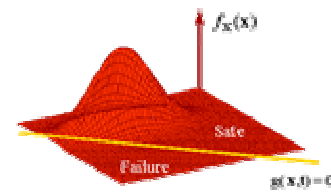


Reliability Based Code Calibration

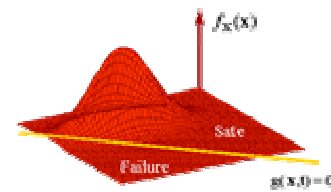
M. H. Faber, ETH Zurich, Switzerland

J.D. Sorensen, AAU, Aalborg, Denmark.



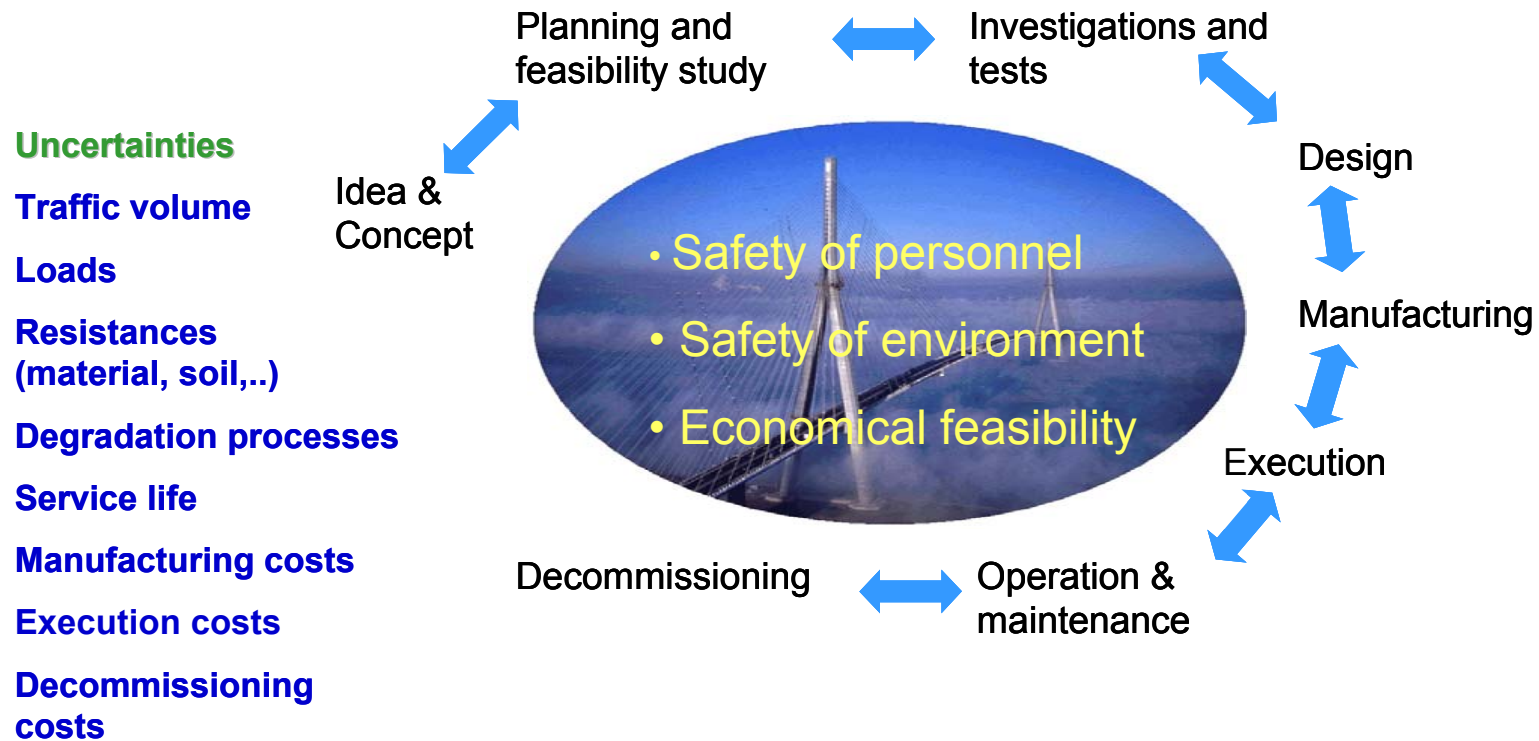
Overview of Presentation

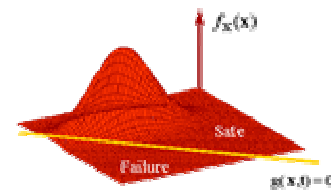
- Design Codes and Code Calibration in the Past
- Principles of Structural Reliability
