# Load bearing capacity of roof trusses

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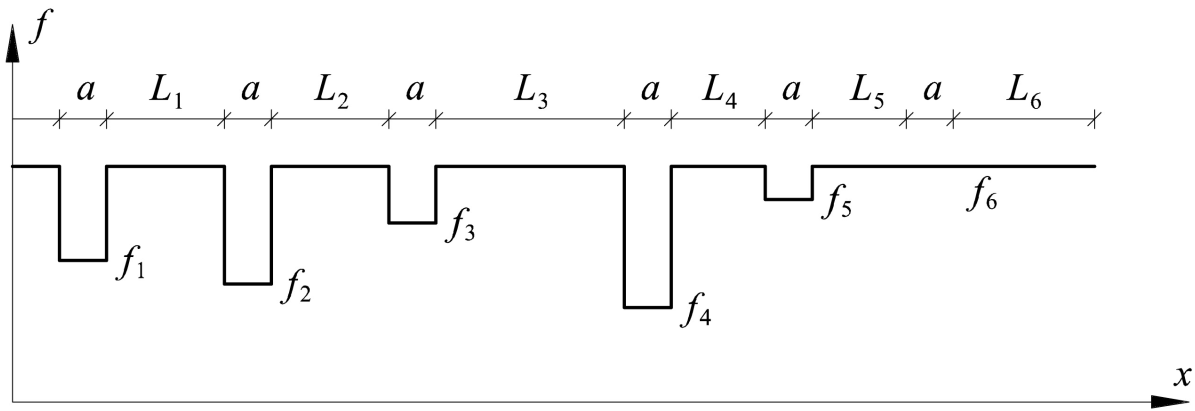
- Stochastic model for strength of timber beam
- Load bearing capacity of roof truss
- Statistical characteristics
- Reliability aspects

# Stochastic model

- Bending strength of timber beam – model 1



# Stochastic model

**Length:**

- weak sections : $a$ length $= 0.15$ m
- strong sections : $L_j$ length of cross-section: Gamma (0.494 m, 0.310 m)

  $L_1$, $L_2$, $L_3$ ,… : independent

**Bending strength:**

- weak sections: $f_{ij}$ = bending strength of weak cross-section $j$ in beam $I$
- strengths in different beams : independent
- strong sections: bending strengths of all strong sections = strength of the strongest weak section

**Modulus of elasticity:**

- $E_{jj} = c\ f_{ij}$ , $c = 3.8\ 10^3$
- Correlation with bending strength: correlation coefficient = 0.8

# Stochastic model

**Bending strength:**

- Lognormal distributed: $f_{ij} = \tau_i \ \varepsilon_{ij}$ $\quad COV = 0.25$

  $\tau_i$ = mean strength of beam no $i$: Lognormal

  $\varepsilon_{ij}$ = difference between mean strength of beam $i$ and strength in cross-section $j$: Lognormal
- 40% and 60% of the variance of $f_{ij}$ are related to $\tau_i$ and $\varepsilon_{ij}$

**Compression strength:**

- Lognormal distributed $\quad COV = 0.15$
- Correlation with bending strengths: correlation coeff. =0.9

**Tension strength:**

- Lognormal distributed $\quad COV = 0.30$
- Correlation with bending strengths: correlation coeff. =0.8

