

Dejan Zupan

IZPITNE NALOGE IN REŠITVE NALOG S POSTOPKOM IZ PREDMETA STATIKA NA
VISOKOŠOLSLEM ŠTUDIJU GRADBENIŠTVA

Igor Planinc

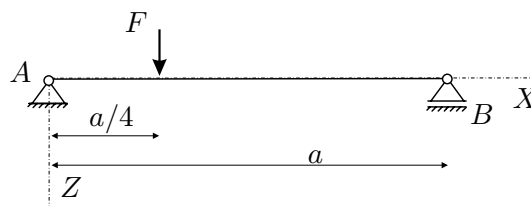
VPRAŠANJA IZ TEORIJE PRI PREDMETU STATIKA NA
VISOKOŠOLSLEM ŠTUDIJU GRADBENIŠTVA

ŠTUDIJSKO LETO: 2004/05

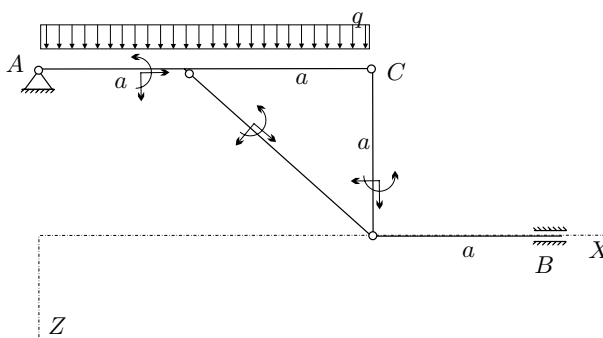
STATIKA (VSŠ) - IZPITNI ROK (31. 01. 2005)

RAČUNSKI DEL IZPITA:

1. Za nosilec na sliki izračunajte in prikažite diagrame notranjih statičnih količin! ((OBVEZNA NALOGA! 15%)



2. Za konstrukcijo na sliki izračunajte stopnjo statične nedoločenosti, reakcije in notranje statične količine (N_x, N_z, M_y)! Rezultate notranjih statičnih količin prikažite z diagrami!

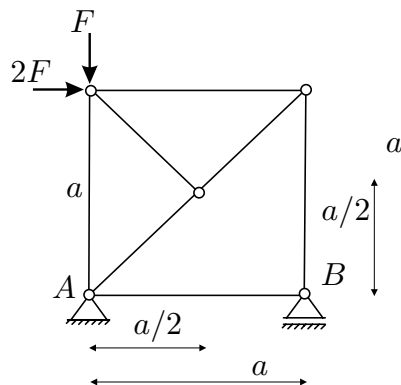


Podatki: $a = 2 \text{ m}$,

$q = 5 \text{ kN/m}$.

(OBVEZNA NALOGA! 50%)

3. Za palično konstrukcijo na sliki izračunajte stopnjo statične nedoločenosti in osne sile v vseh palicah! (35%)



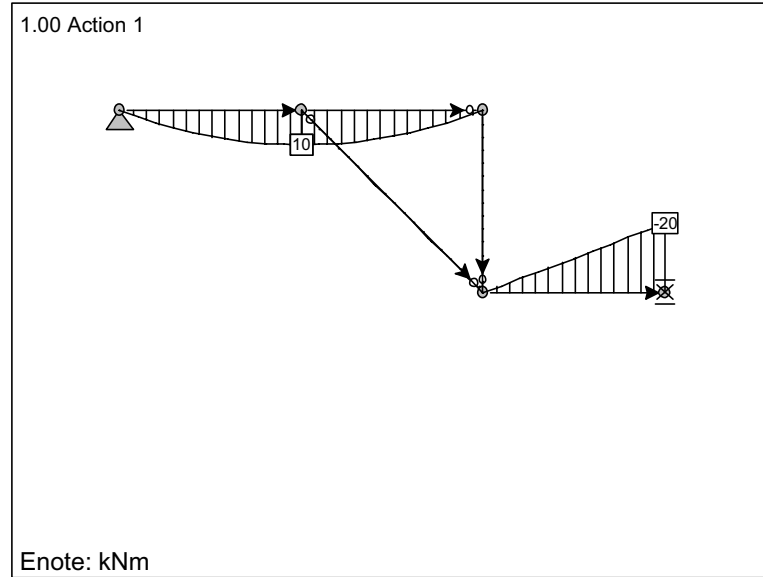
Podatki: $a = 4 \text{ m}$, $F = 10 \text{ kN}$.

TEORETIČNI DEL IZPITA:

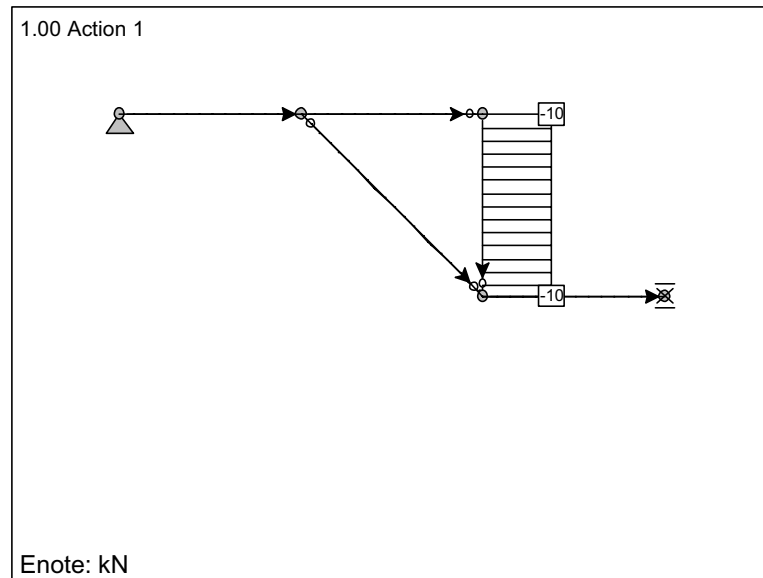
Izmed treh zastavljenih vprašanj si izberete dve, na kateri boste odgovarjali. Izbrani vprašanji jasno označite! Pišite čitljivo.

