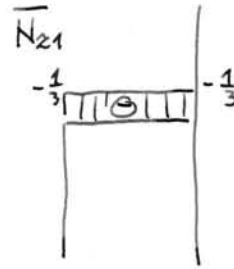
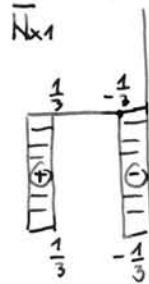
$$\begin{aligned} \uparrow M_y \\ \rightarrow N_x \\ \downarrow N_z \\ \downarrow A_z \end{aligned} \quad \begin{aligned} N_x &= 0 \\ N_z &= -\frac{1}{3} \text{ kN} \\ M_y &= 1 - \frac{1}{3}x \end{aligned}$$

polje III

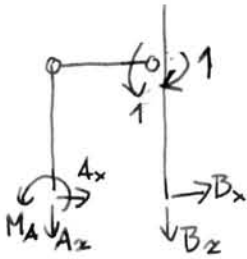


$$N_x = -\frac{1}{3} \text{ kN}$$

DIAGRAMI:



c3.) $X_2 = 1$



$$B_x \cdot h - 1 = 0$$

$$B_x = \frac{1}{5} \text{ kN} \quad A_x = -\frac{1}{5} \text{ kN}$$

$$B_z + A_z = 0$$

$$A_z = \frac{1}{3} \text{ kN}$$

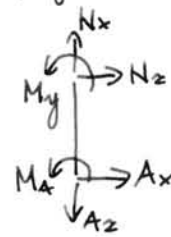
$$M_A + A_x \cdot h = 0$$

$$M_A = +1 \text{ kNm}$$

$$M_A - B_z \cdot a = 0$$

$$B_z = +\frac{1}{3} \text{ kN}$$

polje I:

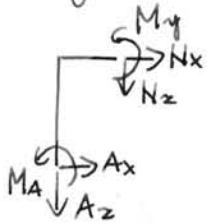


$$N_x = -\frac{1}{3} \text{ kN}$$

$$N_z = \frac{1}{5} \text{ kN}$$

$$M_y = -1 + \frac{1}{5}x$$

polje II



$$N_x = +\frac{1}{5} \text{ kN}$$

$$N_z = +\frac{1}{3} \text{ kN}$$

$$M_y = -1 + \frac{1}{5} \cdot 5 + \frac{1}{3}x$$

polje III

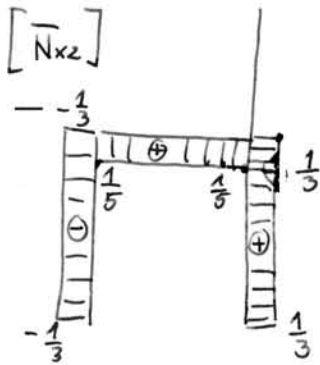


$$N_x = +\frac{1}{3} \text{ kN}$$

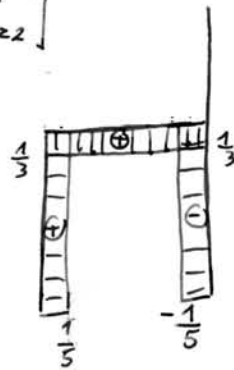
$$N_z = -\frac{1}{5} \text{ kN}$$

$$M_y = -\frac{1}{5}x$$

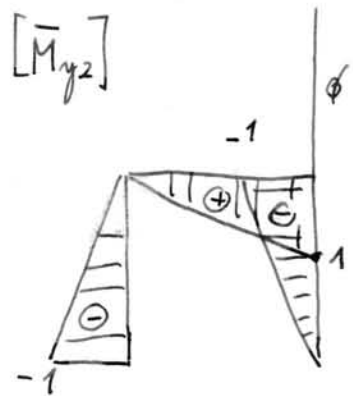
DIAGRAMI



[N_z]



[M_y]



d.) Dobčitev X_1 in X_2

$$EI_y a_{11} = \int_0^h \square^1 \square^1 dx + \int_0^a \triangle^1 \triangle^1 dx = 5 \cdot 1 \cdot 1 + \frac{1}{3} \cdot 3 \cdot 1 \cdot 1 = 6$$

$$EI_y a_{22} = 2 \int_0^h \triangle^1 \triangle^1 dx + \int_0^a \triangle^1 \triangle^1 dx = 2 \cdot 5 \cdot \frac{1}{3} \cdot 1 \cdot 1 + 3 \cdot \frac{1}{3} \cdot 1 \cdot 1 = \frac{13}{3}$$

$$EI_y a_{12} = \int_0^h \square^1 \triangle^{-1} dx + \int_0^a \triangle^1 \triangle^1 dx = 5 \cdot \frac{1}{2} \cdot (-1) + \frac{1}{6} \cdot 3 = -2$$

$$EI_y b_1 = \int_0^a \text{parabola} \cdot \triangle^1 dx = \frac{1}{3} \cdot 3 \cdot \frac{135}{8} = \frac{135}{8}$$

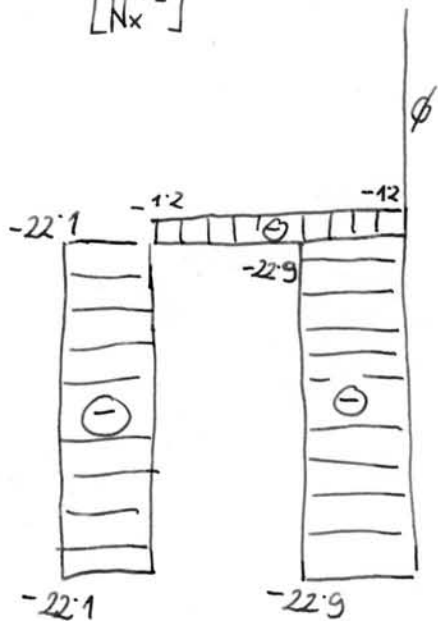
$$EI_y b_2 = \frac{135}{8}$$

$$\begin{bmatrix} 6 & -2 \\ -2 & \frac{13}{3} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = - \begin{bmatrix} \frac{135}{8} \\ \frac{135}{8} \end{bmatrix} \Rightarrow -2X_2 + 13X_2 = -\frac{135}{8} - \frac{135 \cdot 3}{8}$$

