Introduction of Probabilistic Tools in the Evaluation of Wooden Historical Buildings, in terms of Safety: a case study

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ABSTRACT

This paper deals with the evaluation of safety in timber frame historical buildings, using field measurements coupled with numerical methods. The first step of the study aims at identifying the weak points of the system, i.e. those which have a preponderant influence on the system response: an *in situ* diagnostic coupled with an experimental design and a FE calculation provides a qualitative information on the system behavior, which remains deterministic. At this stage, a limited number of weak points have been identified, that should be reinforced. In order to obtain the actual state of the system, a quantitative information on the relationship between the input variables (properties of weak points) and the response, is required. A probabilistic analysis is proposed, in which the input variables are described through probability distribution functions, and the response is therefore also probabilistic. This approach should enable: i) to evaluate the distance to the unsafe state, ii) to quantify the effect of reinforcement. An illustration of the method is proposed throughout the paper. In this example, timber components have no influence on the system behaviour, which is mainly determined by the properties of connections.

1. INTRODUCTION

The maintenance of timber frame historical buildings requires a diagnostic usually made by an architect, based on a global visual inspection, sometimes coupled with local non destructive techniques. Expertise in that field in southern Europe countries (Italy, Portugal, Spain) is recognized, and a number of tools have been developed, such as ultrasonic measurements, vibration or drilling methods. The ongoing COST Action E24 supported by EC deals with the introduction of probabilistic tools in the safety analysis of timber structures, including diagnostic of existing buildings. If, in the design of new buildings, probabilistic concepts will mainly help in code calibration, they should also serve as a decision support tool for the reinforcement of old structures, in complement to *in situ* inspection and other non destructive techniques.

This paper presents a numerical procedure for the diagnostic of a timber frame old building, first tackled in a paper by ANDRE et al (2001). The case study is a bell tower that was built in the middle of the 19^{th} century. The bell tower is surrounded by a masonry, and has been reinforced several time due to excessive horizontal displacements when the bells are in movement. A similar case has been experimentally studied by NIEDERWANGER (1997), who carried out a dynamic test on the tower and determined the eigenfrequencies of the system.

The numerical procedure is divided in two steps: a deterministic qualitative analysis aimed at identifying the weak points of the building, and a probabilistic analysis. This methodology is coupled with a FE calculation.

2. DESCRIPTION OF THE SYSTEM

This study deals with the mechanical behaviour of a wood bell-tower located in Tarn-et-Garonne (France), which is enclosed by an octagonal masonry tower (figure1). When bells are ringing, the whole wood structure undergoes large displacements and hits the masonry. Experts have diagnosed this wood bell-tower, and have replaced damaged wood beams and reinforced some joints with bolted plates (figure2). Nevertheless, the structure was still unreliable.

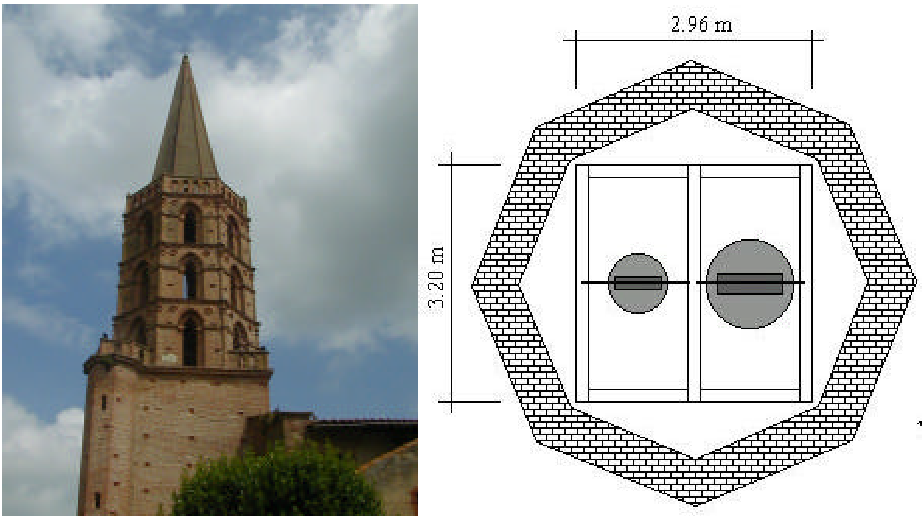

Figure 1 . Bell-Tower of the Church of Finhan (Tarn-et-Garonne)

Figure 2 . reinforcement of a joint with a transverse bolt and with bolted steel plates.

