Probabilistic Modelling of Duration of Load Effects in Timber Structures

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Summary
In present code formats the described load duration effect on the strength of timber material is represented by a modification factor $k_{\text{mod}}$. In the present report this modification factor is calibrated for snow load. The modeling of the load duration effect has been described with particular emphasis on the ‘Damaged Viscoelastic Material’ (DVM) model. The parameters of the model have been estimated as stochastic variables using the Maximum Likelihood method. The snow load process has been modeled by a Poisson-process with triangular load pulses.

Keywords: timber, duration of load, code calibration, snow load.

1. Introduction
Reliability analysis of timber structures for the purpose of code calibration or reliability verification of specific timber structures and components requires that the relevant failure modes are represented in terms of limit state functions. The special behavior of structural timber in regard to long-term load situations, exclude modeling load and resistance as independent variables in the limit state function. That the strength of timber material is influenced by both the intensity and the duration of loads, often referred to as the duration of load (DOL) effect, has been an area of great interest over the last decades in the timber research community. Numerous experimental programs have focused on duration of load effects in full size timber components and a number of different models, mostly based on experimental evidence, have been proposed to describe the phenomenon. The so-called Madison curve [1] is one of those models built on results from tests on clear wood specimens i.e. specimen, with no visible defects, loaded with constant loads. The empirical models have the disadvantage that observable quantities, like load history and environmental conditions, can not directly be utilized for calibration and updating of the parameters in these models.

L. Fuglsang Nielsen suggests a model based on analytical physical considerations in e.g. [2] where the observable and measurable quantities, such as the strength and stiffness properties of structural timber, the creep behavior and load variations, are taken into account. This fracture mechanical model considers structural timber as a damaged viscoelastic material. In the present report this model is applied in the context of reliability analysis where the essential uncertain model parameters are represented by random variables. For the propose of illustration and comparison with other models such as reported in [3] appropriate load duration modification factors $k_{\text{mod}}$ for use in load and resistance factor design (LRFD) are calibrated.

2. Resistance models
2.1 Duration of load
One mechanism leading to the decrease in load carrying capacity of timber components subjected to long-term load is creep-rupture. Specifically, creep rupture arises from the propagation of small damages in the microstructure of the timber at a load level lower than the short-term load carrying capacity. A number of models of creep-rupture, commonly referred to as damage accumulation models, have been proposed by e.g. [4], [5], [6] and [7] to assess the damage accumulation in timber subjected to random load histories. These models have the general form:

$$\frac{d\alpha}{dt} = F(\alpha(t), \sigma_s(t), \sigma_s, \sigma_s^*)$$

(1)
where \( t \) is time, \( a \) is the damage state variable which ranges from 0 (no damage) to 1 (failure), \( s(t) \) is the applied stress, \( s_S \) is the short-term strength, \( s_0 \) is the threshold stress below which damage does not accumulate and \( \mathbf{p} \) is a vector of parameters describing the model. The parameters of the model \( \mathbf{p} \) have to be fitted to experimental data and have no further physical meaning. The damage accumulation models are able to consider loading and unloading in immediate, stepwise and ramp wise form. However, the main assumption is that load histories may be modeled as a sum of increment periods of constant load and that the damage accumulates as a consequence of these only, i.e. no other failure mechanisms besides creep-rupture are considered. Damage accumulation models may be seen entirely as suitable curve fitting tools to experimental data. Such models do not necessarily reflect the physical failure mechanism. A different approach is to try to model the mechanisms in line with the experimental evidence that for the most loading modes structural timber fails as a result of crack growth.

2.2 Timber as a Damaged Viscoelastic Material

A model based on viscoelastic fracture mechanics was proposed by Nielsen in [8]. The main idea behind the ‘Damaged Viscoelastic Material’ (DVM) -model is that structural timber may be seen as an initially damaged material, where the damage is represented by cracks along the fibers. The time dependent behavior of timber under load is modeled by a single crack under stress perpendicular to the crack plane. The crack is modeled as a Dugdale crack with a time dependent modulus of elasticity. The time dependence of the modulus of elasticity is described by a power law expression, i.e. creep is modeled by the normalized creep function \( C(t) \) as:

\[
C(t) = 1 + \left(\frac{t}{\kappa}\right)^b
\]  

(2)

where \( t \) and \( b \) are material parameters depending e.g. on loading mode and moisture history.