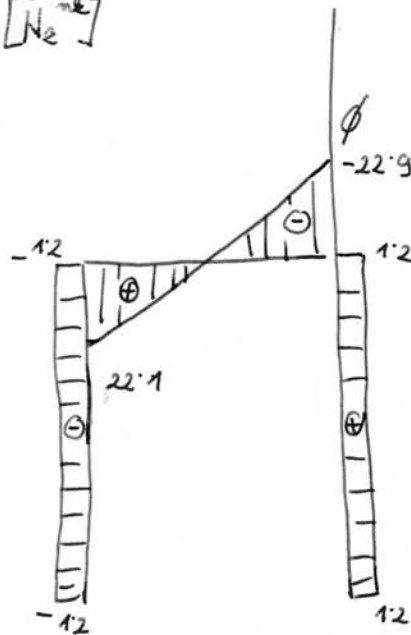
$X_2 = -6 \cdot 1$
 $X_1 = -4 \cdot 8$

e.) Superpozicija

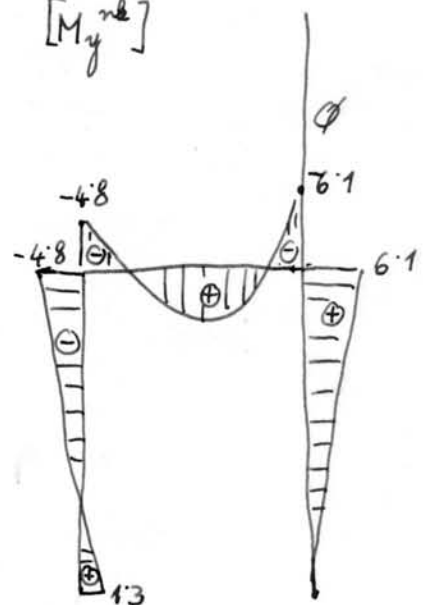
$[N_x^{nd}]$



$[N_z^{nd}]$



$[M_y^{nd}]$



TRDNOST - VSŠ 18.1.2006

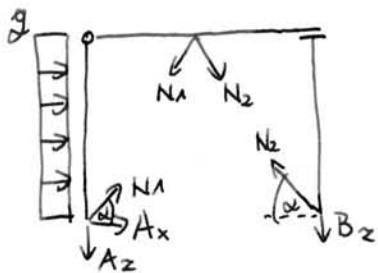
3. NALOGA

a.) Notranji momenti

$$A_x + q \cdot a = 0 \quad \boxed{A_x = -15 \text{ kN}}$$

$$A_z + B_z = 0 \quad \boxed{A_z = 75 \text{ kN}}$$

$$-q \cdot \frac{a^2}{2} - B_z \cdot a = 0 \quad \boxed{B_z = -75 \text{ kN}}$$

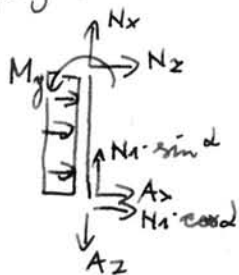


$$\sum x_{BD}: N_2 \cdot \cos \alpha = 0 \Rightarrow \boxed{N_2 = 0}$$

$$\sum M_{AC}^C: N_1 \cdot \cos \alpha \cdot a + q \cdot \frac{a^2}{2} + A_x \cdot a = 0$$

$$\boxed{N_1 \cdot \cos \alpha = 75 \text{ kN}}$$

polje I:



$$M_y + q \cdot \frac{x^2}{2} + A_x \cdot x + N_1 \cdot \cos \alpha \cdot x = 0$$

$$M_y = 75x - 25x^2$$

$$\boxed{M_y(1.5) = 5625 \text{ kNm}}$$

polje II:

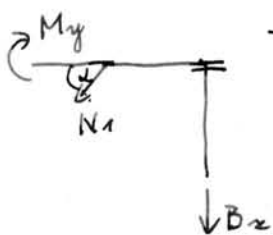


$$-M_y - B_z \cdot x = 0$$

$$M_y = 75x$$

$$\boxed{M_y(1.5) = 1125}$$

polje III

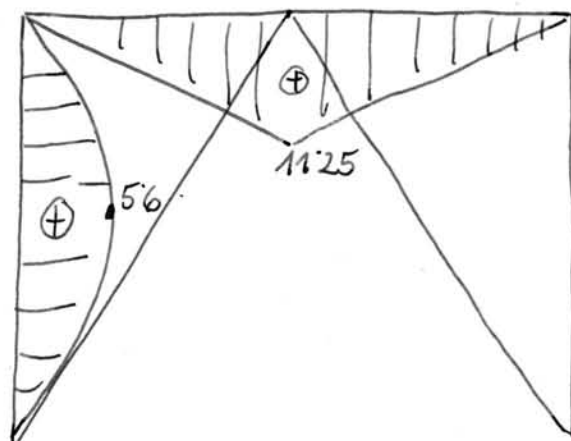


$$-M_y - B_z(a/2 + x) - N_1 \sin \alpha \cdot x = 0$$

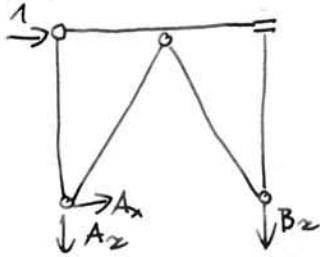
$$M_y = 75 \cdot (1.5 + x) - 75 \cdot x \cdot \tan \alpha$$