This problem raises a lot of questions. Is the structure badly designed or build, is there a lack of mechanical behaviour assessment by the experts ? We developed an expert method consisting first in modelling the bell-tower by the F.E.M. For simplification, joints are modelled by rotational springs with variable stiffness to represent non perfect cantilever and by axial springs with variable stiffness to represent displacements into the joint. Designs of experiment allowed to determine the influent parameters or elements of the structure. The next step was to get information about these elements by non destructive evaluation in order to update the FE model. An optimized solution, proposed by this method, has been applied experimentally on the structure by reinforcing the most influent joints.

3. NUMERICAL ANALYSIS AND DIAGNOSTIC

The first step consists in modeling the bell-tower by F.E. Because the structure is not symmetrical, we made a 3D model composed of 1D beams (figure 3). A simple truss system was not chosen here, because of possible bending of timber components. The most critical points in the truss system are the joints between timber components, which are traditional tenon-mortise joints . In order to reproduce joints gap, we introduced axial and rotatory springs (figure 4).

An experimental and numerical study has been performed by BULLEIT *et al* (1999) for four different design type of traditional joints in order to specify the behaviour and the modeling of wood pegged timber frames. These joints are also modeled by springs combination (two axial springs and a rotatory spring).

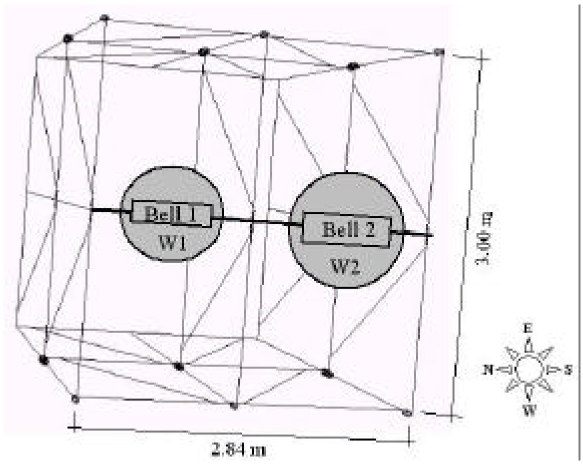

Figure 3 . Top view of the bell tower – 3D FE mesh with an added sketch of the bells

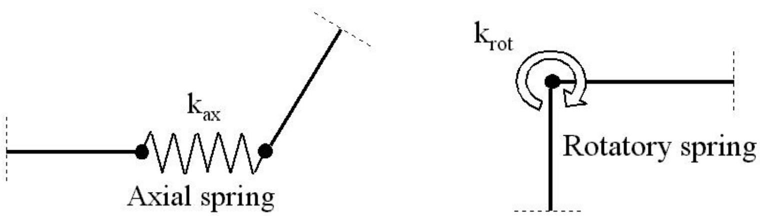

Figure 4 . Axial and rotatory springs between timber elements end nodes

3.1. Modeling of loads

As the biggest bell of the bell-tower could not ring (because of too important horizontal displacements), we studied the load case with this bell in movement. The bell 2, initially brought up to 90 degrees, is let down (initial speed = 0) at this position and reach – 90 degrees (figure 5).

First, we solved a simple pendulum mechanical problem in order to approximate the maximum bell 2 loads, the horizontal and vertical reactions are calculated for the whole bell movement. We obtained the reactions variation as a function of bell 2 (1100 kg and 1.18 m of basis diameter) location during the movement. This shows that horizontal reaction is maximum for a 45° or –45° angle. The corresponding vertical reaction is lower. Reactions values are given in table 1.

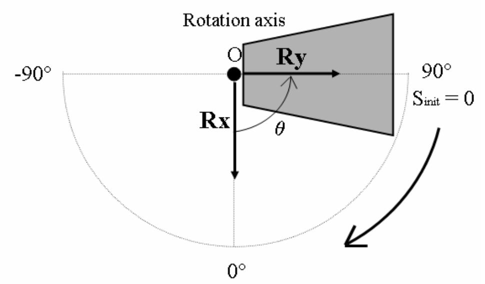

Figure 5 . Schematic diagram of the bell movement

Table 1. Loading parameters due to bell movement

	R_y (H_2)	R_x (V_2)
Bell 2	2.06 W_2	-1.69 W_2

3.2. Numerical design

Initially, a design of experiments is used to reduce the number of experiment when a large number of factors must be tested. The post-treatment enables to optimize the various responses and choose an optimum test. Here, we used numerical design in order to specify the structure mechanical behavior in accordance with input parameters.
Axial spring stiffness (R_{ax}), rotatory spring stiffness (R_{rot}) and beam stiffness (R_{beam}) are input parameters of the model. Horizontal displacements (which are too large on site) are output parameters.