1. Vzporedna prestavitev sile! Kako lahko nadomestimo silo in moment, ki sta med seboj pravokotna? Dokaz!
2. Izpeljite in opišite nadomestne ravnotežne pogoje!
3. Opišite določanje reakcij in notranjih sil statično določenih linijskih konstrukcij z izrekom o virtualnih pomikih (pomagajte si s primerom)

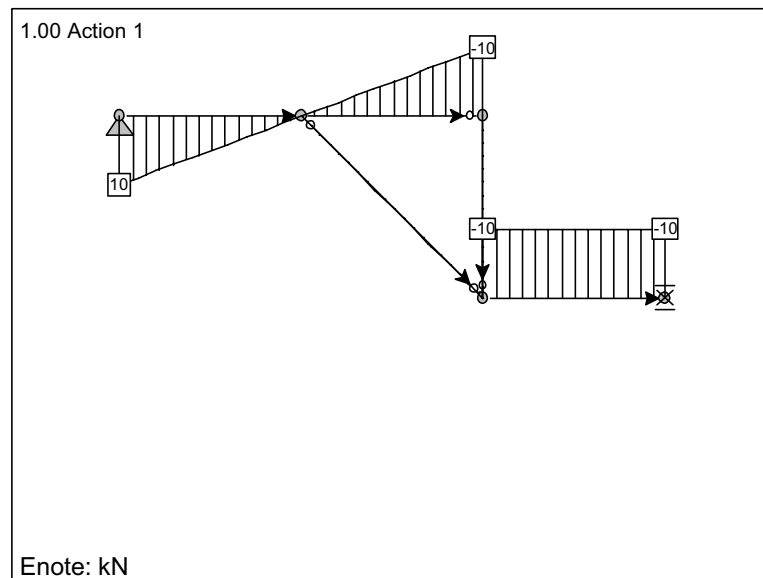
LC1: Load case 2: Upogibni moment My

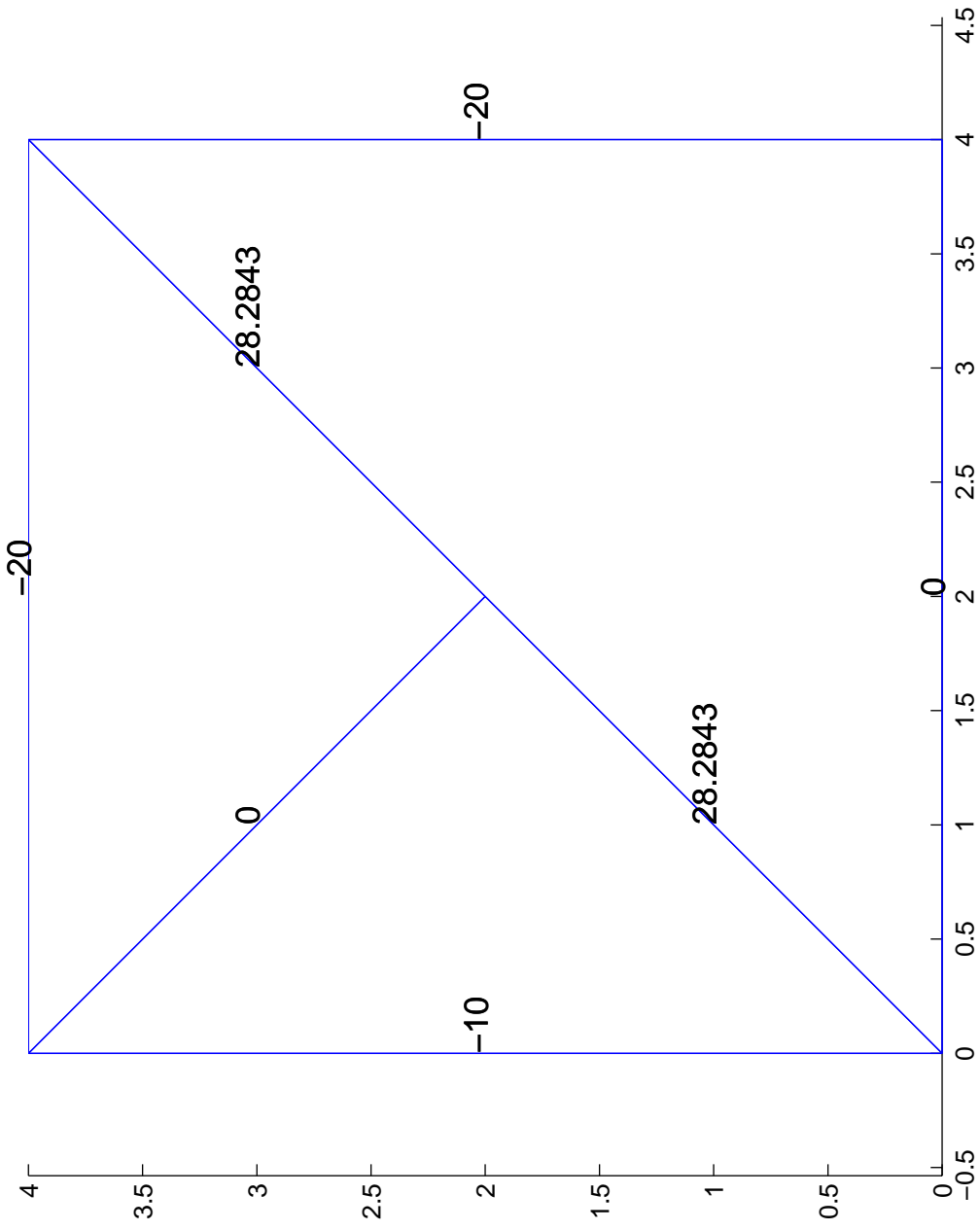


LC1: Load case 2: Osna sila Fx



LC1: Load case 2: Preèna sila Fz

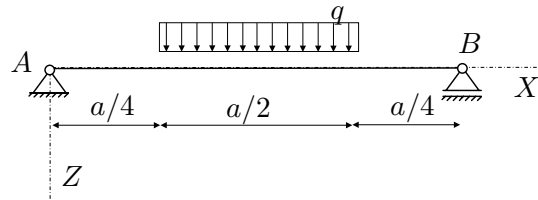




STATIKA (VSŠ) - 1. IZREDNI IZPITNI ROK (14. 3. 2005)

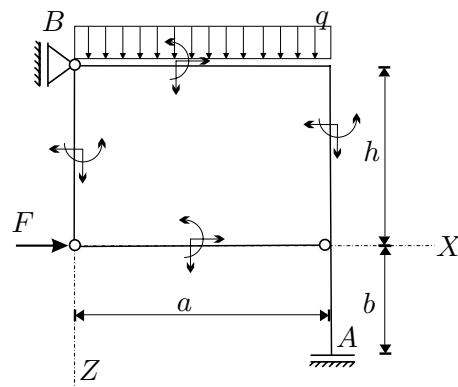
RAČUNSKI DEL IZPITA:

1. Za nosilec na sliki izračunajte reakcije v vezeh ter izračunajte in prikažite diagrame notranjih statičnih količin! (OBVEZNA NALOGA! 20%)



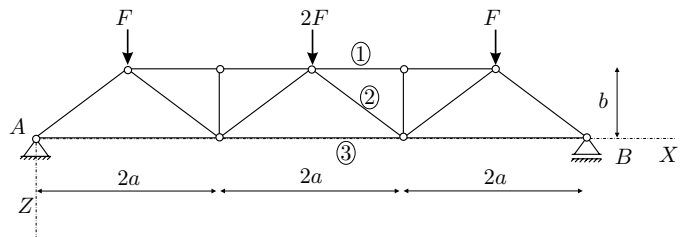
2. Za konstrukcijo na sliki izračunajte stopnjo statične nedoločnosti, reakcije in notranje statične količine (N_x, N_z, M_y)! Rezultate notranjih statičnih količin prikažite z diagrami!