The DVM-Theory for constant load is modelling creep-rupture by considering initiation of crack development at time \( t_s \) and crack development until the crack has grown to a critical length \( l_{crit} \) at time \( t_{crit} \) corresponding to the applied constant load.

For the case of a constant load level, a damage accumulation law can be formulated from the DVM-theory as:

\[
\frac{dk}{dt} = \frac{(\pi \cdot FL)}{8q\tau} \left(\frac{\kappa \cdot SL^2}{(\kappa \cdot SL^2)^3 - 1}\right)^b
\]

(3)

where \( SL \) is the stress level (or load level) defined by the ratio \( s/s_{cr} \) between applied stress \( s \) (load) and short-term strength \( s_{cr} \). The latter measured in a very fast ramp test. \( FL \) is the strength level defined as the ratio \( s_{cr}/s_{cr} \) between short-term strength \( s_{cr} \) as defined above and the intrinsic strength of the (hypothetical) non-cracked material \( s_{cr} \). \( ? = l/l_0 \) is the damage and defined as the ratio between the actual crack length \( l \) and the initial crack length \( l_0 \). \( ? = 1 \) corresponds to no damage and \( ? = SL^2 \) to full damage. \( q \) is given as a function of the creep exponent \( b \) as \( q = (0.5(b+1)(b+2))^{\frac{1}{b}} \) and considers a parabolic increasing crack opening progression.

The time to failure \( t_f \) can be expressed by the time between \( ?(t=0) = 1 \) and \( ?(t_f) = SL^2 \), as

\[
t_f = \frac{8q\tau}{(\pi \cdot FL \cdot SL)} \int_1^{2SL^2} \frac{(\phi - 1)^{b+2}}{\phi} d\phi
\]

(4)

where \( f \) is a damage state variable. The model of crack propagation under constant load by the DVM-theory is of a format similar to the damage accumulation laws mentioned above, with the difference that no stress threshold \( s_0 \) is introduced and the damage \( ? \) is defined as the physical status of the crack length. Based on the solution for constant load the DVM-model was developed to consider rectangular pulsed load histories by...
introducing crack closure and contact effects due to unloading. It has been shown that the failure mechanism in timber is not only due to creep rupture. Frequency dependent fatigue effects in timber have been shown theoretically by Nielsen in [4] and experimentally e.g. in [9] by Clorius. The DVM-theory as presented in [8] is restricted to model harmonic and weakly non-harmonic rectangular pulsed load histories. The representation of damage in timber subject to a random load process of any load pulse shape is not possible with the theory at present state. It can be shown that the solution of the DVM-Model for constant loads is also applicable for the modeling of damage in timber subject to load histories with moderately varying loads which means that if the rate (velocity) of unloading is sufficiently small, the effect of crack closure might be counteracted by the relaxation of the viscoelastic material in the crack front zone. Assuming that the rate of unloading is small, equation 3 can be used to model the increment damage over the time, with

$$\kappa_{n+1} - \kappa_n = \frac{\pi^2 FL^2}{8q} \left( \frac{\kappa_n SL_{n+1}^2}{(\kappa_n SL_{n+1})^2 - 1} \right) (t_{n+1} - t_n)$$

(5)

2.3 Model parameters and their estimation

The parameters required to estimate time-to-failure, the intrinsic strength of a non-cracked material, $s_i$ and the creep parameters $t$ and $b$ are all physical presented in nature. But none of these parameters is readily obtainable by measurements.

The creep modeled in the DVM-theory is a very local phenomenon of the material in front of the crack tip and thus depends on the condition of grain orientation and morphology around the crack tip. The strength of the un-cracked material can only be assessed by theoretical considerations. For these reasons the model parameters are fitted to data obtained from duration of load tests, where constant load is applied until ultimate failure. On the basis of equation 4 and introducing a normal distributed error term $e$ with zero mean and unknown standard deviation, the parameter $t$ can be estimated using the Maximum Likelihood method. The parameters $b$ and $FL$ can not be estimated with the same freedom; only values of e.g. $b = 1/3, 1/4, 1/5...$ will lead to closed solutions of the integral in equation 4. Furthermore, the parameter $FL$ is dependent on $t$ and can therefore not be estimated individually. Based on test results from Hoffmeyer [10], the parameter $t$ and the standard deviation of the error term are estimated as normal distributed random variables; $b$ and $FL$ are estimated as constants. All parameters are given in table 1.

