The effect of machine strength grading on the strength distribution of sawn timber based on numerical simulation

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1. Machine strength grading in accordance with prEN 14081 (April 2002)

This paper is based on an upcoming standard prEN 14081-2, which defines the procedure for determination of grading parameters in machine strength grading of sawn timber. The procedure has two steps: first the real grade of each specimen is determined (optimum grading) and secondly the rules are given for determination of the grading parameters for machine strength grading when assigning timber to the grades.

Optimum grading

- The limit value for bending strength is set so, that the modified requirement for the characteristic value of the best class is met (Table 1). All the specimens which have a bending strength better or equal to the limit value are assigned to that class.
- The limit value for modulus of elasticity is set so, that the modified requirement for the mean value of the best class is met. All the specimens which have a modulus of elasticity better or equal to the limit value are assigned to that class.
- The limit value for density is set so, that the modified requirement for the characteristic value of the best class is met. All the specimens which have a density better or equal to the limit value were assigned to that class.

If a specimen is assigned to the best class by all three criteria, then the optimum class of that specimen is the best class. If any of the criteria was not fulfilled, then the specimen is not assigned to the class. After that the specimens which were assigned to the class are excluded and the procedure is repeated for the remaining specimens to be assigned to the next best class.

Table 1. Modified requirements for some grades in accordance with prEN 14081.

Grade	f_k N/mm^2	E_{mean} N/mm^2	ρ_k kg/m^3
C40	40,0	13300	420
C30	26,8	11400	380
C24	21,4	10450	350
C18	16,1	8550	320

Assigned grades

A limit value of grading parameter (f_{model}) is determined for each grade in such a way that the modified requirement on bending strength, modulus of elasticity and density are met. This procedure is repeated for each sample. The final limit value of f_{model} is taken as the mean value of at least four samples and each specimen of the total sample is re-graded in accordance with this limit value. The results of this procedure is illustrated in Table 2. The grade so determined for each timber piece is its assigned grade.

According to prEN 14081-2 the mean setting value shall not be less than 85 % of the most conservative sub-sample setting value, and according to the revised version (September 2002) the lowest setting value shall not be less than 50 % of the setting value related to the required bending strength in the lowest class. These rules have not been exactly followed in the simulated examples of this paper.

Global cost matrix

Goodness of grading is rated by forming a global cost matrix. The matrix is determined by multiplying the values in each cell of the size matrix (Table 2) by given factors depending on the magnitude of error in grading and dividing by the total number of specimens in the relevant assigned grade. None of the cells in the global cost matrix that indicate specimens wrongly upgraded is allowed to be greater than 0.20.

2. Simulation of mechanical properties

In order to demonstrate how the grading (determination of settings of the machine) is made, we have simulated values of bending strength (f), modulus of elasticity (E) and density (ρ), and used these data sets for virtual determination of settings, and then for virtual grading of material.

The following correlations between bending strength f, modulus of elasticity E and density ρ were used in the simulation of data sets for ungraded timber:

$$E = 4820 + 165f + e, \quad e \in N(0,\sigma) \qquad \sigma=1400 \ (r^2=0.646) \qquad (1)$$

$$\rho = 362 + 2.03f + e, \quad e \in N(0,\sigma) \qquad \sigma=35 \ (r^2=0.313) \qquad (2)$$

Bending strength f is assumed to be normally distributed with μ = 45,2 and σ = 11,4 MPa (COV = 0,25). These relations are based on experiments made with Finnish spruce timber (45x150) (Ranta-Maunus et al 2001).

Three quality levels of grading method are used corresponding to the low and high ends of the practice of today, and a more advanced one which is not yet available. The relationship between grading parameter (f_{model}) and real bending strength (f) was adopted as follows:

$$f_{\text{model}} = f + e \ , \quad e \in N(0,\sigma) \qquad (3)$$

where σ= 15, 8.5 or 5.5 corresponding to r^2=0.36, 0.64 or 0.81, consequently.