3.3. Influence of input variables on the system response

We wanted to specify the global influence for each of the three input parameters. We made a full factorial design with three levels. Input parameters are coded, input variables take three values : -1 is the low value, 0 is the medium value and +1 is the high value. Each experiment of the test matrix is reproduced in the FE model, injecting the corresponding parameter value. For example, -1 is the MOE beam low value in the DOE, 5000 MPa is the MOE beam corresponding low value (0 equivalent to 10000 MPa and 1 equivalent to 15000 MPa). In the following, we only work with input parameters stiffness.

Analysis of this full factorial DOE gives the following relationship between input parameters and the horizontal displacement :

$$h = -0.027*(1 + 0.34*R_{rot} + 1.37*R_{ax} + 0.06*R_{beam} ...+ \text{interactions})$$

This shows that R_{beam} factor can be neglected (figure 8). So beams material properties do not influence the horizontal displacements of the bell-tower. R_{ax} factor is very influent : axial springs stiffness influence the horizontal displacement of the structure.

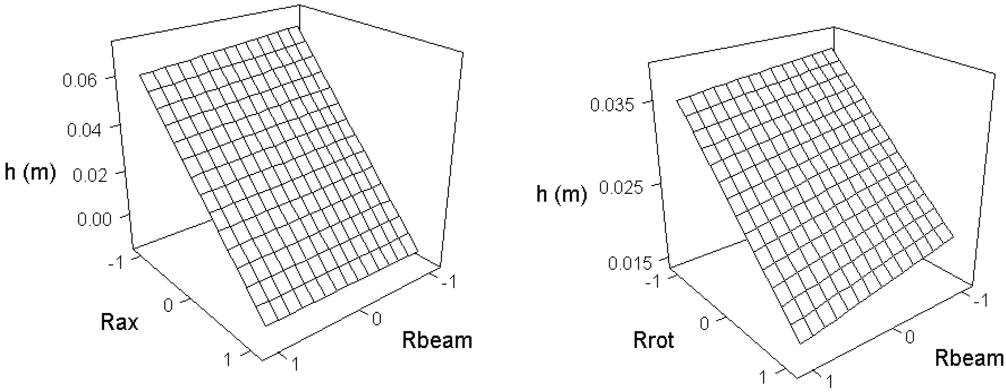

Figure 6 . Response surface graphs for R_{ax}, R_{rot}, and R_{beam}

3.4. Determination of the most influent axial springs

We decided to study the 35 axial springs by determining successively the most influent of them. As the axial springs are numerous, we made a Plackett-Burmann (two levels) numerical design.

The first DOE contains 35 axial springs. DOE post treatment indicates the most influent axial spring. We reinforce numerically this axial spring and we consider that it stiffness is endless. Now, the model only contains 34 axial springs. The method is applied until horizontal displacements are not significantly reduced. This step is summarised in table 2 and figure 7.

Table 2. Reinforcement of the most influent joints and its effect on the system response

	Horizontal displacements		
	h1 (m)	h2 (m)	h3 (m)
without reinforcement	0.0787	0.0776	0.0443
S75	0.0718	0.0511	0.0379
S88	0.0649	0.0248	0.0314
S73	0.0462	0.0241	0.0278
S86	0.0242	0.0232	0.0234
S80	0.0022	0.0032	0.0026
S81	0.0012	0.0018	0.0008

This priority order is based on the assumption that every axial spring can create a gap. We only have taken into account the previous reinforcements. We must update these informations by measuring in-situ gaps of the influent joints.