# Stochastic model

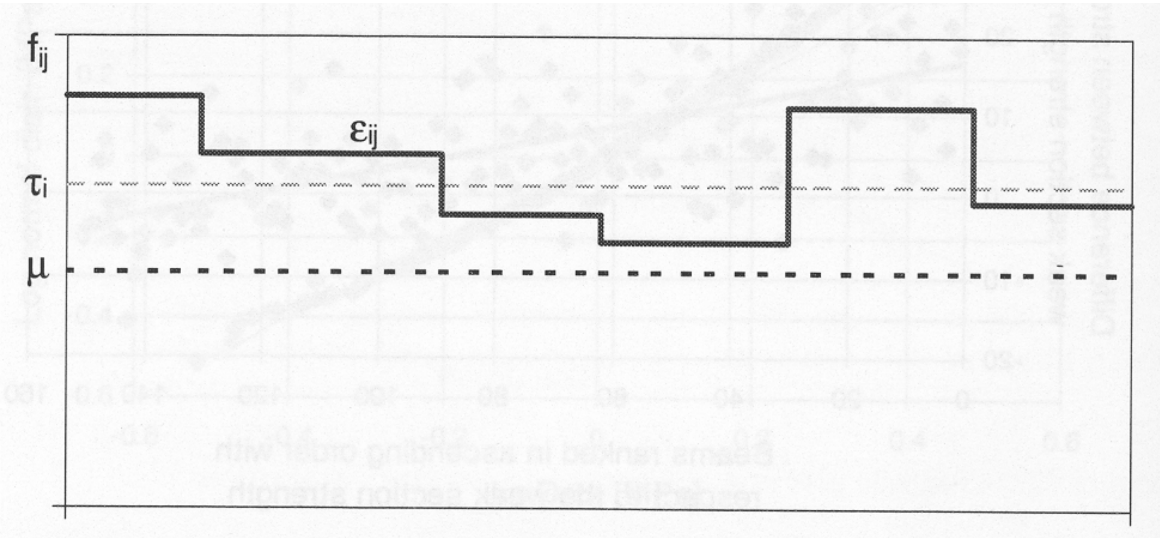
Characteristic values (MPa) and coefficient of variation (*COV*):

| | K14 | K18 | K24 | K30 | *COV* |
|---|---|---|---|---|---|
| Bending | 14 | 18 | 24 | 30 | 0.25 |
| Compression | 12 | 15 | 20 | 26 | 0.15 |
| Tension | 8 | 10 | 16 | 20 | 0.30 |
| Modulus of elasticity | | | | | 0.13 |

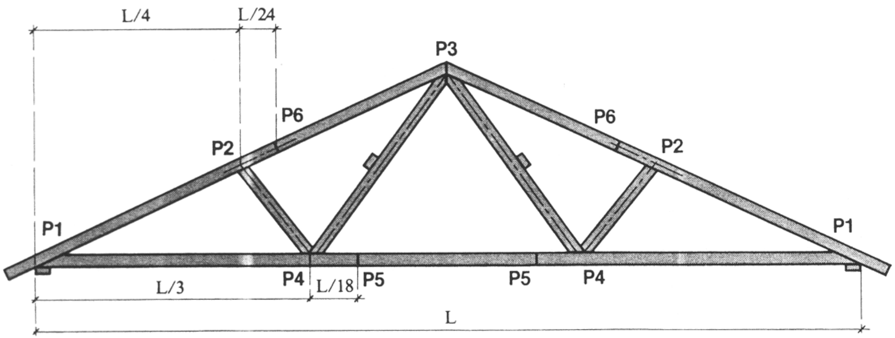
Stochastic model: calibrated to standard test specimens

# Stochastic model

- Bending strength of timber beam – model 2



# Example 1 – roof truss




Load: permanent + snow

# Example 1 – roof truss

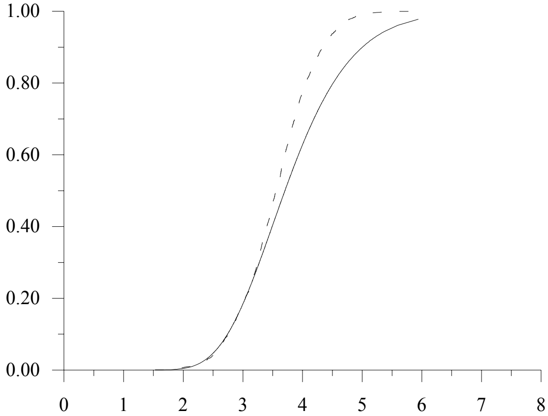
**Stochastic model 1:** $L$ = 12.45 m:

| Load: | permanent | | snow | | permanent + snow | |
|---|---|---|---|---|---|---|
| | COV | $P_{0.05}$ | COV | $P_{0.05}$ | COV | $P_{0.05}$ |
| Non-parametric | 0.13 | 2.51 | 0.12 | 2.80 | 0.13 | 2.33 |
| LogNormal | 0.17 | 2.51 | 0.14 | 2.79 | 0.17 | 2.32 |
| Weibull-2p | 0.10 | 2.53 | 0.09 | 2.81 | 0.11 | 2.33 |
| $P_{0.05}$ | | 2.47 | | 2.88 | | 2.08 |