Podatki: $a = 4 \text{ m}$, $h = 3 \text{ m}$, $b = 2 \text{ m}$,
 $q = 16 \text{ kN/m}$, $F = 10 \text{ kN}$.
 (OBVEZNA NALOGA! 50%)



3. Za palično konstrukcijo na sliki izračunajte stopnjo statične nedoločnosti in osne sile v palicah 1, 2 in 3! (30%)

Podatki: $a = 2 \text{ m}$, $b = 1.5 \text{ m}$,
 $F = 5 \text{ kN}$.

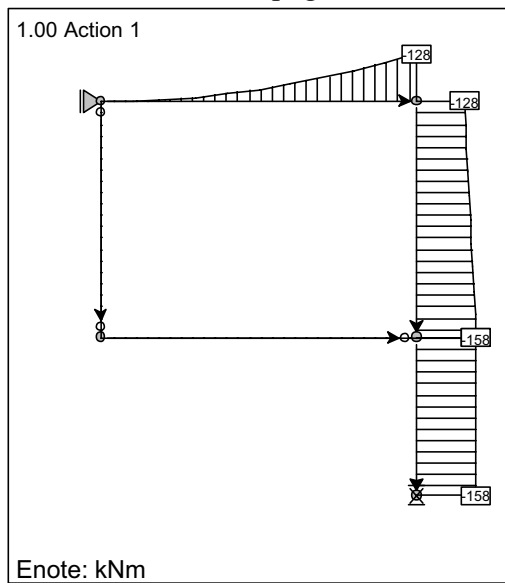


TEORETIČNI DEL IZPITA:

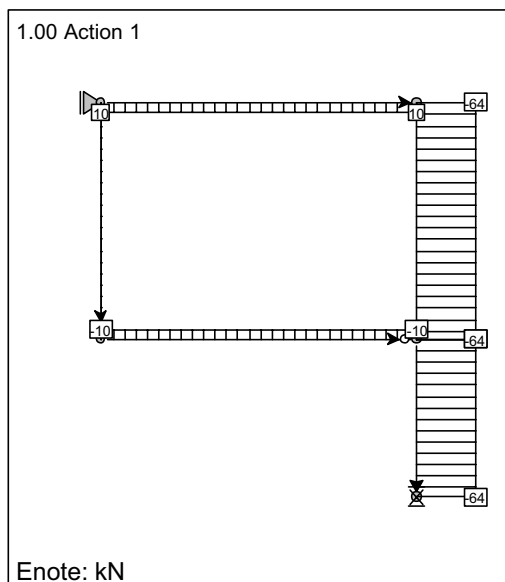
Izmed treh zastavljenih vprašanj si izberete dve, na kateri boste odgovarjali. Izbrani vprašanji jasno označite! Pišite čitljivo.

1. Izpeljite in opišite obe obliki nadomestnih ravnotežnih pogojev (rezultate ilustrirajte na prostoležečem nosilcu s točkovno silo na sredini razpona)!
2. Izpeljite in opišite izraz za število odvzetih prostostnih stopenj, ki jih vez odvzame k nepovezanim telesom! Obravnavaj tudi primer, ko imajo vsa telesa na mestu vezi enake nekatere kinematične količine, preostale količine pa so možne za vsa telesa!
3. Opišite določanje reakcij in notranjih sil statično določenih linijskih konstrukcij z izrekom o virtualnih pomikih (ilustracija s primerom)!