- Reliability and Partial Safety Factors
- The Code Calibration Decision Problem
- Optimality and Target Reliabilities
- A Practical Code Calibration Procedure
- Conclusions and Future Challenges



Design Codes and Code Calibration in the Past

Structural safety is a matter to be considered throughout all phases of the life of a structure





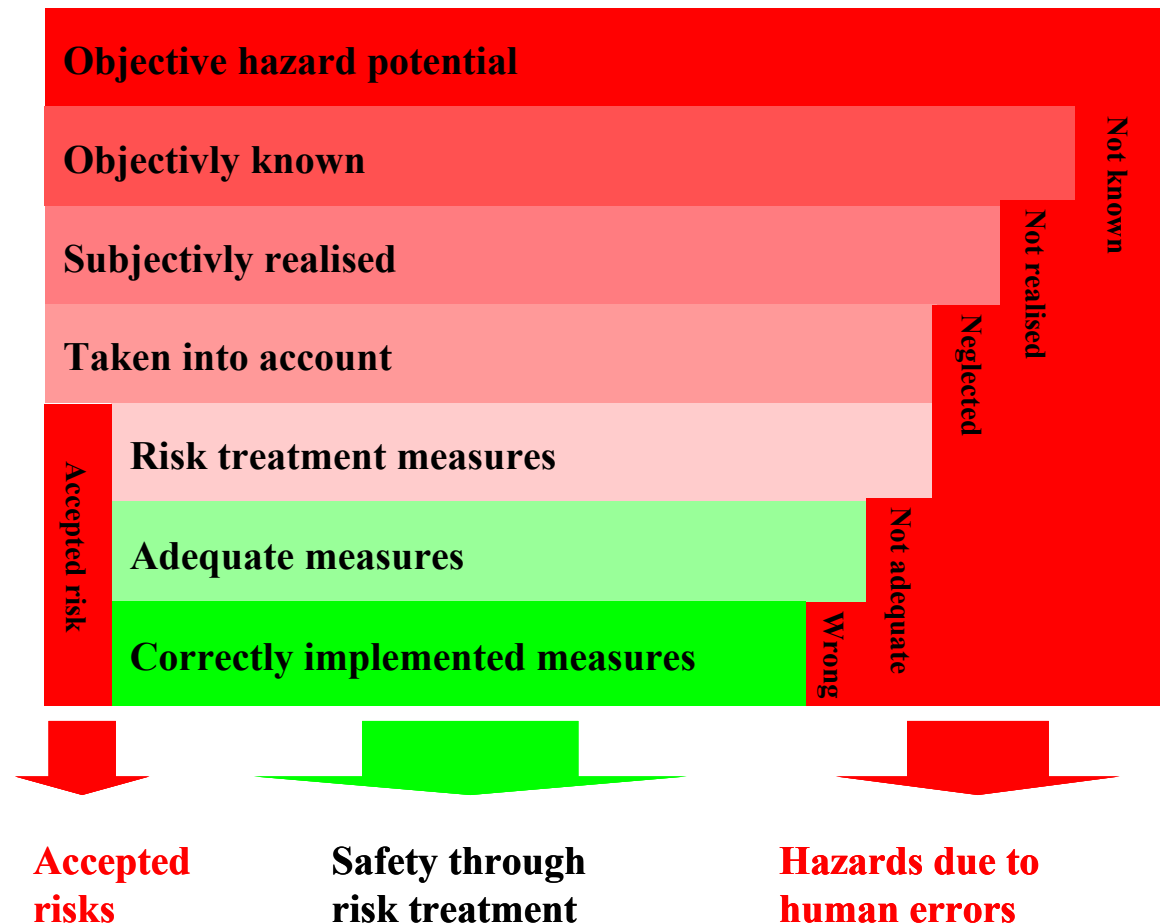
Design Codes and Code Calibration in the Past