3. Demonstration of grading

The determination of settings proceeds in the following steps:

1. Based on material properties *f, E* and ρ, it is determined to which grade each piece belongs (optimum grading). In the first exercise, 2000 simulated specimens were graded to C40, C30 and C18. Results are shown in Tables 2 to 4.
2. Secondly, the limits for grading parameter (f_{model}) were set such that maximum number of specimens goes to C40, the rest is graded so that maximum number goes to C30 etc. The CEN rules are followed concerning the limits given for *f, E* and ρ as well as the number of specimens graded to high.

Number of specimens belonging to each grade in optimum grading and in machine grading is given in Table 2. The limits used for grading are shown in Table 3. We observe that 1 reject specimen goes both to C30 and to C18. This is just acceptable to the CEN procedure. The characteristic values of each grade are shown in Table 4. The 5% fractile of the ungraded population was 26 MPa, and that of each grade: 46,7 for C40, 30,4 for C30 and 18,4 for C18. As a tail shape measure the ratio of $f_{0.005}/f_{0.05}$ is used. For the ungraded population the ratio was 0.62, whereas for graded timber from 0.75 to 0.88.

Table 2. Size matrix: number of simulated 2000 specimens belonging to each grade according to the properties (optimum grade) and according to the machine settings (assigned grade) when r^2 = 0.64.

	Assigned machine grade				
Optimum grade	C40	C30	C18	reject	total
40	313	783	16	0	1112
30	29	453	102	1	585
18	1	156	134	6	297
reject	0	1	1	4	6
total	343	1393	253	11	2000
yield (%)	17,2	69,7	12,7	0,6	

Table 3. Actual limits used for material properties and machine settings (f_{model}) in grading for results in Table 2.

Grading		C40	C30	C18
Optimum	f	38,3	24,5	13,0
	E	10750	9600	5000
	ρ	425	380	300
Machine	f-model	59	29	8

Table 4. Characteristic values of graded samples and limits given in CEN standards.

		C40	C30	C18
Results	$f_{0.05}$	46,70	30,40	18,24
	$f_{0.005}$	40,90	22,75	14,58
	$f_{0.005}/f_{0.05}$	0,88	0,75	0,80
	E_{Mean}	14460	12200	9867
	$\rho_{0.05}$	420	389	362
Requirements	f-Modified	40,0	26,8	16,1
	E-EN338	14000	12000	9000
	E-Modified	13300	11400	8550
	$\rho_{0.05}$	420	380	320

3. Grading simulations

Normally, when machine settings are determined, a smaller sample size than 2000 is used. Therefore we next simulated values for 500 specimens in accordance with eqns (1) to (3). This was repeated until we were able to grade 6 sets of values into 3 grades C40, C30 and C18. Setting parameters (f_{model}) were determined separately for these 6 populations. The means of f_{model} were determined and considered as settings of the grading machine. When doing so, the criteria of the latest CEN-standard version concerning the highest and lowest setting values obtained for individual samples were not met in case of the lowest grade. Then each of 6 sets of grading parameters and their means are applied in grading of 2000 specimens, the same values as demonstrated earlier. In Table 5 the results concerning individual simulations 1 to 6 are given in order to demonstrate the variability of results and the results corresponding to the means of f_{model} are close to the values which would be obtained when settings are determined according to the present CEN-method. However, for C18 the simulated results may be less conservative than the exact following of CEN-procedure would result in. In case of the poorest grading quality, the grading into three grades may be impossible, when the CEN-method is followed, but the time did not allow us to check it for this paper.