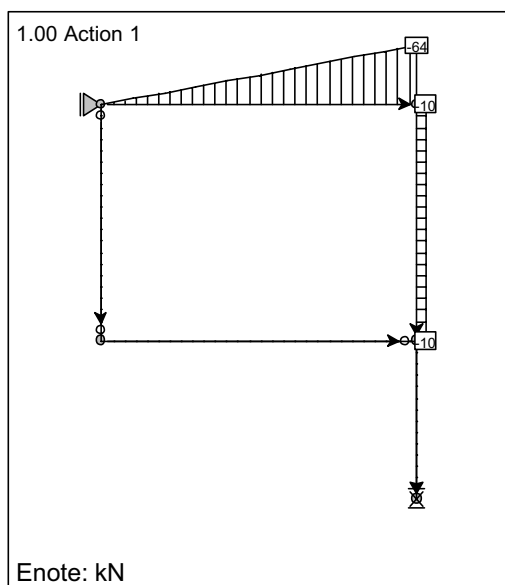
LC1: Load case 2: Upogibni moment My

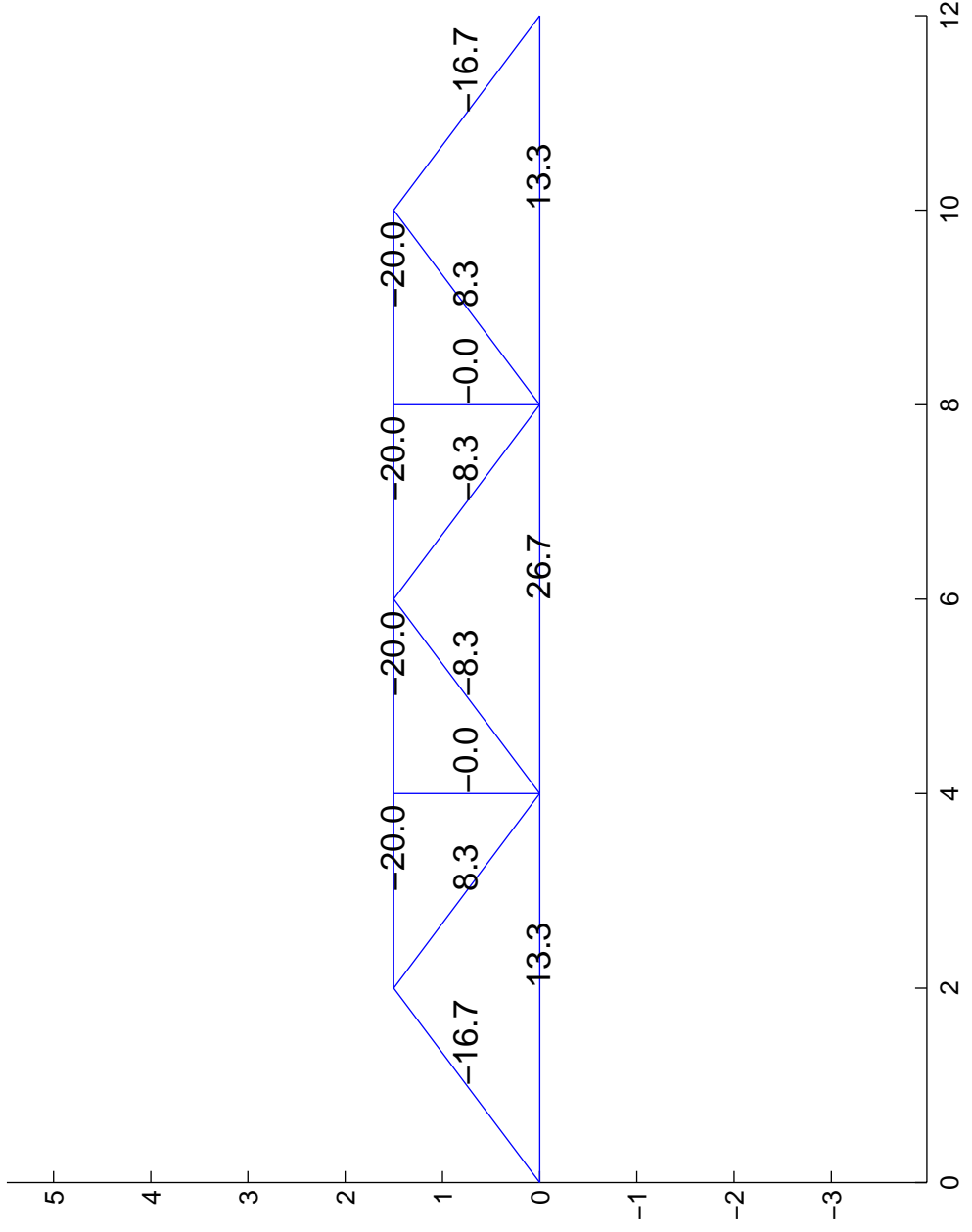


LC1: Load case 2: Osna sila Fx



LC1: Load case 2: Preèna sila Fz



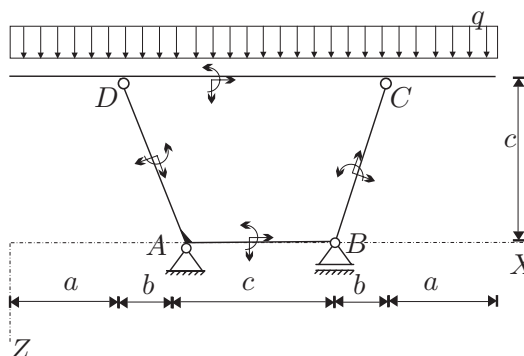


STATIKA (VSŠ) - 1. IZPITNI ROK (10. 6. 2005)

RAČUNSKI DEL IZPITA:

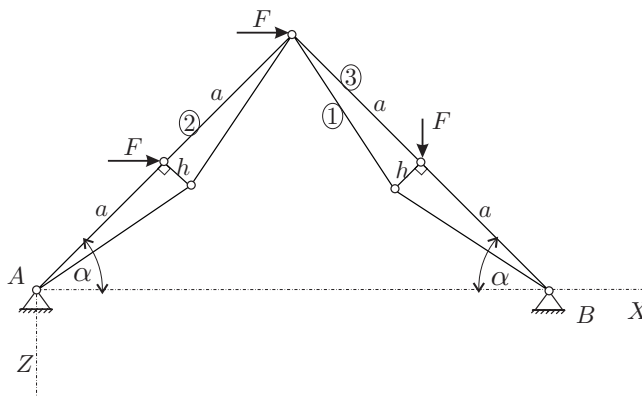
1. Za konstrukcijo na sliki izračunajte stopnjo statične nedoločenosti, reakcije in notranje statične količine (N_x, N_z, M_y)! Rezultate notranjih statičnih količin prikažite z diagrami!

Podatki: $a = 2\text{ m}, b = 1\text{ m}, c = 3\text{ m},$
 $q = 5\text{ kN/m}.$
 (OBVEZNA NALOGA! 40%)



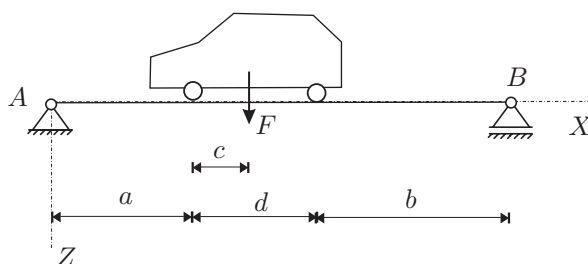
2. Za palično konstrukcijo na sliki izračunajte stopnjo statične nedoločenosti in osne sile v palicah 1, 2 in 3! (30%)

Podatki: $a = 4\text{ m}, h = 0.5\text{ m},$
 $\alpha = 45^\circ, F = 5\text{ kN}.$



3. Izračunajte in narišite diagram upogibnih momentov v mostu na sliki! Most (prostoležeči nosilec) je obremenjen le s težo F vozila, ki ima prijemališče na razdalji c od prve osi. (30%)

Podatki: $a = 20\text{ m}, b = 7.5\text{ m},$
 $c = 1\text{ m}, d = 2.5\text{ m}, F = 15\text{ kN}.$



TEORETIČNI DEL IZPITA:

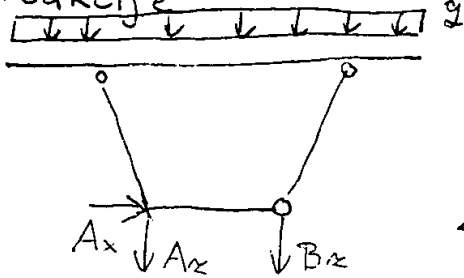
Izmed treh zastavljenih vprašanj si izberete dve, na kateri boste odgovarjali. Izbrani vprašanji jasno označite! Pišite čitljivo.

1. Ravnotežni par sil in dvojica sil!
2. Račun reakcij in sil v vezeh pri statično določenih linijskih konstrukcijah (pomagajte si s primerom)!
3. Definicija virtualnega pomika! Definicijo ilustrirajte s primerom!