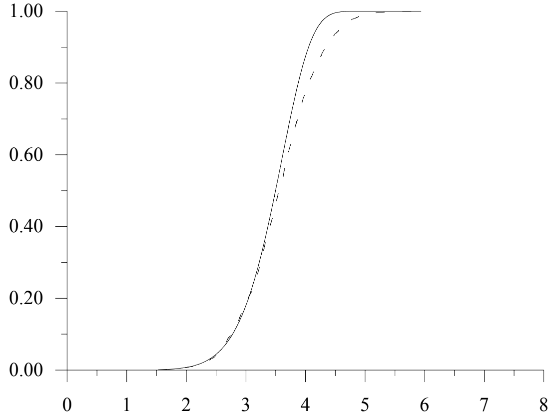
**Stochastic model 1:** $L$ = 8.85 m:

| Load: | permanent | | snow | | permanent + snow | |
|---|---|---|---|---|---|---|
| | COV | $P_{0.05}$ | COV | $P_{0.05}$ | COV | $P_{0.05}$ |
| Non-parametric | 0.12 | 3.23 | 0.11 | 2.89 | 0.13 | 2.41 |
| LogNormal | 0.17 | 3.22 | 0.14 | 2.89 | 0.16 | 2.40 |
| Weibull-2p | 0.10 | 3.25 | 0.09 | 2.90 | 0.10 | 2.42 |
| $P_{0.05}$ | | 3.17 | | 2.98 | | 2.17 |

# Example 1 – roof truss




Lognormal




Weibull

# Example 1 – roof truss

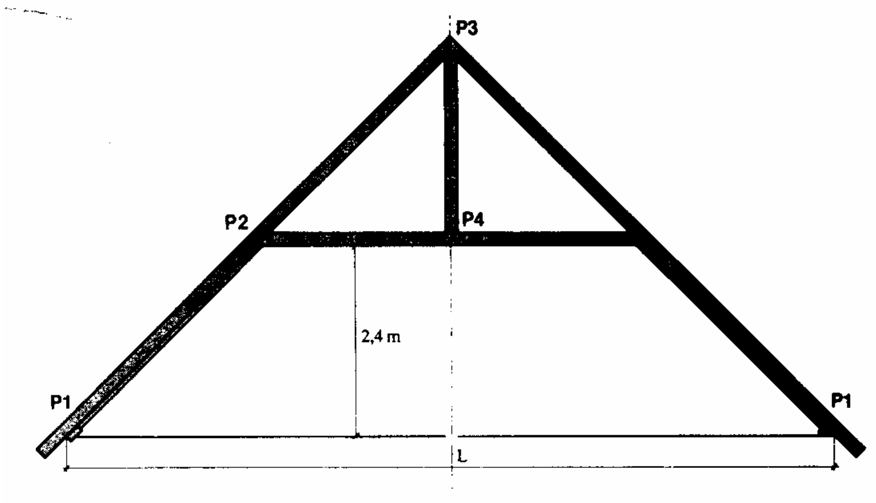
## Stochastic model 1: $L$ = 8.85 m:

| Load: | permanent | | snow | | permanent + snow | |
|---|---|---|---|---|---|---|
| | COV | $P_{0.05}$ | COV | $P_{0.05}$ | COV | $P_{0.05}$ |
| Non-parametric | 0.12 | 3.23 | 0.11 | 2.89 | 0.13 | 2.41 |
| LogNormal | 0.17 | 3.22 | 0.14 | 2.89 | 0.16 | 2.40 |
| Weibull-2p | 0.10 | 3.25 | 0.09 | 2.90 | 0.10 | 2.42 |
| $P_{0.05}$ | | 3.17 | | 2.98 | | 2.17 |