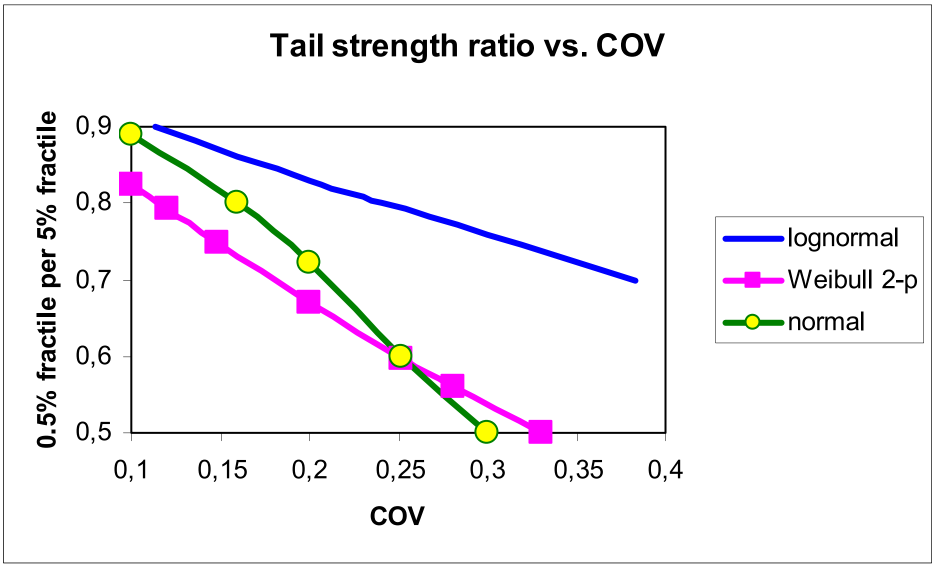

Figure 1. Relation of the tail strength ratio ($f_{0.005}/f_{0.05}$) and coefficient of variation (tail-COV) in populations with Normal, Lognormal and two-parameter Weibull distributions.

Table 5. Grading results of population of 2000 when the settings are based on 6 samples with N=500 with three different qualities of grading. When the yield is less than 10% (200 pieces) the 0.5% fractile has not been determined.

Grading quality	Simulation	C40			C30			C18		
		yield [%]	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	yield [%]	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	yield [%]	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
Optimum	**grading**	**56**	**1.01**	**0.96**	**28**	**0.98**	**0.87**	**15**	**1.13**	**0.73**
exellent $r^2 = 0.81$	1	36	1.12	0.88	22	1.24	0.81	42	1.23	0.65
	2	10	1.32	0.83	87	0.93	0.73	3	0.87	
	3	28	1.16	0.90	43	1.18	0.83	27	1.20	0.69
	4	18	1.23	0.90	67	1.07	0.84	14	1.10	0.62
	5	20	1.21	0.88	52	1.19	0.83	27	1.14	0.66
	6	16	1.25	0.88	56	1.19	0.83	26	1.20	0.69
	mean	**19**	**1.21**	**0.87**	**59**	**1.14**	**0.81**	**21**	**1.11**	**0.72**
good $r^2 = 0.64$	1	15	1.17	0.86	54	1.13	0.74	30	1.22	0.72
	2	11	1.20	0.86	64	1.10	0.76	25	1.16	0.72
	3	19	1.15	0.89	57	1.09	0.76	23	1.17	0.70
	4	6	1.27		85	1.00	0.73	9	0.97	
	5	13	1.19	0.87	51	1.19	0.79	36	1.26	0.70
	6	29	1.10	0.80	61	0.96	0.71	7	1.02	
	mean	**15**	**1.17**	**0.87**	**64**	**1.08**	**0.76**	**21**	**1.14**	**0.70**
poor $r^2 = 0.36$	1	4	1.10		81	0.97	0.66	14	1.18	0.42
	2	6	1.11		57	1.07	0.75	36	1.28	0.50
	3	10	1.02	0.82	57	1.04	0.71	33	1.28	0.49
	4	5	1.10		67	1.04	0.70	28	1.19	0.48
	5	16	1.00	0.82	49	1.04	0.72	35	1.26	0.50
	6	11	1.01	0.84	53	1.06	0.75	36	1.26	0.51
	mean	**8**	**1.09**		**61**	**1.04**	**0.71**	**30**	**1.27**	**0.48**

5. Strength distributions before and after grading

Strength distribution, especially in the lower tail area, has an effect on the structural reliability. Here a single parameter is adopted to describe the slope of the tail: ratio of the 0.5% fractile to the 5% fractile ($f_{0.005}/f_{0.05}$). The ratio tells which distribution function (COV as parameter) fits to the data in this regime. Another important parameter is obviously the 5% fractile which is used as characteristic value in design of structures.