1. NALOGA

(i) $\tilde{m}_P = 3 \cdot 3 - 1 - 2 - 3 \cdot 2 = 0$

(ii) reakcije



$\Sigma X: A_x = 0$

$\Sigma Z: A_z + B_z + q(2a + 2b + c) = 0$

$\Sigma M^A: -B_z \cdot c - q \frac{(a+b+c)^2}{2} + q \frac{(a+b)^2}{2} = 0$

$B_z = \frac{1}{3} \left(\frac{5}{2} (3)^2 - \frac{5}{2} 6^2 \right)$

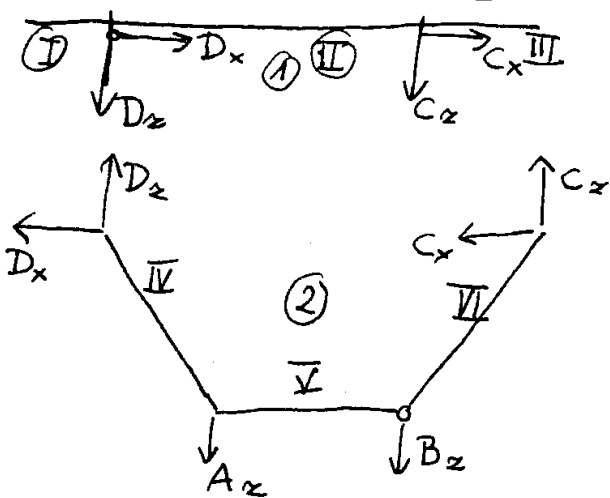
$= -\frac{5}{6} (36 - 9)$

$B_z = -22.5 \text{ kN}$

$A_z = -5(4 + 2 + 3) + 22.5$

$A_z = -22.5 \text{ kN}$

(iii) razrez



②:

$\Sigma X: C_x + D_x = 0$

$\Sigma Z: C_z + D_z = A_z + B_z$

$\Sigma M^D: -A_z \cdot 3 - B_z \cdot 6 + C_z \cdot 9 = 0$

$C_z = \frac{1}{9} (A_z \cdot 3 + B_z \cdot 6)$

$C_z = -22.5 \text{ kN}$

$D_z = -22.5 \text{ kN}$

$\Sigma M^B_{BC}: C_x \cdot 3 + C_z \cdot 1 = 0$

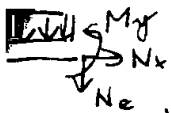
$C_x = -C_z \cdot \frac{1}{3}$

$C_x = +7.5 \text{ kN}$

$D_x = -7.5 \text{ kN}$

iv) notranje sile

polje I



$$N_x = 0$$

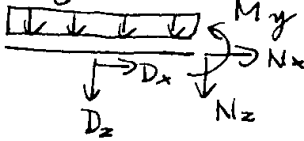
$$N_z = -5x$$

$$M_y = -\frac{g x^2}{2} = -\frac{5x^2}{2}$$

$$N_x(2) = -10$$

$$M_y(2) = -10$$

polje II



$$N_x = -D_x$$

$$N_z = -g(x+2) - D_z$$

$$M_y = -g \frac{(x+2)^2}{2} - D_z \cdot x$$

$$N_x = 7.5 \text{ kN}$$

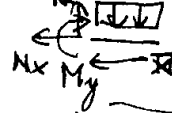
$$N_z = 12.5 - 5x$$

$$M_y = -10 + 12.5x - \frac{5}{2}x^2$$

ekstrem $x = \frac{12.5}{5} = 2.5$ $M_y(5) = 0$

$$M_y(2.5) = 5.625$$

polje III

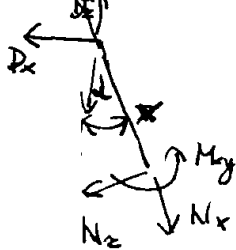


$$N_x = 0$$

$$N_z = g x = 5x$$

$$M_y = -\frac{g x^2}{2} = -\frac{5}{2}x^2$$

polje IV



$$\sum x: +N_x - D_z \cos \alpha - D_x \sin \alpha = 0$$

$$\sum z: +N_z - D_z \sin \alpha + D_x \cos \alpha = 0$$

$$\sum M^I: -M_y - D_x x \cos \alpha + D_z x \sin \alpha = 0$$

$$\tan \alpha = \frac{1}{3}$$

$$\alpha = 18.435^\circ$$

$$x = \sqrt{1+9} = \sqrt{10}$$

$$N_x = -7.5 \sin \alpha - 22.5 \cos \alpha$$

$$N_z = 7.5 \cos \alpha - 22.5 \sin \alpha$$

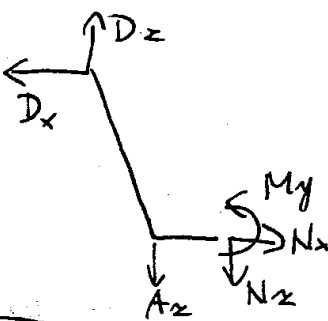
$$M_y = N_z \cdot x$$

$$N_x = -23.7 \text{ kN}$$

$$N_z = 0$$

$$M_y = 0$$

polje V



$$N_x = D_x = -7.5 \text{ kN}$$

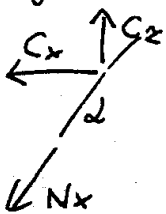
$$N_z = D_z - A_z = 0$$

$$M_y + A_z \cdot x - D_z \cdot (x+1) + 3D_x = 0$$

$$M_y = 3 \cdot 7.5 - 22.5 = 0$$

$$M_y = 0$$

polje VI



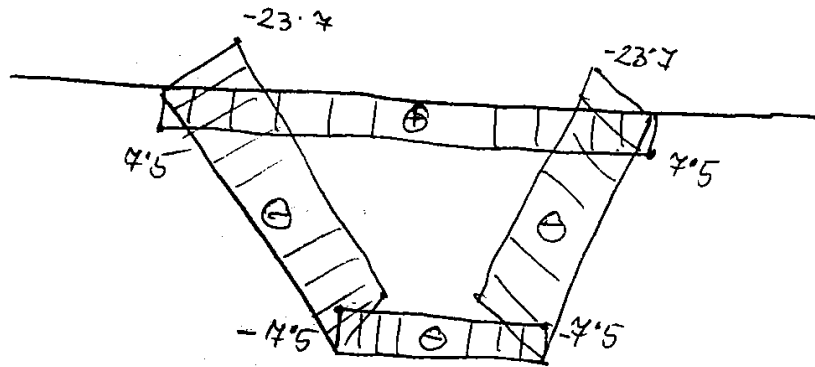
$$N_x = C_x \cos \alpha - C_z \sin \alpha$$