$$\boxed{M_y = 1125 - 75x}$$

$[M_y]$



b.) $F=1$



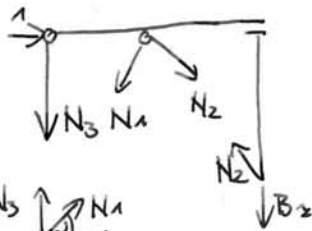
$$A_x = -1$$

$$A_z + B_z = 0$$

$$-a - B_z \cdot a = 0$$

$$A_z = 1$$

$$B_z = -1$$



$$N_2 = 0$$

$$N_1 \cdot \cos \alpha + A_x = 0$$

$$N_1 \cdot \cos \alpha = 1$$

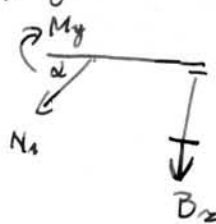
polje III:



$$M_y = -B_z \cdot x$$

$$M_y = x$$

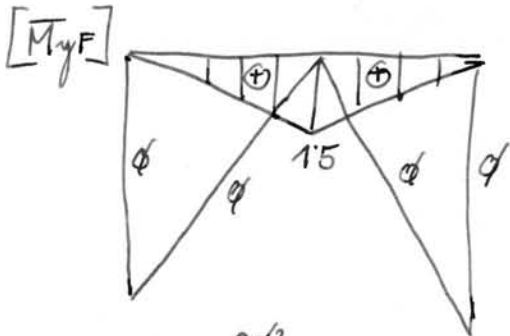
polje II



$$-M_y - N_1 \sin \alpha \cdot x - B_z \cdot \left(x + \frac{a}{2}\right)$$

$$M_y = -x^2 + x + 15$$

$$M_y = 15 - x$$

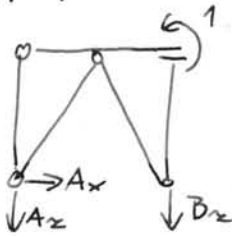


$$w_c = \frac{1}{EI_y} \cdot 2 \int_0^{a/2} \left(\frac{1}{2} \cdot 11.25 \cdot x \right) dx = \frac{1}{EI_y} \cdot 2 \cdot \frac{1}{2} \cdot \frac{3}{2} \cdot 11.25 \cdot 15$$

$$= \frac{1}{2,1000 \cdot 7908} \cdot 11.25 \cdot 15 = 0.00102 \text{ m}$$

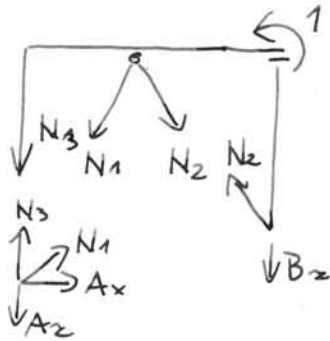
$$= 0.1 \text{ cm}$$

d. $M=1$

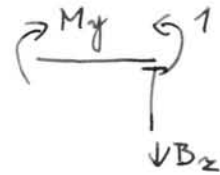


$$\begin{aligned} A_x &= 0 \\ A_z + B_z &= 0 \\ -B_z \cdot a + 1 &= 0 \end{aligned}$$

$$\begin{aligned} A_z &= -\frac{1}{3} \text{ kN} \\ B_z &= \frac{1}{3} \text{ kN} \end{aligned}$$

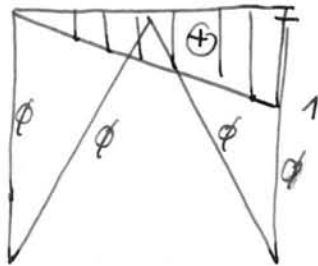


$$\begin{aligned} N_2 &= 0 \\ N_1 &= 0 \end{aligned}$$



$$\begin{aligned} M_y &= 1 - B_z \cdot x \\ M_y &= 1 - \frac{x}{3} \end{aligned}$$

$[M_y]$



$$\varphi_D = \frac{1}{EI_y} \left(\int_0^{a/2} \text{triangle} \, dx + \int_0^{a/2} \text{rectangle} \, dx \right)$$

$$= \frac{1}{2.1 \cdot 4308} \cdot \left(\frac{1}{3} \cdot \frac{3}{2} \cdot 1125 \cdot 0.5 + \frac{1}{6} \cdot \frac{3}{2} \cdot (1+1) \cdot 1125 \right) =$$

$$= 0.000508 \text{ radiana}$$

$$= 0.029 \text{ stopinje}$$

TRDNOST (VŠŠ) - 2. KOLOKVIJ (19. 01. 2007)

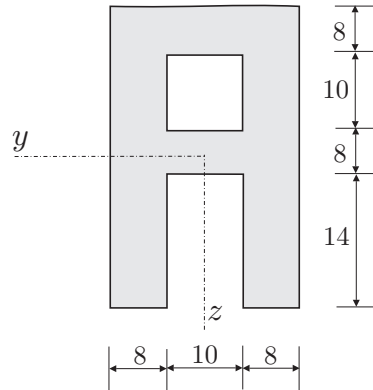
Pazljivo preberite besedilo vsake naloge!

Naloge imajo različno težo (25%, 25% in 50%), ob tem je 3. naloga še obvezna!

Pišite čitljivo! Uspešno reševanje!

1. Prerez na sliki je obremenjen s prečno silo $N_z = 12 \text{ kN}$. Določite nekaj značilnih vrednosti in skicirajte diagram strižne napetosti σ_{xz} v tem prerezu! (25%)

Podatki za prerez so v centimetrih.