3. Load model

The solutions of the DVM-theory for constant load can be used to model load histories with randomly varying loads if the rate of the unloading is small. Snow on roofs of buildings is considered as such a load and a stochastic load model is formulated in accordance with the load model presented in [3]. Snow load on the ground is modeled by triangularly shaped load pulses (snow packs). This is illustrated in figure 2. Based on Danish snow data over 32 years from five locations the following load process parameters have been estimated. The occurrence of a snow package at times $X_1, X_2, ...$ is modeled by a Poisson-process. The duration between snow packages is therefore exponential distributed with expected value $1/\gamma$, where $\gamma$ is annually the expected number of snow packages. The magnitude of each snow package $P_m$ is assumed to be Gumbel distributed. The duration of a snow package $T$ is assumed to be related to the snow package magnitude with $T = X_T \cdot P_m$ where the factor $X_T$ is exponential distributed. The load history on the ground $P(t)$ can be simulated and transformed to the roof load $Q(t)$ by introducing a Gumbel distributed shape factor $C$ with $Q(t) = C \cdot P(t)$. The parameters for the stochastic models are found in table 1.
Table 1: Overview of the stochastic Models used in this report.

<table>
<thead>
<tr>
<th>Stochastic Variable</th>
<th>Distribution Type</th>
<th>Mean Value</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Short-term strength</td>
<td>( R_0 )</td>
<td>Log-normal</td>
<td>1</td>
</tr>
<tr>
<td>Snow load (50 y max)</td>
<td>( Q )</td>
<td>Gumbel max</td>
<td>1</td>
</tr>
<tr>
<td>Model uncertainty</td>
<td>( X_R )</td>
<td>Log-normal</td>
<td>1</td>
</tr>
<tr>
<td>Relaxation time [h]</td>
<td>( t )</td>
<td>Normal</td>
<td>875.4</td>
</tr>
<tr>
<td>stdvia. Error term log[d]</td>
<td>( s_r )</td>
<td>Normal</td>
<td>0.51</td>
</tr>
<tr>
<td>Strength level ( FL )</td>
<td>Constant</td>
<td>0.25</td>
<td>-</td>
</tr>
<tr>
<td>Creep power</td>
<td>( b )</td>
<td>Constant</td>
<td>0.2</td>
</tr>
<tr>
<td>Duration between snow pack [y]</td>
<td>( db )</td>
<td>Exponential</td>
<td>1/1.175</td>
</tr>
<tr>
<td>Max. per snow pack [kN/m²]</td>
<td>( P_m )</td>
<td>Gumbel max</td>
<td>0.33</td>
</tr>
<tr>
<td>Duration pack factor [d/kN/m²]</td>
<td>( X_T )</td>
<td>Exponential</td>
<td>75</td>
</tr>
<tr>
<td>Roof shape factor [-]</td>
<td>( C )</td>
<td>Gumbel max</td>
<td>1</td>
</tr>
</tbody>
</table>

4. Calibration of the modification factor \( k_{mod} \)

In present code formats as the EC5 the described load duration effect of on the load carrying capacity of timber components is represented by a modification factor \( k_{mod} \). The following design equation can be found in the codes.

\[
\frac{zR_0}{\gamma_m} k_{mod} - \gamma_Q Q_t = 0
\]  

(6)

where \( z \) is the design variable, \( R_0 \) and \( Q_t \) are the characteristic values of short-term strength (5% quantile) and variable load (98% quantile of one year maximum distribution). \( \gamma_m \) and \( \gamma_Q \) are the partial safety factors for material parameter, and variable load. \( k_{mod} \) is the modification factor. The limit state function for the long-term load carrying capacity can be formulated as

\[
g = \kappa_{ul} (SL, t, z) - \kappa(SL, t, FL, \tau, b, z)
\]  

(7)

where \( \kappa_{ul} \) is the ultimate damage at time \( t \) and load level \( SL \). For the calibration of the modification factor \( k_{mod} \) the following limit state function for short-term load carrying capacity is used.

\[
g = zR_0 X_R - CQ
\]  

(8)

where \( z \) is the design parameter, \( R_0 \) the short-term strength, \( X_R \) is the model uncertainty, \( Q \) is the snow load on the ground and \( C \) is the shape factor.