In this study, normal distribution is used for the strength prior to grading (COV = 0.25). This gives the tail ratio $f_{0.005}/f_{0.05} = 0.62$ in the simulated population with N = 2000.

In the populations graded nearly in accordance with CEN standard, the tail ratio is 0.87, 0.76 and 0.70, for grades C40, C30 and C18, respectively, when the correlation between grading parameter and strength is up to the best industrial practice ($r^2 = 0.64$). This indicates that lognormal distribution with COV = 0.16, 0.30 or 0.38, depending on the grade, fits to the tail distribution. At the same time, the 5% fractile is 8 to 17% higher than the characteristic value used in the design of structures, which gives some extra safety to the system. The basic condition for these results is that the grading machine has been calibrated to the same population as it is used for grading.

When the correlation between grading parameter and strength is lower ($r^2 = 0.36$) which is typical of visual grading and in some cases also of machine grading, the tail ratio is 0.71 for C30 and 0.48 for C18. For C40 it was not determined because of too small yield to this class. For all grades the tail ratio is smaller than in case of a better grading method ($r^2 = 0.64$), and the fitting of a lognormal distribution would give COV = 0.37 for C30 and a very high COV for C18. Results are illustrated also in Table 6 where also a comparison to the test data is made. Both experiments and the simulation show that tail ratio has been improved due to grading.

Table 6. Comparison the test data to which the simulation model is based. Grading in test is made only to C30 instead of three grades in simulation.

	C30			ungraded	
	yield [%]	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$
Tested	84	1.02	0.75	0.92	0.56
Simulated $r^2 = 0.64$	64(+15)	1.08	0.76	0.87	0.62
Simulated $r^2 = 0.36$	61(+8)	1.04	0.71	0.89	0.56

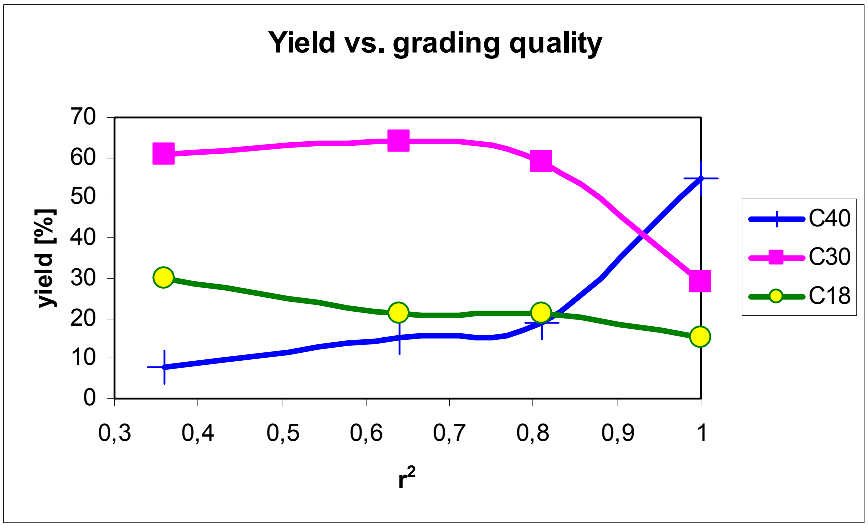

Figure 2. Yield to various strength classes as a function of the quality of grading method according to simulation. $r^2 = 1$ in this figure refers to optimum grading, not only to eqn (3).