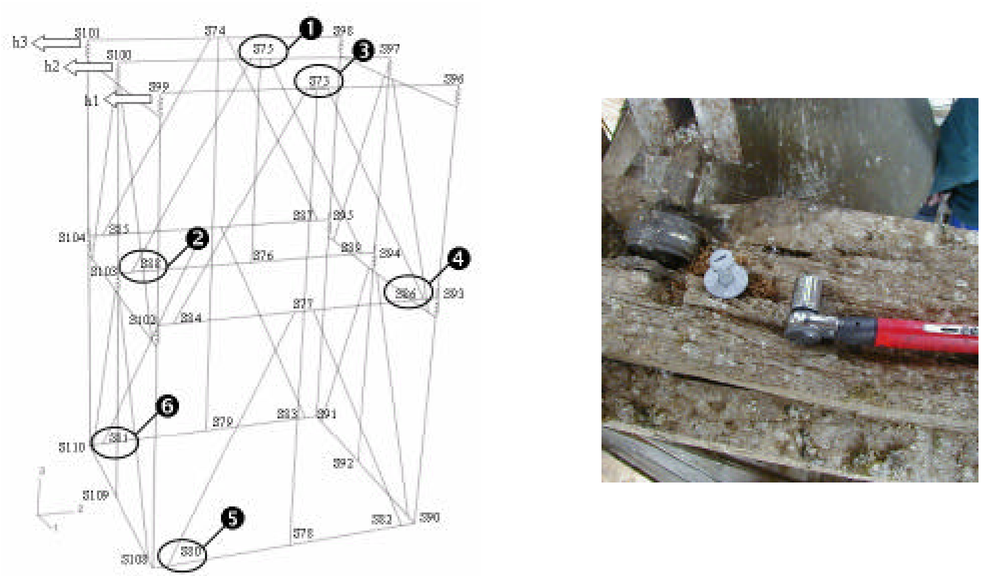

Figure 7 . Spring order influence on displacements as calculated by successive reinforcements (*in situ reinforcement* –photo on the right)

3.5. *In situ* diagnostic and reinforcement

It results from the experimental measurements that only S_{75}, S_{73} and S_{74} joints have a considerable gap (about 8 mm). We updated the FE bell-tower model in order to identify other influent joints. In this new procedure, three joints are influent : S_{75}, S_{73} and S_{74} respectively. Reinforcing these joints should be enough to minimize horizontal displacements.

We decided to reinforce the three joints by 16 mm diameter and 300 mm length screw spikes (see figure 7). After putting in place every screw spikes (figure10), we measure the horizontal displacements h_1, h_2 and h_3 : 1 mm, 2 mm and about 0 mm respectively. With a very limited repair operation, we succeeded in the restarting of bell 2.

4. SYSTEM RELIABILITY ASSESSMENT

Based on the previous information, we will now concentrate on the weak connections, and the corresponding variables, *i.e.* the displacements u_i and rotations w_i at these points. As described in the previous section, these displacements are represented by an equivalent spring compliance S_i and a rotation compliance ω_i,, leading to a static equilibrium of the whole system. The relationship between S_i and w_i, on one hand, u_i and w_i on the other hand, are defined by the equilibrium of the tower, which means that, even if all random variables S_i (resp. ω_t) are assumed to be independent, the resulting displacements u_i and w_i are correlated through the system behavior, *i.e.*

$u_i = F(\omega_j, S_j , j = 1, \dots k)$

Let P_f be the probability of failure of the system. As we know Pf may be written as (Goyet 1994)

$$P_f = \int_{h \geq h_{max}} p(u_1,..., u_k, w_1, ..., w_k) du_1...du_k dw_1...dw_k$$

The calculation of P_f requires the expression of the joint pdf of u_i and w_i, which is not known. Moreover, the limit state function is not explicit with respect to the input variables, which is often the case. We therefore used Monte Carlo simulations in order to obtain the probability of failure. However, the implementation into a FE code leads to heavy calculation costs, especially when the target probability is low. We found more efficient to describe the system response as a quadratic function of the input variables, as shown in the previous section. We used the FE analysis only for the validation of the method.