2. Za paličje na sliki smo že določili pomike vozlišč 1, 2 in 3! Določite reakcije v podporah A, B in C! (25%)

Podatki: $a = 1.5 \text{ m}$, $b = 2 \text{ m}$,

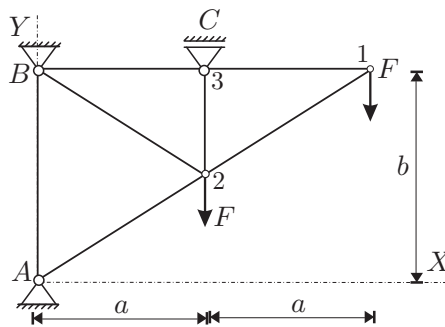
$E = 2 \cdot 10^5 \text{ MPa}$, $A_p = 0.01 \text{ m}^2$,

$F = -10 \text{ MN}$,

$\vec{u}_1 = (0.02250, -0.08515) \text{ m}$,

$\vec{u}_2 = (-0.00977, -0.00746) \text{ m}$,

$\vec{u}_3 = (0.01125, 0.00000) \text{ m}$.



3. Za konstrukcijo na sliki izračunajte notranje statične količine po metodi sil!

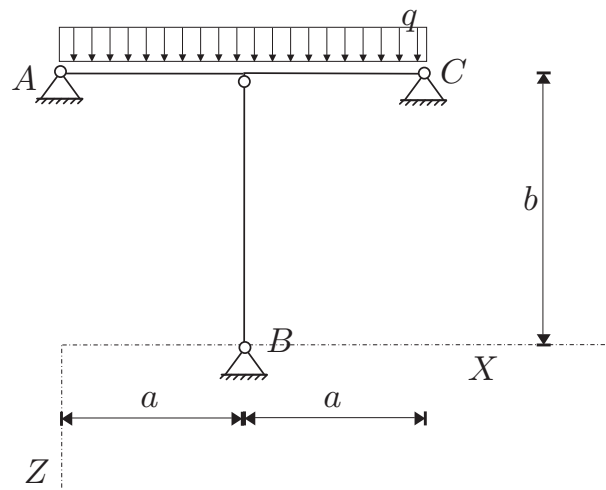
Upoštevajte tudi vpliv osnih sil!

(OBVEZNA NALOGA! 50%)

Podatki: $a = 2 \text{ m}$, $b = 3 \text{ m}$, $q = 5 \text{ kN/m}$

$E = 21000 \text{ kN/cm}^2$, $A_x = 80 \text{ cm}^2$,

$J_y = 6200 \text{ cm}^4$.



1. NALOGA

$$G_{xz} = -\frac{N_z}{I_y} \cdot \frac{1}{b^2} S_y^*$$

$$\begin{aligned} \text{a.) } A^{\text{I}} &= 26 \cdot 40 & A^{\text{II}} &= 10 \cdot 10 & A^{\text{III}} &= 10 \cdot 14 \\ z^{\text{I}} &= 20 & z^{\text{II}} &= 13 & z^{\text{III}} &= 33 \\ I^{\text{I}} &= 26 \cdot \frac{40^3}{12} & I^{\text{II}} &= \frac{10 \cdot 10^3}{12} & I^{\text{III}} &= \frac{10 \cdot 14^3}{12} \end{aligned}$$

$$A = 800 \text{ cm}^2, z_T = \frac{z^{\text{I}} A^{\text{I}} - z^{\text{II}} A^{\text{II}} - z^{\text{III}} A^{\text{III}}}{A} = 18.6 \text{ cm}$$

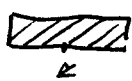
$$I_y = I^{\text{I}} - I^{\text{II}} - I^{\text{III}} + z^{\text{I}2} A^{\text{I}} - z^{\text{II}2} A^{\text{II}} - z^{\text{III}2} A^{\text{III}} = 382187 \text{ cm}^4$$

$$I_y^T = I_y - z_T^2 A = 105419 \text{ cm}^4$$

$$\text{b.) } -\frac{N_z}{I_y} = -0.1138 \frac{\text{N}}{\text{cm}^4}$$

$$\text{c.) } S_y^*(z)$$

$$\text{c1.) } z \in [-18.6, -10.6]$$



$$S_y^* = \int_{-13}^{13} \int_{-18.6}^z z \, dz \, dy = 13 \cdot z^2 \Big|_{-18.6}^z = \underline{13z^2 - 4497}$$

$$S_y^*(-18.6) = 0 \quad S_y^*(-10.6) = -3037 \text{ cm}^3$$

$$\text{c2.) } z \in [-10.6, -0.6]$$

$$S_y^* = -3037 + 2 \cdot \int_{-10.6}^z z \, dz = -3037 + \frac{16}{2} \cdot z^2 \Big|_{-10.6}^z = -3037 + 8(z^2 - 0.6^2)$$

$$= \underline{-8z^2 - 3936}$$

$$S_y^*(-10.6) = -3037 \text{ cm}^3 \quad S_y^*(-0.6) = -3932 \text{ cm}^3$$

$$\text{c3.) } z \in [-0.6, 7.4]$$

$$S_y^* = -3932 + \int_{-13}^{13} \int_{-0.6}^z z \, dz \, dy = -3932 + 13 \cdot z^2 \Big|_{-0.6}^z = -3932 + 13(z^2 - 0.6^2)$$

$$= \underline{13z^2 - 3937}$$

$$S_y^*(-0.6) = -3932 \text{ cm}^3 \quad S_y^*(7.4) = -3225 \text{ cm}^3$$

c3.) $z \in [7.4, 21.6]$

$$S_y^* = -3226 + 2 \int_{5.74}^{13.2} z dz = -3226 + 8 z^2 \Big|_{7.4}^{13.2}$$

$$= 8 z^2 - 3664$$

$$S_y^*(7.4) = -3226 \text{ cm}^3 \quad S_y^*(21.6) = 0$$

d.) tabeliranje

z^*	b^*	S_y^*	σ_{xz}^* [N/cm ²]
-18.6	26	0	0
-10.6	26	-3037	13.3
-10.6	16	-3037	21.6
-0.6	16	-3932	28.0
-0.6	26	-3932	17.3
7.4	26	-3226	14.1
7.4	16	-3226	22.9
21.6	16	0	0