$$= -22.5 \cos \alpha - 7.5 \sin \alpha$$

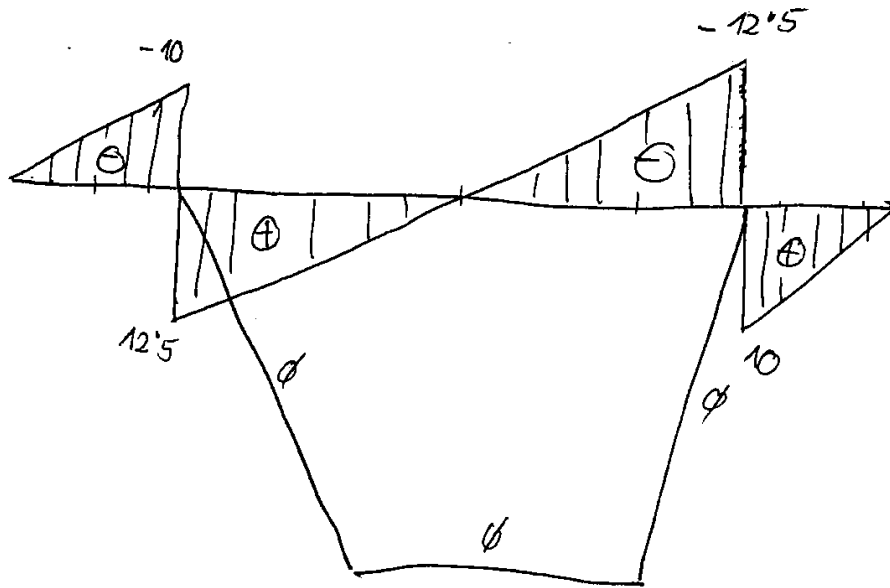
$$N_x = -23.7 \text{ kN}$$

$$N_z = M_y = 0$$

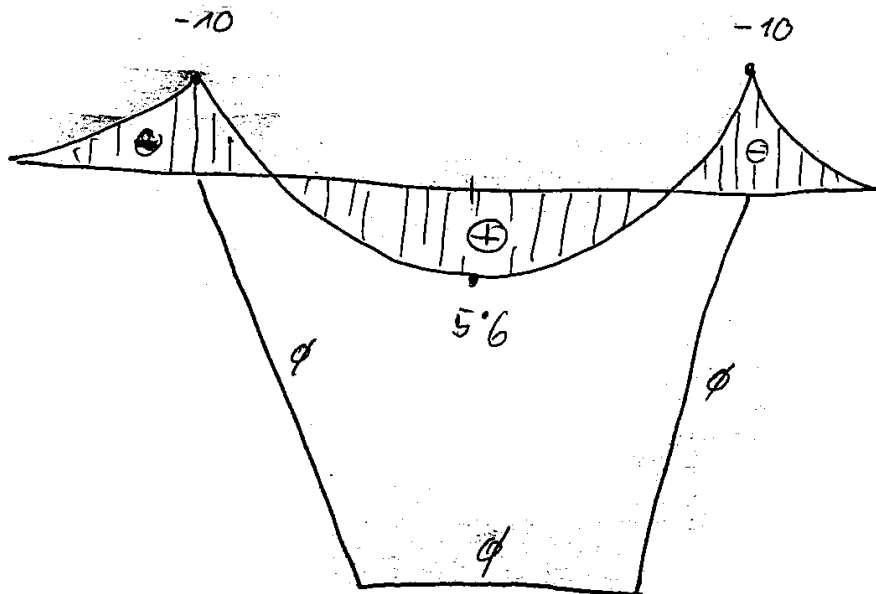
~) diagrammi



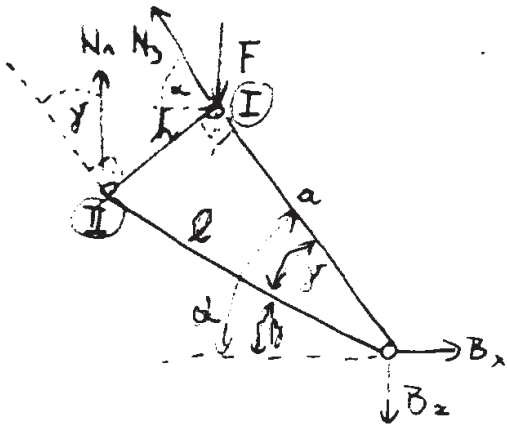
[N_z]



[M_y]



iv.) razrezi 1-3 2. NAČIN



$$\Sigma M^I: -N_1 \cos \gamma \cdot h - B_z \cdot a \cos \alpha + B_x \cdot a \sin \alpha = 0$$

$$N_1 = \frac{a}{h \cos \gamma} (B_x \sin \alpha - B_z \cos \alpha)$$

$$N_1 = \frac{4}{0.5 \cos \gamma} \left(-F \cdot \frac{\sqrt{2}}{2} + \frac{3}{2} F \frac{\sqrt{2}}{2} \right)$$

$$N_1 = 14.25 \text{ kN}$$

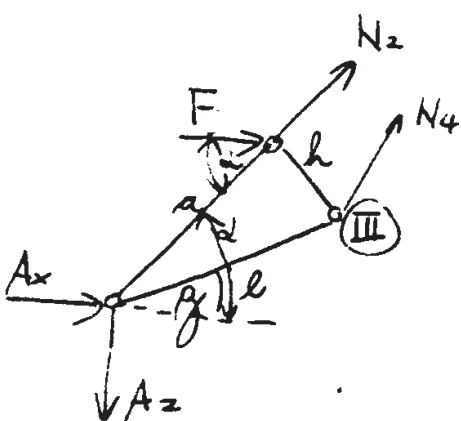
$$\Sigma M^{II}: N_3 \cdot h - F \cdot h \sin \alpha - B_z \cdot l \cos \beta + B_x \cdot l \sin \beta = 0$$

$$N_3 = \frac{1}{h} (F \cdot h \sin \alpha + B_z \cdot l \cos \beta - B_x \cdot l \sin \beta)$$

$$N_3 = \frac{1}{0.5} \left(5 \cdot 0.5 \frac{\sqrt{2}}{2} - \frac{3}{2} \cdot F \cdot l \cos \beta + F \cdot l \sin \beta \right)$$

$$N_3 = -19.4 \text{ kN}$$

v.) rez (2)-(4)



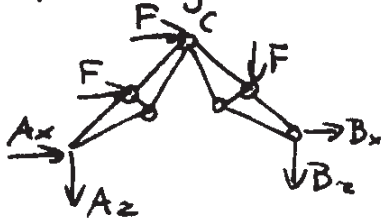
$$\Sigma M^{III}: -N_2 \cdot h - F \cdot \cos \alpha \cdot h + A_z \cdot l \cos \beta + A_x \cdot l \sin \beta = 0$$

$$N_2 = \frac{1}{h} (-F \cdot h \cos \alpha + A_z \cdot l \cos \beta + A_x \cdot l \sin \beta) = 0$$

$$N_2 = -12.4 \text{ kN}$$

(i) $\tilde{m}_{ps} = 10 - 25 = 0$

(ii) reakcije



$$\sum X: A_x + B_x = -2F$$

$$\sum Z: A_z + B_z = -F$$

$$\sum M^A: -B_z \cdot 4a \frac{\sqrt{2}}{2} - F \cdot a \frac{\sqrt{2}}{2} - F \cdot 2a \frac{\sqrt{2}}{2} - F \cdot 3a \frac{\sqrt{2}}{2} = 0$$

$$4B_z = -6F$$

$$B_z = -\frac{3}{2}F = -4.5 \text{ kN}$$

$$A_z = \frac{1}{2}F = 2.5 \text{ kN}$$

$$\sum M_{BC}^C: -F \cdot a \frac{\sqrt{2}}{2} - B_z \cdot 2a \frac{\sqrt{2}}{2} + B_x \cdot 2a \frac{\sqrt{2}}{2} = 0$$