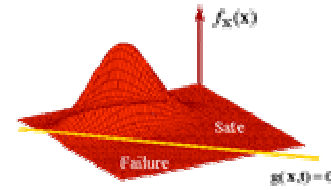
The risk potential associated with a given structure may be subdivided as

Structural reliability is ensured by means of selecting

- appropriate dimensions
- appropriate control

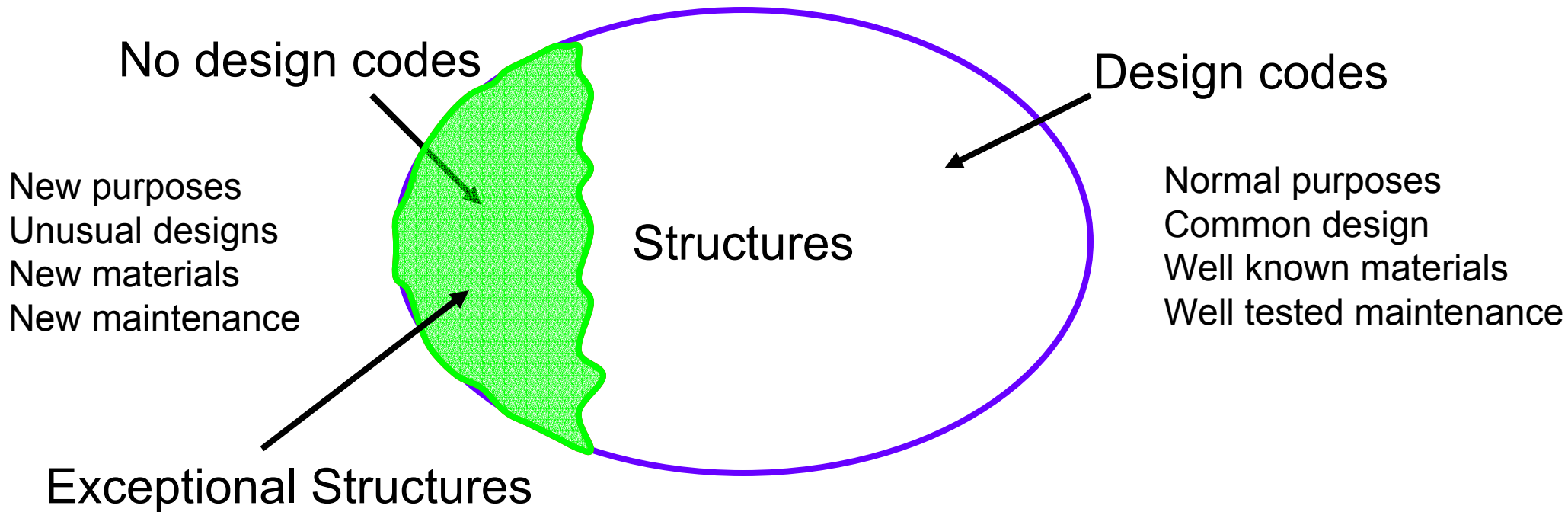
Accepted risks is a matter of choice – based on cost benefit considerations