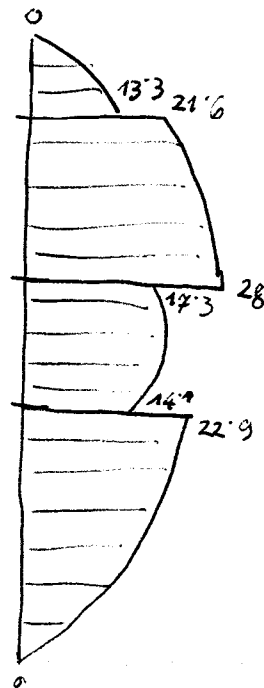
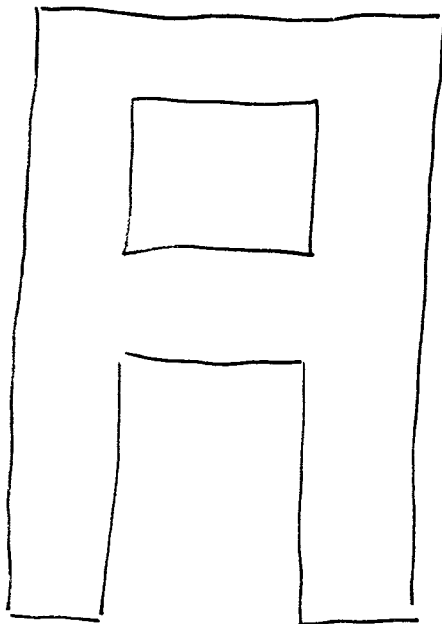


TABELA DOLŽIN, KOSINUSOV IN OSNIH TOGOSTI ZA PODANO PALIČJE

palica	vozel1	vozel2	dolzina	cos(a_ij)	cos(b_ij)	k_ij
1	1	2	1.803	-0.832	-0.555	1109.400
2	1	3	1.500	-1.000	0.000	1333.333
3	2	3	1.000	0.000	1.000	2000.000
4	2	4	1.803	-0.832	-0.555	1109.400
5	2	5	1.803	-0.832	0.555	1109.400
6	3	5	1.500	-1.000	0.000	1333.333
7	4	5	2.000	0.000	1.000	1000.000

TOGOSTNA MATRIKA PALIČJA

-2101.380	-512.031	768.046	512.031	1333.333	0.000	0.000	0.000	0.000
0.000	0.000							
-512.031	-341.354	512.031	341.354	0.000	0.000	0.000	0.000	0.000
0.000	0.000							
768.046	512.031	-2304.139	-512.031	0.000	0.000	768.046	512.031	0.000
768.046	-512.031							
512.031	341.354	-512.031	-3024.062	0.000	2000.000	512.031	341.354	0.000
-512.031	341.354							
1333.333	0.000	0.000	0.000	-2666.667	0.000	0.000	0.000	0.000
1333.333	0.000							
0.000	0.000	0.000	2000.000	0.000	-2000.000	0.000	0.000	0.000
0.000	0.000							
0.000	0.000	768.046	512.031	0.000	0.000	-768.046	-512.031	0.000
0.000	0.000							
0.000	0.000	512.031	341.354	0.000	0.000	-512.031	-1341.354	0.000
0.000	1000.000							
0.000	0.000	768.046	-512.031	1333.333	0.000	0.000	0.000	0.000
-2101.380	512.031							
0.000	0.000	-512.031	341.354	0.000	0.000	0.000	0.000	1000.000
512.031	-1341.354							

POMIKI IN REAKCIJE VOZLIŠČ DANEGA PALIČJA

vozel	u_x	u_y	R_x	R_y
1	0.02250	-0.08515		
2	-0.00977	-0.00746		
3	0.01125	0.00000		14.910
4	0.00000	0.00000	11.317	7.545

```
-----  
5          0.00000          0.00000          -11.317          -2.455  
-----
```

TABELA OSNIH SIL ZA PODANO PALIČJE

```
=====
```

palica	vozel1	vozel2	N_ij
1	1	2	-18.028
2	1	3	15.000
3	2	3	14.910
4	2	4	-13.602
5	2	5	-4.426
6	3	5	15.000
7	4	5	0.000

```
=====
```

RESITEV PREK RAVNOTEŽNIH ENAČB

a.) OSNE SILE

$$N_{AB} = 0$$

$$N_{23} = k_{23} [\cos \alpha_{23} \cos \beta_{23}] \begin{bmatrix} u_3 - u_2 \\ w_3 - w_2 \end{bmatrix}$$

$$= \frac{2 \cdot 10^5 \cdot 10^{-2}}{1} \frac{\text{MN}}{\text{m}} \cdot \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} u_3 - u_2 \\ +0.00746 \end{bmatrix} = 2 \cdot 10^3 \cdot 7.46 \cdot 10^{-3}$$

$$N_{23} = 14.92 \text{ MN}$$

$$N_{3B} = k_{3B} [\cos \alpha_{3B} \cos \beta_{3B}] \begin{bmatrix} -u_3 + 0 \\ -w_3 + 0 \end{bmatrix}$$

$$= \frac{2 \cdot 10^5 \cdot 10^{-2}}{1.5} \frac{\text{MN}}{\text{m}} \begin{bmatrix} -1 & 0 \end{bmatrix} \begin{bmatrix} -0.01125 \\ 0 \end{bmatrix} = 15 \text{ MN}$$

$$N_{2B} = k_{2B} [\cos \alpha_{2B} \cos \beta_{2B}] \begin{bmatrix} 0 + 0.00977 \\ 0 + 0.00746 \end{bmatrix}$$