## Stochastic model 2: $L$ = 8.85 m:

| Load: | permanent | | snow | | permanent + snow | |
|---|---|---|---|---|---|---|
| | COV | $P_{0.05}$ | COV | $P_{0.05}$ | COV | $P_{0.05}$ |
| Non-parametric | 0.12 | 3.07 | 0.11 | 2.89 | 0.13 | 2.31 |
| LogNormal | 0.15 | 3.05 | 0.14 | 2.89 | 0.16 | 2.30 |
| Weibull-2p | 0.10 | 3.08 | 0.09 | 2.90 | 0.10 | 2.32 |
| $P_{0.05}$ | | 3.17 | | 2.98 | | 2.17 |

# Example 2 – collar tie roof truss




Load: permanent + snow + imposed

# Example 2 – collar tie roof truss

| Load: | Permanent load | | Imposed load | | Snow load | | Imposed + permanent load | | Snow + imposed + permanent load | |
|---|---|---|---|---|---|---|---|---|---|---|
| | COV | $P_{0.05}$ | COV | $P_{0.05}$ | COV | $P_{0.05}$ | COV | $P_{0.05}$ | COV | $P_{0.05}$ |
| Non-par. | 0.09 | 3.19 | 0.16 | 5.70 | 0.13 | 9.97 | 0.18 | 4.69 | 0.15 | 7.87 |
| LogNormal | 0.13 | 3.17 | 0.18 | 5.67 | 0.16 | 9.87 | 0.21 | 4.66 | 0.20 | 7.77 |
| Weibull-2p | 0.08 | 3.19 | 0.11 | 5.72 | 0.10 | 9.94 | 0.13 | 4.70 | 0.13 | 7.84 |
| $P_{0.05}$ | | 2.99 | | 5.55 | | 9.80 | | 3.65 | | 6.14 |

# Reliability aspects

Limit state function: $g = zRX_R - ((1-\alpha)G + \alpha Q)$

| Variable | Distribution type | Expected value | COV | Quantile value |
|---|---|---|---|---|
| Permanent load | N | 1 | 0.10 | 50 % |
| Variable load (environmental load) | G | 1 | 0.40 | 98 % |
| Variable last (imposed load) | G | 1 | 0.20 | 98 % |
| Strength | LN | 1 | $VR$ | 5 % |
| Model uncertainty | N | 1 | 0.05 | 50 % |

Danish code: $VR$ = 20% for structural timber

# Reliability aspects

Design equations: $z = \max(z_1, z_3)$

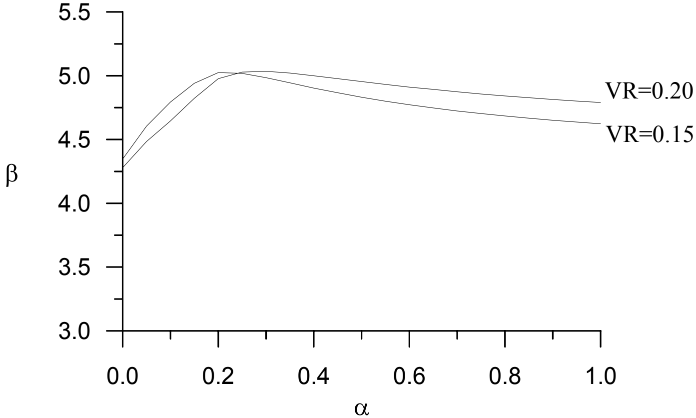
LC 2.1: $z_1 R_c / \gamma_R - \left((1-\alpha)\gamma_{G_1} G_c + \alpha \gamma_{Q_1} Q_c\right) = 0$