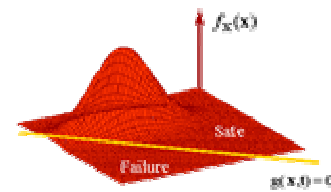




Design Codes and Code Calibration in the Past

- „Normal structures“ are designed according to structural design codes





Design Codes and Code Calibration in the Past

- Exceptional structures is often associated with structures of

„Extreme Dimensions“

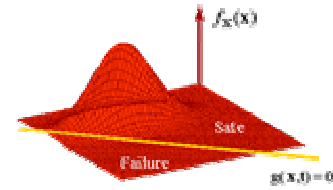


Great Belt Bridge under Construction



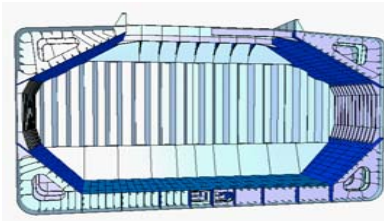
Concept drawing of the Troll platform

Design Codes and Code Calibration in the Past

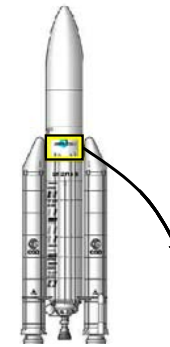


- or associated with structures filfilling

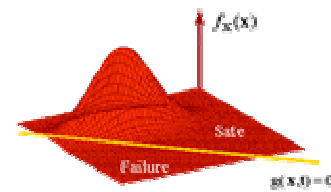
„New and Innovative Purposes“



Concept drawing of
Floating Production, Storage and Offloading unit

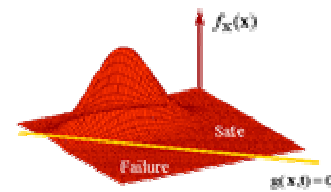


Illustrations of the ARIANE 5 rocket



Principles of Structural Reliability

- Structural performance is subject to uncertainty due to
 - Natural variability in material properties and loads or load effects
 - Statistical uncertainties due to lack of or insufficient data
 - Model uncertainties due to idealisations and lack of understanding in the physical modelling of the structural performance

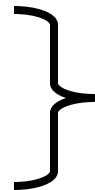


Principles of Structural Reliability

- Structural performance may be treated in probabilistic terms by means of **limit state functions**

i.e. defining the events of special concern (large consequences) – **failure events** - such as e.g.

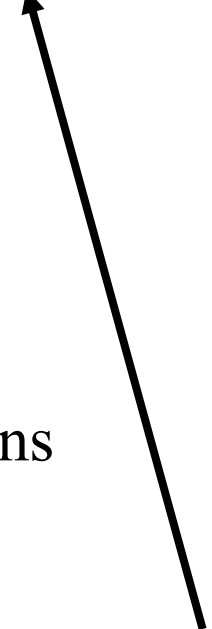
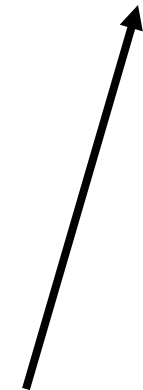
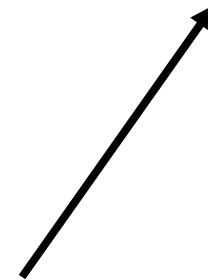
- Collapse
- Inserviceability
- Deterioration