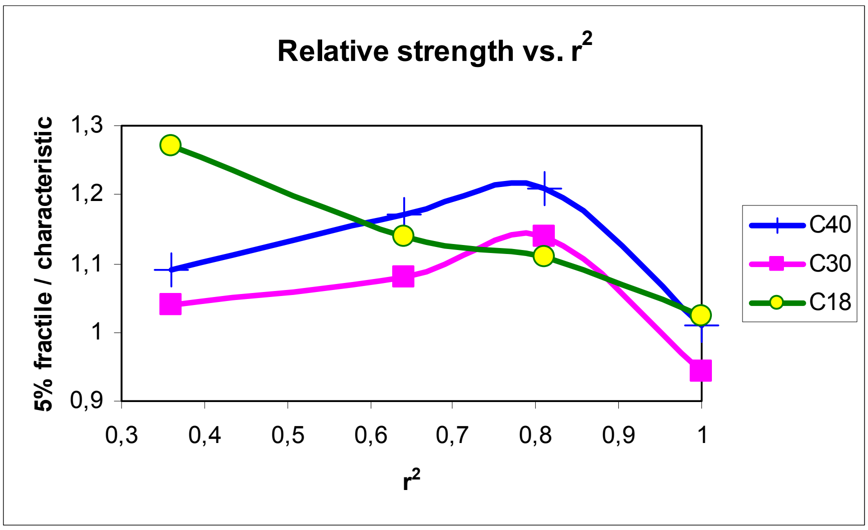

Figure 3. 5% fractile of strength in various strength classes as a function of the quality of grading method according to simulation. $r^2 = 1$ in this figure refers to optimum grading, not only to eqn (3).

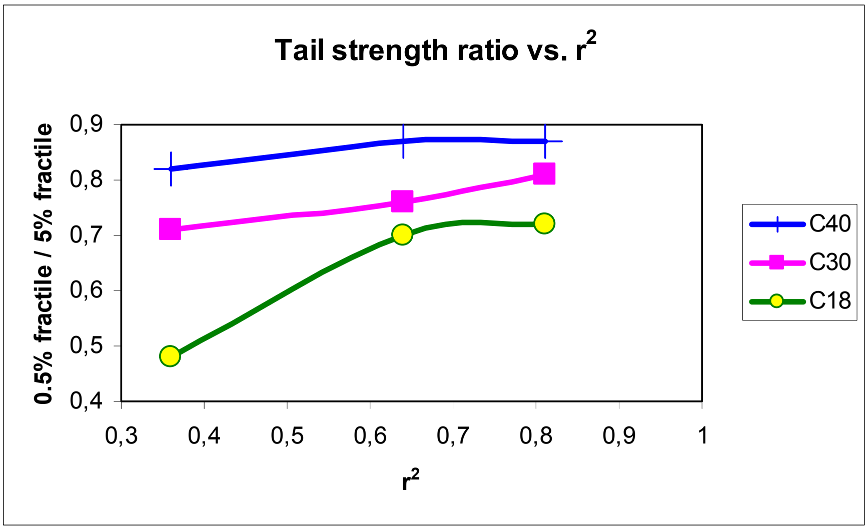

Figure 4. Tail strength ratio in various strength classes as a function of the quality of grading method according to simulation. $r^2 = 1$ in this figure refers to optimum grading, not only to eqn (3).

Conclusions

In this paper the strength grading of Nordic spruce simultaneously to three grades is studied by the use of numerical simulation. It was observed that simultaneous grading to three grades is not always possible according to CEN-procedure, if the correlation between strength and grading parameter id not good. As to the strength of the simulated graded material the conclusions were as follows:

1. The 5% fractiles exceeded the characteristic strength of the strength class nearly in all cases.
2. The lower tail strength values are better than those of ungraded material and generally similar to the test results obtained earlier for graded timber.
3. For the best grade used (C40) the relative tail strength was considerably better than for the lower grades, corresponding to lognormal distribution with COV = 16% fitted to the tail at 0.5% and 5% fractile values.

References

prEN 14081-1 (April 2002). Timber structures - Strength graded structural timber with rectangular cross section - Part 1: General requirements.

prEN 14081-2 (April 2002). Timber structures - Strength graded structural timber with rectangular cross section - Part 2: Machine Grading - Additional requirements for initial type testing.

Ranta-Maunus A., Fonselius M., Kurkela J., Toratti T., 2001, Reliability analysis of timber structures. VTT Research Notes 2109. Espoo, Finland. 102 p + app. 3 p

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