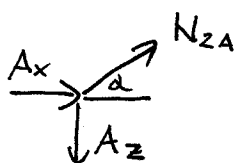
$$= \frac{2 \cdot 10^5 \cdot 10^{-2}}{\sqrt{1.5^2 + 1^2}} \cdot \begin{bmatrix} -\frac{1.5}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 9.77 \\ 7.46 \end{bmatrix} \cdot 10^{-3}$$

$$= \frac{2}{1.5^2 + 1} (-1.5 \cdot 9.77 + 7.46) = -4.43 \text{ MN}$$

$$N_{2A} = k_{2A} [\cos \alpha_{2A} \cos \beta_{2A}] \begin{bmatrix} 0.00977 \\ 0.00746 \end{bmatrix}$$

$$= \frac{2}{1.5^2 + 1} \begin{bmatrix} -1.5 & -1 \end{bmatrix} \begin{bmatrix} 9.77 \\ 7.46 \end{bmatrix} = -13.6 \text{ MN}$$

b.) REAKCIJE



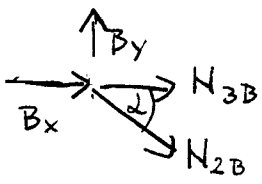
$$A_x = -N_{2A} \cdot \cos \alpha$$

$$A_z = N_{2A} \sin \alpha$$

$$\text{tg } \alpha = \frac{1}{1.5}$$

$$A_x = 11.3 \text{ MN}$$

$$A_z = -7.54 \text{ MN}$$

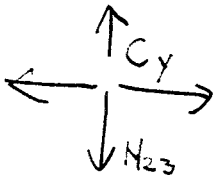


$$B_x = N_{3B} + N_{2B} \cos \alpha$$

$$B_y = N_{2B} \sin \alpha$$

$$B_x = -11.3 \text{ MN}$$

$$B_y = -2.5 \text{ MN}$$



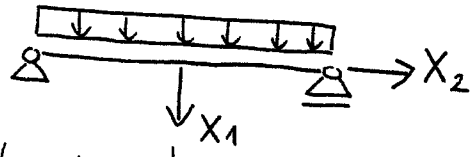
$$C_y = + N_{23}$$

$$C_y = 14.9 \text{ MN}$$

3. NALOGA

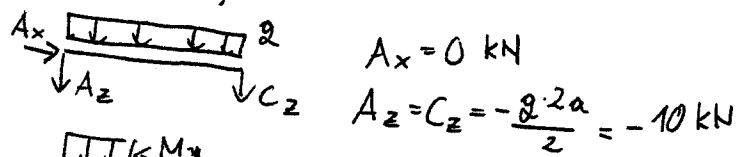
a.) $m = 3 \cdot 2 + 2 - 3 \cdot 2 = 2$

b.) sprostitev:

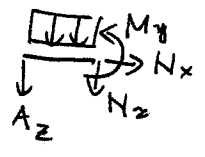


c.) notranje sile nadomestne konstrukcije

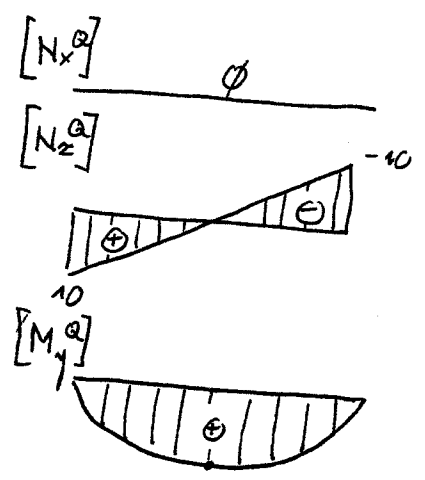
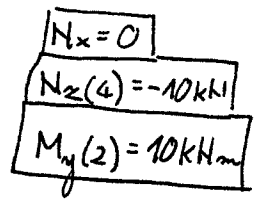
c.1.) zunanja obtežba



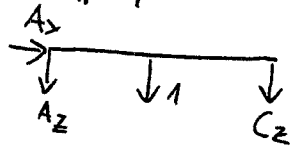
$A_x = 0 \text{ kN}$
 $A_z = C_z = -\frac{q \cdot 2a}{2} = -10 \text{ kN}$



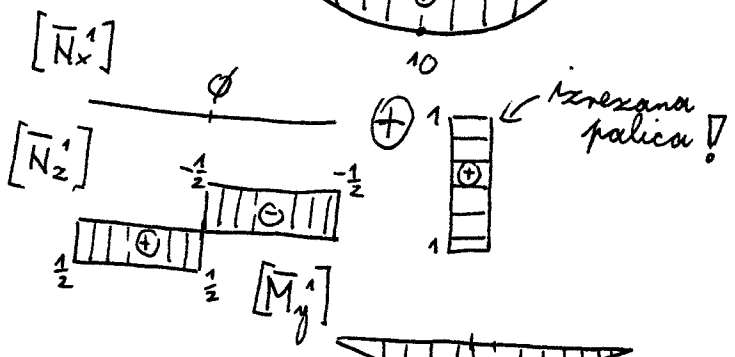
$N_x = 0$
 $N_z = 10 - qx$
 $M_y = 10x - x^2 \frac{q}{2}$