Failure Events

as function of the most important uncertainties – the **basic random variables**

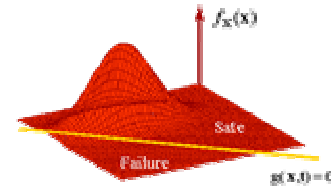
$$F = \{g(\mathbf{x}) \leq 0\}$$



Limit state functions

Outcome of basic random variables

Principles of Structural Reliability



- The fundamental case**

$$P_F = P(R \leq S) = P(R - S \leq 0) =$$

$$\int_{-\infty}^{\infty} F_R(x) f_S(x) dx$$

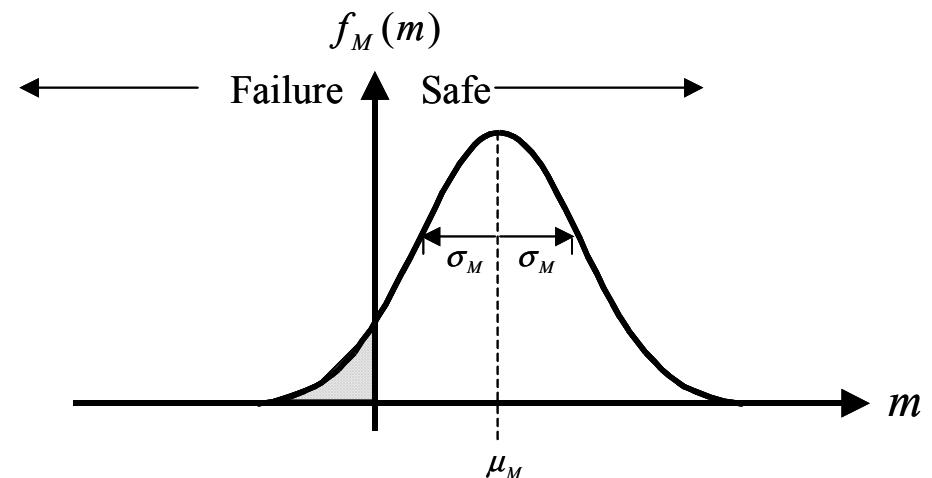
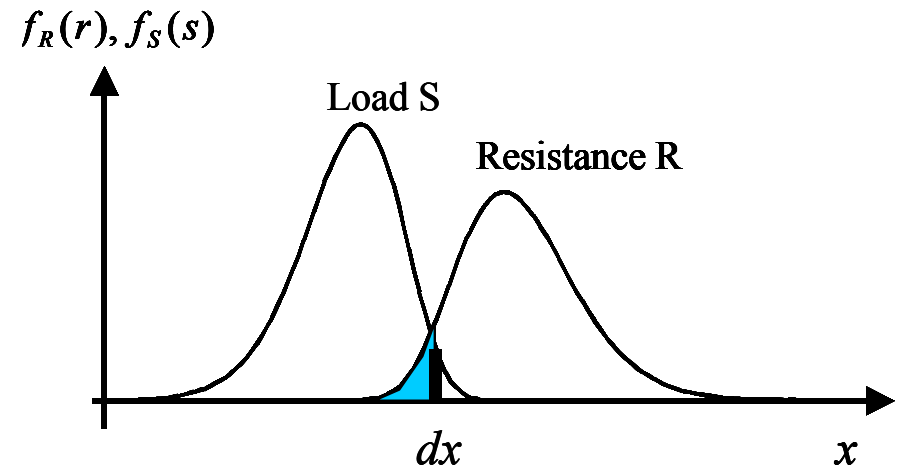
- Normal distributed safety margin**

$$P_F = P(R - S \leq 0) = P(M \leq 0)$$

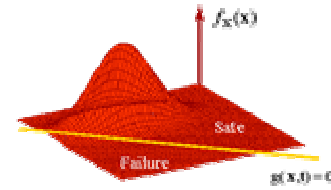
$$\mu_M = \mu_R - \mu_S$$

$$\sigma_M = \sqrt{\sigma_R^2 + \sigma_S^2} \quad \mu_M / \sigma_M = \beta$$

$$P_F = \Phi\left(\frac{0 - \mu_M}{\sigma_M}\right) = \Phi(-\beta)$$



Principles of Structural Reliability



- The probability of failure in regard to

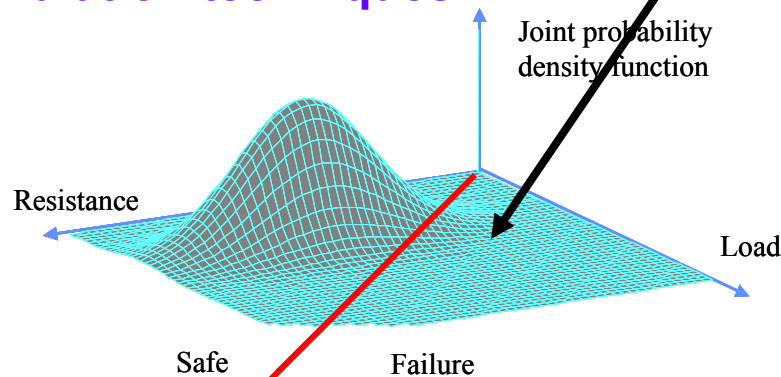
- Ultimate collapse
- Loss of serviceability
- Excessive deterioration