LC 2.3: $z_3 R_c / \gamma_R - \left((1-\alpha)\gamma_{G_3} G_c + \alpha \gamma_{Q_3} Q_c\right) = 0$

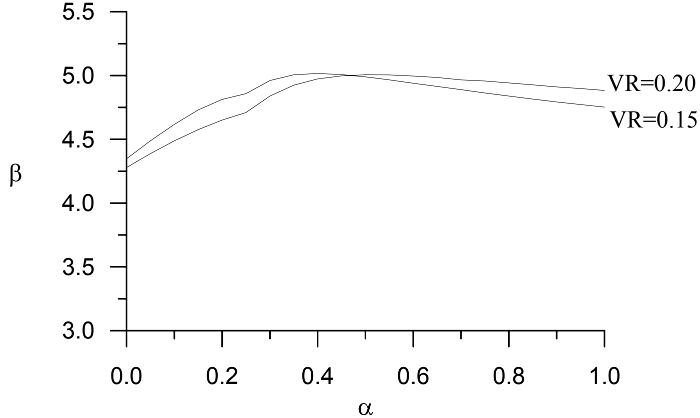
| | Partial safety factor | |
|---|---|---|
| | LC 2.1 | LC 2.3 |
| Permanent load | $\gamma_{G_1} = 1.0$ | $\gamma_{G_3} = 1.15$ |
| Variable load (environmental load) | $\gamma_{Q_1} = 1.5$ | $\gamma_{Q_3} = 1.0$ |
| Variable last (imposed load) | $\gamma_{Q_1} = 1.3$ | $\gamma_{Q_3} = 1.0$ |
| strength | $\gamma_R = \gamma_2$ | |

Danish code: $\gamma_R = 1.65$ for structural timber

# Reliability aspects – reliability index

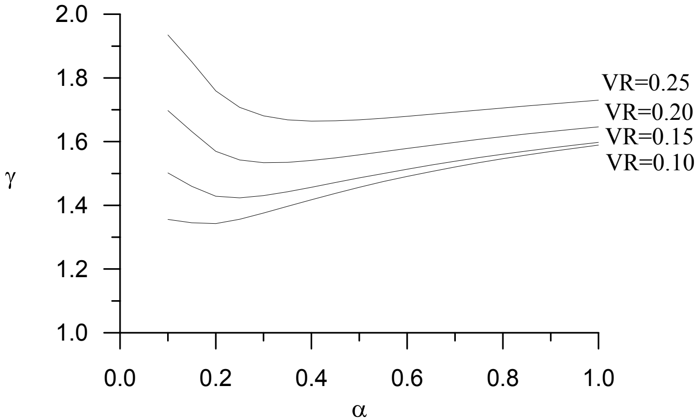



Imposed load

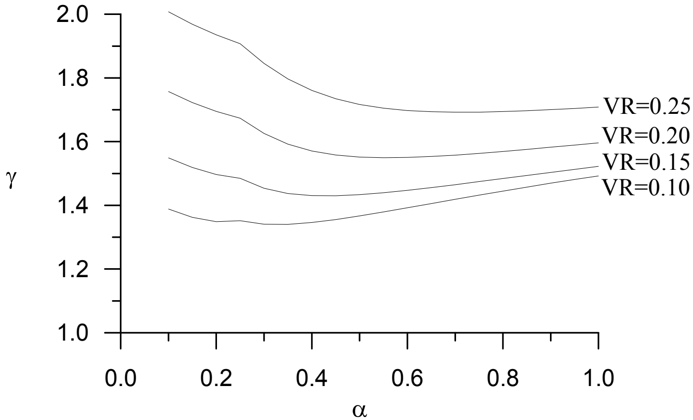



Environmental load

# Reliability aspects - partial factor for strength




Imposed load




Environmental load

# Summary

- Stochastic model for strength of timber beams
- Load-bearing capacity: COV is approximately 15% → system factor = 1.1
- Compared to the DK code based values: characteristic values are at least 10% higher → system factor = 1.1
- System factor = 1.2 for design load bearing capacity