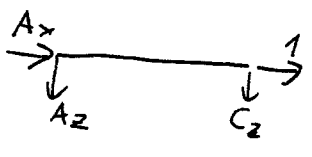
c.2.) $X_1 = 1$



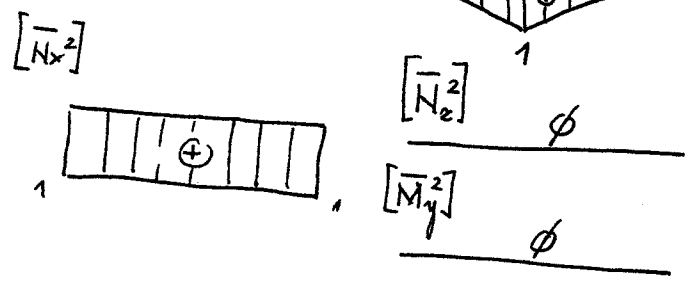
$A_x = 0$
 $A_z + C_z = -1$
 $C_z = -\frac{1}{2}, A_z = -\frac{1}{2}$



c.3.) $X_2 = 1$



$A_x = -1$
 $A_z = C_z = 0$



d.) določitev sil X_1 in X_2

$a_{11} = \frac{2}{EI_y} \int_0^a \Delta^1 \Delta^1 dx + \frac{1}{EA} \int_0^a \square^1 \square^1 dx = \frac{2}{EI_y} \cdot \frac{1}{3} \cdot 2 \cdot 1 \cdot 1 + \frac{1}{EA} \cdot 3 \cdot 1 \cdot 1 = \frac{4}{3EI_y} + \frac{3}{EA}$

$a_{22} = \frac{1}{EA_x} \int_0^a \square^1 \square^1 dx = \frac{2}{EA_x}$

$a_{12} = 0$

$b_1 = \frac{2}{EI_y} \int_0^a \Delta^{10} \Delta^1 dx = \frac{2}{EI_y} \cdot \frac{5}{42} \cdot 2 \cdot 10 \cdot 1 = \frac{50}{3EI_y}$

$b_2 = 0$

$$\begin{bmatrix} \frac{4}{3EI_y} + \frac{3}{EA} & 0 \\ 0 & \frac{2}{EA} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = - \begin{bmatrix} \frac{50}{3EI_y} \\ 0 \end{bmatrix}$$

$$\boxed{X_2 = 0}$$

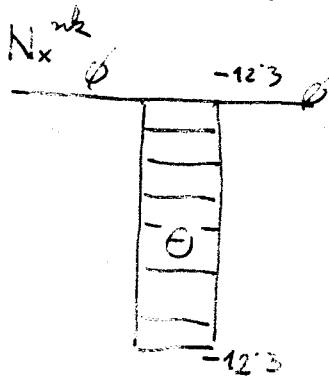
$$\left(\frac{4}{3EI_y} + \frac{3}{EA}\right) X_1 = -\frac{50}{3EI_y}$$

$$X_1 = -\frac{50}{3EI_y \left(\frac{4}{3}I_y + \frac{3}{A}\right)} = -\frac{50}{4 + \frac{9I_y}{A}}$$

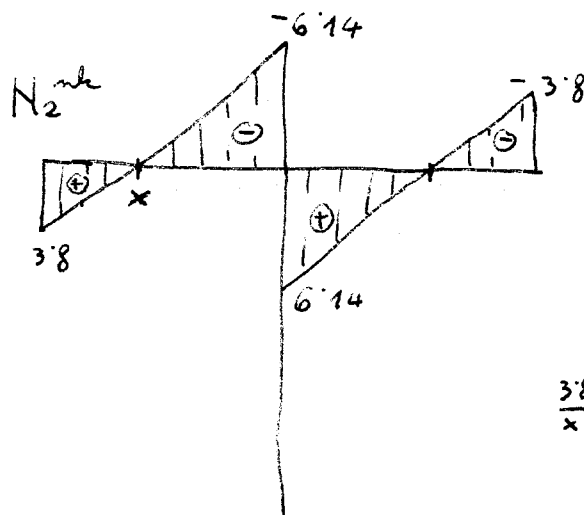
$$X_1 = -\frac{50}{4 + \frac{9 \cdot 6200 \cdot 10^{-4}}{80}} = -12.29$$

$$\boxed{X_1 = -12.29}$$

e.) Superpozicija



M_y^{nh}



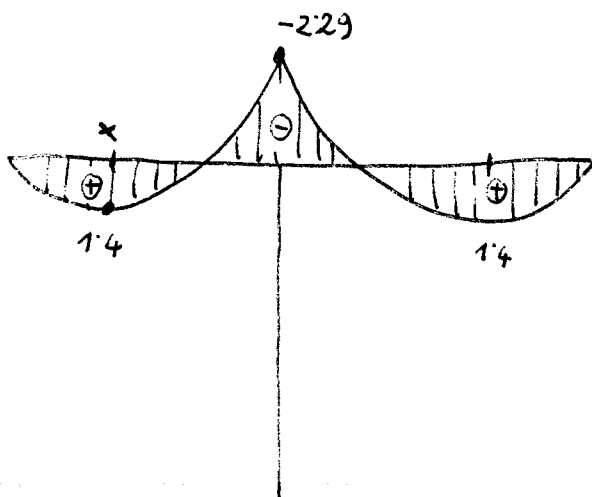
5

$$\frac{3.8}{x} = \frac{6.14}{2-x}$$

$$3.8(2-x) = 6.14x$$

$$x(6.14 + 3.8) = 2 \cdot 3.8$$

$$x = 0.77$$



$$M_y^{nh} = M_y^2 - X_1 \bar{M}_y^1 - 0 \cdot \bar{M}_y^2$$

$$M_y^{nh}(0.77) = 10 \cdot 0.77^2 - \frac{5}{2} \cdot 0.77^2 - 12.5 \cdot \frac{0.77}{2}$$

$$= 1.4 \text{ kNm}$$

TRDNOST (VŠŠ) - 2. KOLOKVIJ (18. 01. 2007)

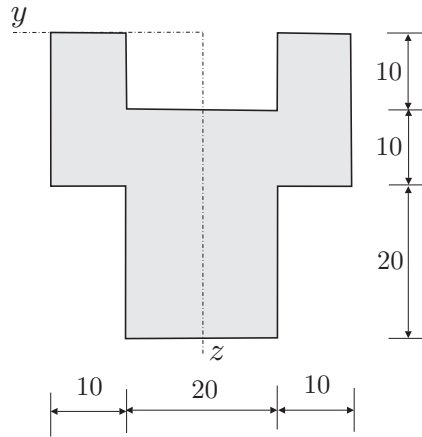
Pazljivo preberite besedilo vsake naloge!