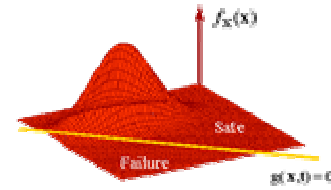
may then be assessed by evaluation of probability integrals using e.g. **FORM/SORM or simulation techniques**

$$P_f = \int_{g(\mathbf{x}) \leq 0} f_{\mathbf{x}}(\mathbf{x}) d\mathbf{x}$$

Failure event

Joint density function for the basic random variables





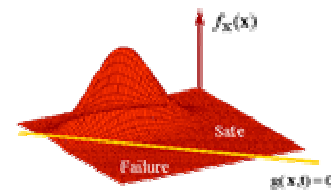
The Code Calibration Decision Problem

- The code calibration problem can be seen as a decision problem with the objective to maximize the life-cycle benefit obtained from the structures by „calibrating“ (adjusting) the partial safety factors

$$\begin{aligned} \max_{\gamma} \quad & W(\gamma) = \sum_{j=1}^L w_j [B_j - C_{Ij}(\gamma) - C_{Rj}(\gamma) - C_{Fj} P_{Fj}(\gamma)] \\ \text{s.t.} \quad & \gamma_i^l \leq \gamma_i \leq \gamma_i^u, i = 1, \dots, m \end{aligned}$$

- The „optimal“ design is determined from the design equations

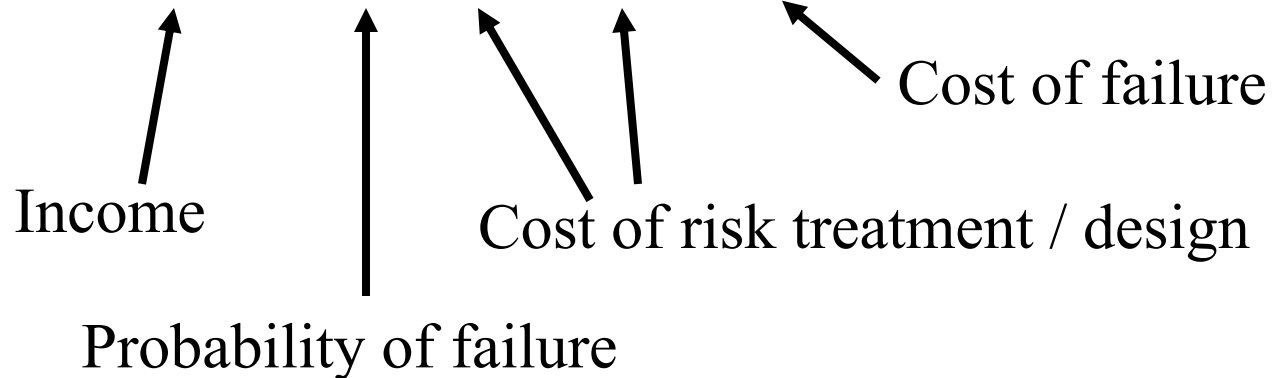
$$\begin{aligned} \min_{\gamma} \quad & C_{Ij}(\mathbf{z}) & G_j(\mathbf{x}_c, \mathbf{p}_j, \mathbf{z}, \gamma) &\geq 0 \\ \text{s.t.} \quad & G_j(\mathbf{x}^c, \mathbf{p}_j, \mathbf{z}, \gamma) &\geq 0 \\ & z_i^l \leq z_i \leq z_i^u, i = 1, \dots, N \end{aligned}$$



Optimality and Target Reliabilities

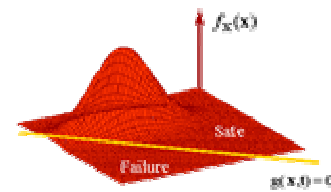
- Acceptance criteria may be established on the basis of cost benefit considerations

$$E[B] = I \cdot (1 - P_F(C_D)) - C_D - C_F \cdot P_F(C_D) = I - C_D - (I + C_F)P_F(C_D)$$



- Optimality may be determined from

$$\frac{\partial E[B]}{\partial C_D} = -1 - (I + C_F) \cdot \frac{\partial P_F(C_D)}{\partial C_D} = 0$$