$$B_x = \frac{1}{2}(2B_z + F) = -\frac{3}{2}F + \frac{F}{2}$$

$$B_x = -F = -5 \text{ kN}$$

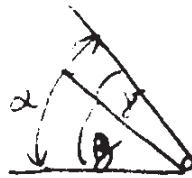
$$A_x = -F = -5 \text{ kN}$$

(iii) geometrija

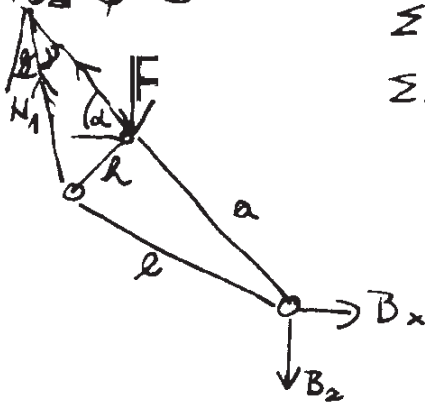
$$l = \sqrt{a^2 + h^2} = \sqrt{16 + 0.25} = 4.03$$

$$\gamma = \arctan \frac{h}{a} = \arctan \frac{0.5}{4} = 7.1^\circ$$

$$\beta = \frac{\pi}{4} - \gamma = 37.875^\circ$$



(iv) rez 1-3



$$\sum X: N_1 \sin \beta + N_3 \cos \beta = B_x \cdot \frac{1}{\cos \beta}$$

$$\sum Z: N_1 \cos \beta + N_3 \sin \beta = B_z + F \sin \beta$$

$$N_3 (\cos \beta \cos \beta - \sin \beta \sin \beta) = B_x \cos \beta - (B_z + F) \sin \beta$$

$$N_3 = \frac{-5 \cos \beta + \frac{1}{2} 5 \sin \beta}{\frac{\sqrt{2}}{2} (\cos \beta - \sin \beta)}$$

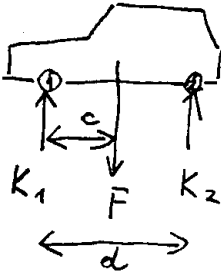
$$N_3 = -19.4 \text{ kN}$$

$$N_1 = \frac{B_x - N_3 \cdot \frac{\sqrt{2}}{2}}{\sin \beta}$$

$$N_1 = 14.2 \text{ kN}$$

3. NALOGA

i.) vozilo



$$\sum Z: K_1 + K_2 = F$$

$$\sum M^{\odot} -F \cdot c + K_2 \cdot d = 0$$

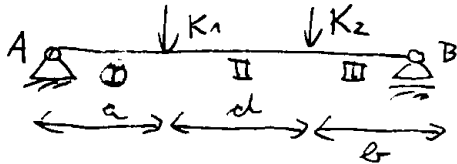
$$K_2 = F \cdot \frac{c}{d}$$

$$K_1 = F \left(1 - \frac{c}{d}\right)$$

$$K_2 = \frac{15}{25} = 6 \text{ kN}$$

$$K_1 = 9 \text{ kN}$$

(ii) nosilec



$$\sum X: A_x = 0$$

$$\sum Z: A_z + B_z + K_1 + K_2 = 0$$

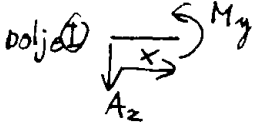
$$\sum M^{\odot}: -K_1 a - K_2 (a+d) - B_z (a+b+d) = 0$$

$$B_z \cdot 30 = -9 \cdot 20 - 6 \cdot 22.5$$

$$B_z = -10.5 \text{ kN}$$

$$A_z = -4.5 \text{ kN}$$

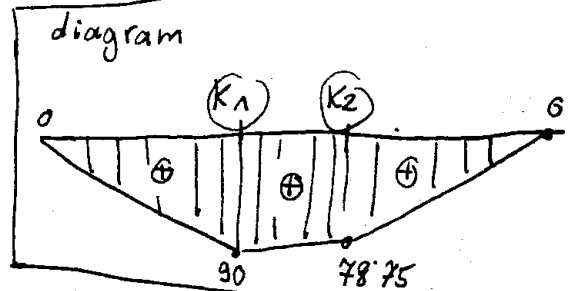
(iii) momenti



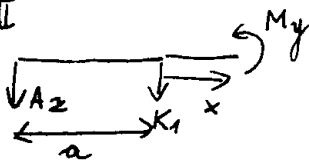
$$M_y = -A_z x$$

$$M_y = 4.5 x$$

$$M_y(20) = 90 \text{ kNm}$$



polje II

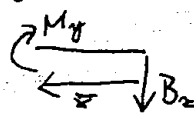


$$M_y = -A_z x - A_z a - K_1 x$$

$$M_y = 90 - 4.5 x$$

$$M_y(25) = 78.75 \text{ kN}$$

polje III



$$M_y = -B_z x$$

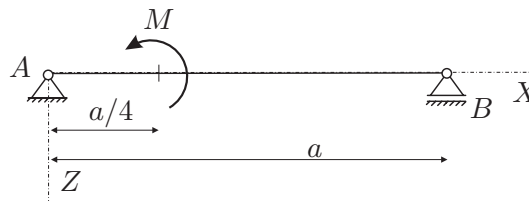
$$M_y = 10.5 x$$

$$M_y(25) = 78.75 \text{ kN}$$

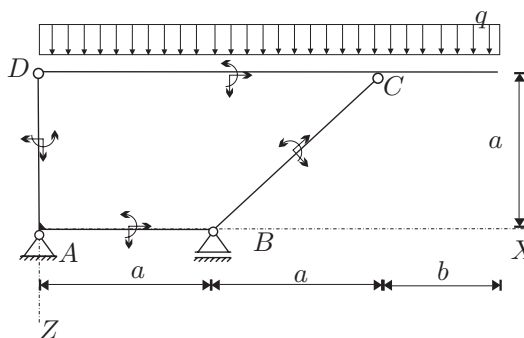
STATIKA (VSŠ) - 2. IZPITNI ROK (27. 6. 2005)

RAČUNSKI DEL IZPITA:

1. Za nosilec na sliki izračunajte reakcije v podporah ter izračunajte in prikažite diagrame notranjih statičnih količin! (OBVEZNA NALOGA! 20%)

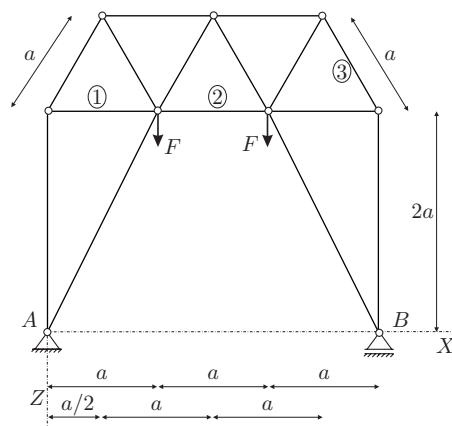


2. Za konstrukcijo na sliki izračunajte stopnjo statične nedoločnosti, reakcije in notranje statične količine (N_x, N_z, M_y)! Rezultate notranjih statičnih količin prikažite z diagrami!