The modification \( k_{mod} \) can be calibrated as follows. The partial safety factor for the short-term strength \( \gamma_{M,S} \) is calibrated to a target reliability index, using the limit state function (8) and the design equation (6) with \( k_{mod} = 1 \) and fixed \( \gamma_Q \). The partial safety factor for the long-term strength \( \gamma_{M,L} \) is calibrated using the design equation (6) with \( k_{mod} = 1 \) and fixed \( \gamma_Q \). A snow load history of 50 years and the hereby-resulted damage in the timber is simulated and the time to failure \( T_F \) is determined. After a sufficient number of simulations the probability of failure \( P_F \) is estimated by the number of failures during the design life of 50 years and the total number of realizations. The corresponding reliability index is determined from \( \beta = - F^{-1}(P_F) \). \( \gamma_{M,L} \) has to be modified until the target reliability index is reached. \( k_{mod} \) is then estimated by

\[
k_{mod} = \frac{\gamma_{M,S}(\beta)}{\gamma_{M,L}(\beta)}
\]  

(9)
5. Observations and Discussion

The modification factor $k_{\text{mod}}$ is calibrated as previously described. A value of $k_{\text{mod}} = 0.8$ was found. This result is consistent with the results presented in Sorensen et al [3], where the same snow load model but different damage accumulation models were used in the calibration procedure. To illustrate the effect of damage to the strength of the timber material, the residual strength $SR$ is defined by

$$SR = \frac{1}{\sqrt{k}}$$

In figure 3, the maximum load level of each snow package and the residual strength of the timber is plotted over lifetime. Failure occurs when the residual strength drops as a consequence of an extreme snow load, i.e. when $SR < SL$. In the case illustrated in figure 3 failure occurs after a time of approximately 32 years. The load history prior to failure seems not to be of significance to the damage and the residual strength respectively. A similar observation was made in [11] where other damage accumulation models were used. In figure 4, a snow load process is used where the standard deviation of the maximum load level of each snow package is lower compared to the example shown in figure 3. It is seen that the residual strength drops as a consequence of major snow loads several times until failure occurs. The damage accumulates with the number of major snow events. Depending on the load level as well as the temporal characteristics of the underlying stochastic load process failure may be considered time independent. Rather than simulating complete load histories and incrementally accumulating damage, a time independent limit state function can be formulated. Only the load pulse that cause ‘significant’ damage, may be considered for reliability analysis with the considered snow load characteristics. If the standard deviation of the maximum load level of a snow package is getting smaller, damage accumulates several times (figure 4) and the time to failure has to be evaluated by simulation as described in this report. The DVM-model does not include a threshold value $s_0$ below which no damage accumulates as some other damage accumulation models. However, through the highly non-linear properties of the model an ‘effective’ threshold can be observed by analyzing the results of the simulations.

6. Conclusion and prospect for further studies

The ‘Damaged Viscoelastic Material’ model (DVM) has been used for the calibration of the modification factor $k_{\text{mod}}$ which considers the duration of load effect in load and resistance factor design (LRFD). Snow load is considered and a stochastic model is formulated. The results correspond with the results from other investigations where the same load model but different resistance models were used. It has been shown that for the considered load situation the load history before failure is not significant and that failure occurs as a consequence of a combination of an extreme snow load event and the resulting damage.
In further studies the combination of different loads should be considered and a classification of load situations should be undertaken for taking damage accumulation effects into account. Further the DVM-model should be developed to also account for fatigue-effects due to stochastic load histories. The model as is at the present restricted to harmonic and weakly non harmonic load processes.

7. Acknowledgements

The work described in the present report was conducted as part of the European research project ‘Reliability design of timber structures’ under the EC Action COST E24. The financial support from the Federal Office of education and science, Switzerland is gratefully acknowledged.

8. References