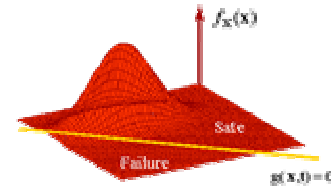
Optimality and Target Reliabilities

- Target reliabilities for Ultimate Limit State verification

Relative cost of safety measure	Minor consequences of failure	Moderate consequences of failure	Large consequences of failure
High	$\beta=3.1$ ($P_F \approx 10^{-3}$)	$\beta=3.3$ ($P_F \approx 5 \cdot 10^{-4}$)	$\beta=3.7$ ($P_F \approx 10^{-4}$)
Normal	$\beta=3.7$ ($P_F \approx 10^{-4}$)	$\beta=4.2$ ($P_F \approx 10^{-5}$)	$\beta=4.4$ ($P_F \approx 5 \cdot 10^{-5}$)
Low	$\beta=4.2$ ($P_F \approx 10^{-5}$)	$\beta=4.4$ ($P_F \approx 10^{-5}$)	$\beta=4.7$ ($P_F \approx 10^{-6}$)

- Target reliabilities for Serviceability Limit State verification

Relative cost of safety measure	Target index (irreversible SLS)
High	$\beta=1.3$ ($P_F \approx 10^{-1}$)
Normal	$\beta=1.7$ ($P_F \approx 5 \cdot 10^{-2}$)
Low	$\beta=2.3$ ($P_F \approx 10^{-2}$)



A Practical Code Calibration Procedure

- A seven step approach

1. Definition of the scope of the code

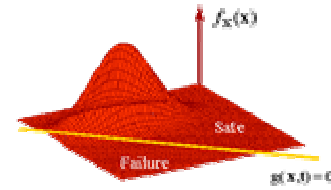
- Class of structures and type of failure modes

2. Definition of the code objective

- Achieve target reliability/probability

3. Definition of code format

- how many partial safety factors and load combination factors to be used
- should load partial safety factors be material independent
- should material partial safety factors be load type independent
- how to use the partial safety factors in the design equations
- rules for load combinations



A Practical Code Calibration Procedure

- A seven step approach

4. Identification of typical failure modes and of stochastic model

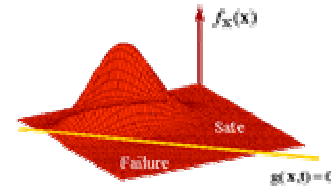
- relevant failure modes are identified and formulated as limit state functions/design equations
- appropriate probabilistic models are formulated for uncertain variables

5. Definition of a measure of closeness

- the objective function for the calibration procedure is formulated e.g.

$$\min_{\gamma} W(\gamma) = \sum_{j=1}^L w_j (\beta_j(\gamma) - \beta_t)^2$$

$$\min_{\gamma} W'(\gamma) = \sum_{j=1}^L w_j (P_{Fj}(\gamma) - P_F^t)^2$$



A Practical Code Calibration Procedure

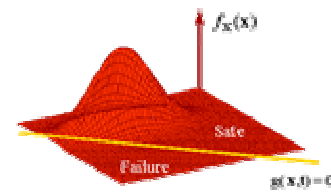
- A seven step approach

6. *Determination of the optimal partial safety factors for the chosen code format*

$$\begin{aligned}
 \min \quad & C(\mathbf{z}) \\
 \text{s.t.} \quad & c_i(\mathbf{x}_c, \mathbf{p}_j, \mathbf{z}, \gamma) = 0 \quad , i = 1, \dots, m_e \\
 & c_i(\mathbf{x}_c, \mathbf{p}_j, \mathbf{z}, \gamma) \geq 0 \quad , i = m_e + 1, \dots, m \\
 & z_i^l \leq z_i \leq z_i^u \quad , i = 1, \dots, N
 \end{aligned}$$

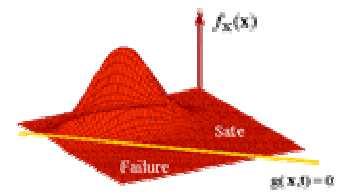
7. *Verification*

- incorporating experience of previous codes and practical aspects



Conclusions and remarks

- Simplified and straight forward formulations exist for the calibration of deterministic LRFD safety formats
- Traditionally the safety formats have only allowed for a rather limited differentiation in regard to failure consequences
 - the codes have origin in „one man – one structure“ situations
- In the modern society the failure of one structure may be associated with consequences reaching far beyond the „interest“ of the individual
- More work is needed to include more realistically the consequences of failure from a societal point of view – including the situations
 - one extreme event affecting one structure (of extreme importance)
 - one extreme event affecting many structures (of moderate importance)



Thanks for your attention !