Naloge imajo različno težo (25%, 25% in 50%), ob tem je 3. naloga še obvezna!

Pišite čitljivo! Uspešno reševanje!

1. Prerez na sliki je obremenjen s prečno silo $N_z = 20 \text{ kN}$ in upogibnim momentom $M_y = 10 \text{ kNm}$. Določite nekaj značilnih vrednosti in skicirajte diagrama osne napetosti σ_{xx} in strižne napetosti σ_{xy} v tem prerezu! (25%)

Podatki za prerez so v centimetrih.

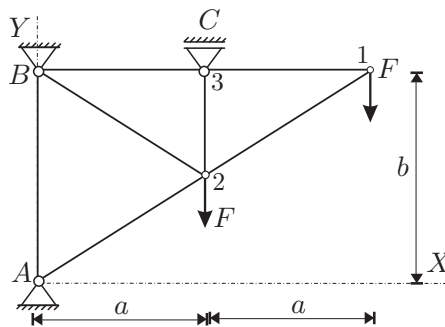


2. Za paličje na sliki določite pomike vozlišč in notranje sile v palicah po metodi pomikov! Namig: veliko prostostnih stopenj je podprtih. (25%)

Podatki: $a = 2 \text{ m}$, $h = 4 \text{ m}$,

$F = 30 \text{ MN}$, $G = 10 \text{ MN}$,

$E = 2 \cdot 10^5 \text{ MPa}$, $A = 0.01 \text{ m}^2$.



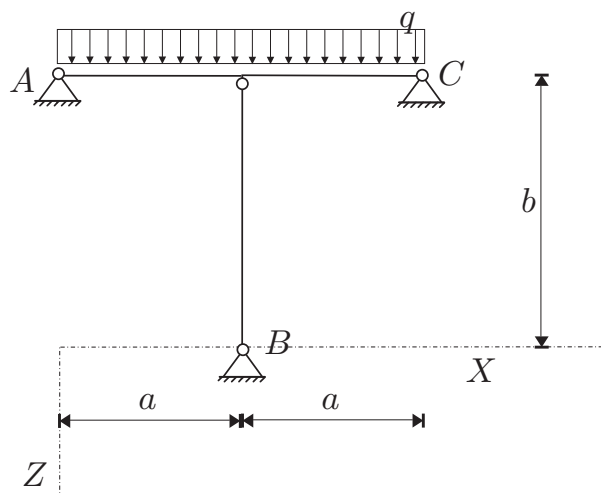
3. Za konstrukcijo na sliki izračunajte notranje statične količine po metodi sil! Upoštevajte tudi vpliv osnih sil!

(OBVEZNA NALOGA! 50%)

Podatki: $a = 2 \text{ m}$, $b = 3 \text{ m}$, $q = 5 \text{ kN/m}$

$E = 21000 \text{ kN/cm}^2$, $A_x = 80 \text{ cm}^2$,

$J_y = 6200 \text{ cm}^4$.



TRDNOST (VŠŠ) - 2. KOLOKVIJ (23. 01. 2009)

Pazljivo preberite besedilo vsake naloge!

Naloge imajo različno težo (25%, 50% in 25%), ob tem je 2. naloga še obvezna!

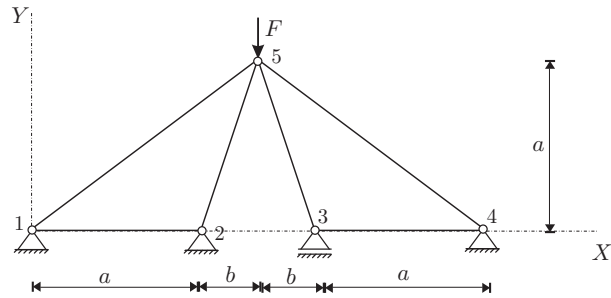
Pišite čitljivo! Uspešno reševanje!

1. Za paličje na sliki določite pomike vozlišč in notranje sile v palicah po metodi pomikov! Namig: veliko prostostnih stopenj je podprtih. (25%)

Podatki: $a = 3\text{ m}$, $b = 1\text{ m}$,

$F = 30\text{ MN}$,

$E = 2 \cdot 10^5\text{ MPa}$, $A = 0.01\text{ m}^2$.



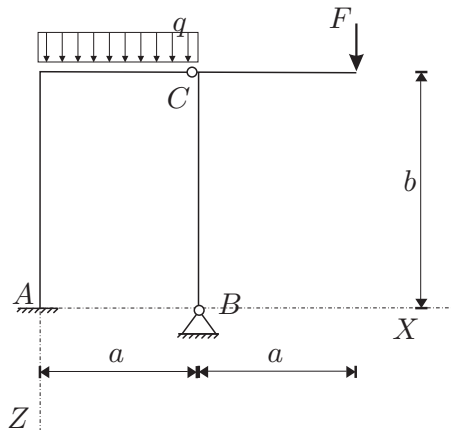
2. Za konstrukcijo na sliki izračunajte notranje statične količine po metodi sil! Vpliva osnih in prečnih sil ni potrebno upoštevati!

(OBVEZNA NALOGA! 50%)

Podatki: $a = 2\text{ m}$, $b = 3\text{ m}$, $q = 4\text{ kN/m}$,

$F = 5\text{ kN}$, $E = 21000\text{ kN/cm}^2$,

$A_x = 80\text{ cm}^2$, $J_y = 6200\text{ cm}^4$.



3. Za konstrukcijo na sliki izračunajte vertikalni pomik v točki B in zasak v točki D!

(25%)

Podatki: $a = 2\text{ m}$, $q = 2\text{ kN/m}$,

$E = 1000\text{ kN/cm}^2$, $A_x = 240\text{ cm}^2$,

$J_y = 8000\text{ cm}^4$.