4.1. Probabilistic description of S_i

The connections between timber components can be completely rigid, *a fortiori* when they have been reinforced, semi-rigid – limited displacements or rotations are possible – or weak – large displacements are possible. The uncertainty on the actual state of a connection leads to the choice of an exponential pdf, see figure 8:

$$f(x) = \frac{1}{b} \exp\left(-\frac{1}{b} x \right)$$

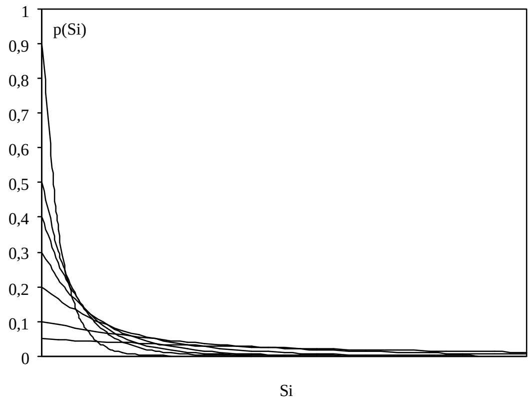

Figure 8 . Probability distribution functions for the equivalent compliance of weak connections

Changing the value of b, or the expected average displacement, will change the shape of the distribution.

Various shapes of probability density functions have been tested, but at the end two types were used: one for rigid connections, and one for weak connections.

4.2. Calculation of a performance function

Focusing on the horizontal displacement h at the top of the system, the performance function may be written as

$$g(h) = h_{max} – h > 0$$

The limit state equation is then

$$g(h) = h_{max} – h = 0$$

Assuming, as indicated before, that near the most probable limit state, u may be expanded into a quadratic function of S_i, *i.e.*

$$g(h) = h_{max} - \sum_i l_i S_i - \sum_i l_{ii} S_i^2 - \sum_i \sum_j l_{ij} S_i S_j$$

then it is possible to integrate easily the performance function and calculate the probability of failure. Note that three terms are introduced: a linear term, a quadratic term, and an interaction term. For simplicity, the uncertainty on the parameters of the function (model uncertainty) are not taken into account here. An equivalent equation can be written for the rotations w_i.

4.3. Illustration of the method

In order to illustrate the outputs of the method, a simple quadratic function with only two random variables has been used, representing two weak connections in a system. It is assumed that both connections have a positive linear effect on the response, a positive interaction effect, but a negative non linear effect (quadratic term). Moreover, one of the connections is less rigid than the other, and therefore has a greater influence on the response. The probability density function of the performance functionwas computed using Monte Carlo simulations. Some results are illustrated in figure 9. On the left hand side of the figure, the initial state of the system is described: the performance function is positive, with an average normalized value of 0.05; the probability of failure (colored bar) is significant. On the right hand side of the figure, a restriction on the weak connection is simulated by changing the parameters of the pdf: the pdf of the performance function exhibits a different shape, with a most probable value of 0.09. However, the probability of failure, in this example, is not significantly affected.

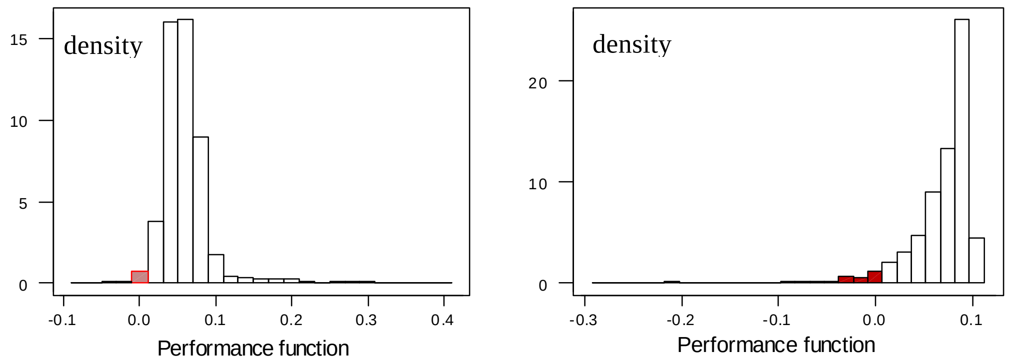

Figure 9 . Simulated histograms of the performance function before (left) and after (right) restriction of the displacements on the most critical connection – illustration on a case study with 2 connections

5. CONCLUSIONS

In the evaluation of timber structures, careful attention must be paid to the connections. In the example of the paper, timber components have no significant effect on the overall response. A deterministic approach based on an experimental design coupled with a FE method enables to detect the weak points of the system. The results have been verified *in situ* by visual inspection, and the weak connections were reinforced. The effective performance of the building can be quantified by probabilistic methods, which require quantitative information on the behavior of components. Probabilistic tools show that, even if reinforcement may increase the average performance of a system, the probability of failure is not necessarily affected.

This method will be validated on different types of timber systems.

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