Podatki: $a = 3\text{ m}$, $b = 2\text{ m}$,
 $q = 2\text{ kN/m}$.
 (OBVEZNA NALOGA! 50%)

3. Za palično konstrukcijo na sliki izračunajte stopnjo statične nedoločnosti in osne sile v palicah 1, 2 in 3! (30%)



Podatki: $a = 2\text{ m}$, $F = 10\text{ kN}$.

TEORETIČNI DEL IZPITA:

Izmed treh zastavljenih vprašanj si izberete dve, na kateri boste odgovarjali. Izbrani vprašanji jasno označite! Pišite čitljivo.

1. Rezultanta sil in rezultanta momentov. Dokaži, da smernica rezultante sil poteka skozi točko, na katero računamo rezultanto momentov! Kdaj sta dva sistema sil statično enakovredna?
2. Izpeljite ravnotežne pogoje za linijski element z ravno osjo!
3. Opišite določanje reakcij in notranjih sil statično določenih linijskih konstrukcij z izrekom o virtualnih pomikih (ilustracija s primerom)!

STATIKA - VSŠ

27.6.2005

1. NALOGA

a.) reakcije



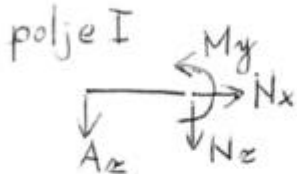
$$\sum X: A_x = 0$$

$$\sum Z: A_z + B_z = 0$$

$$\sum M^A: M - B_z \cdot a = 0$$

$$B_z = \frac{M}{a} \quad A_z = -\frac{M}{a}$$

b.) notranje sile



$$N_x = 0$$

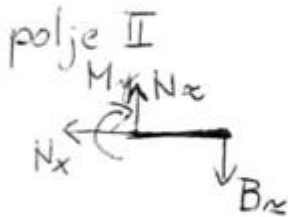
$$N_z = -A_z$$

$$N_z = \frac{M}{a}$$

$$M_y = -A_z x$$

$$M_y = -\frac{M}{a} x$$

$$M_y\left(\frac{a}{4}\right) = \frac{M}{4}$$



$$N_x = 0$$

$$N_z = B_z$$

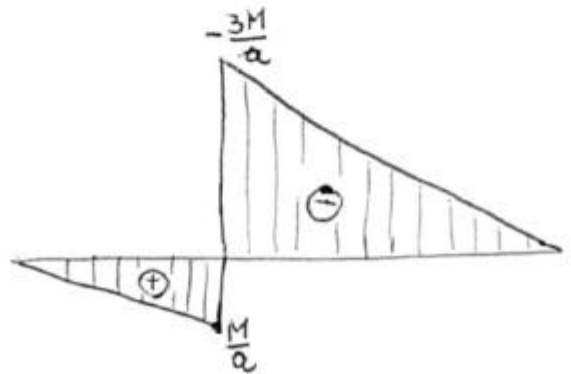
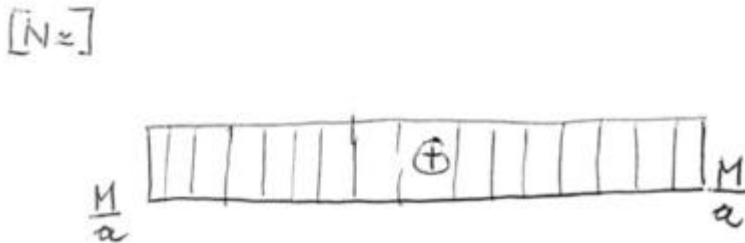
$$N_z = \frac{M}{a}$$

$$M_y = -B_z x$$

$$M_y = -\frac{M}{a} x$$

$$M_y\left(\frac{3a}{4}\right) = -\frac{3M}{4}$$

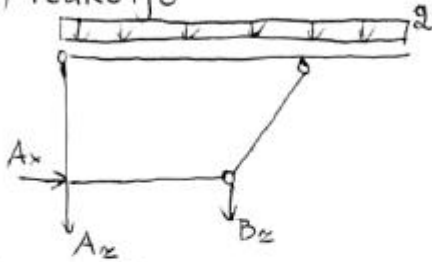
c.) diagrami



2. NALOGA

a.) $\tilde{n}_{ps} = 3 \cdot 3 - 2 - 1 - 3 \cdot 2 = 0 \checkmark$

b.) reakcije



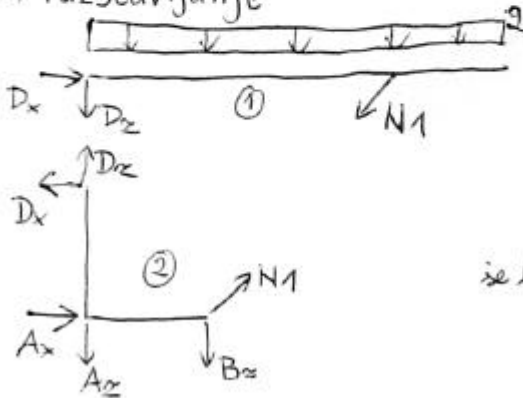
$\sum X: A_x = 0$

$\sum Z: A_z + B_z + q \cdot (2a + b) = 0$

$\sum M^A: -B_z \cdot a - q \cdot (2a + b) \cdot \frac{2a + b}{2} = 0$

$B_z = -21\sqrt{3} \text{ kN}$
$A_z = +5\sqrt{3} \text{ kN}$

c.) razstavljanje



②: $\sum X: A_x - D_x + N_1 \cdot \frac{\sqrt{2}}{2} = 0$

$\sum Z: A_z + B_z - D_z - N_1 \cdot \frac{\sqrt{2}}{2} = 0$

$\sum M^A: -B_z \cdot a + D_x \cdot a + N_1 \cdot \frac{\sqrt{2}}{2} \cdot a = 0$ ← za kontrolo

ie bolje $\sum M^D: A_x \cdot a - B_z \cdot a + N_1 \cdot \frac{\sqrt{2}}{2} \cdot a + N_1 \cdot \frac{\sqrt{2}}{2} \cdot a = 0$

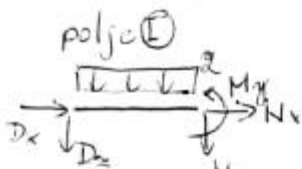
$N_1 \sqrt{2} = B_z - A_x$

$N_1 = -15.08 \text{ kN}$

$D_x = -10.6 \text{ kN}$

$D_z = -5.3 \text{ kN}$

d.) notranje sile



$N_x = -D_x \Rightarrow N_x = 10.7 \text{ kN}$

$N_z = -D_z - q \cdot x$

$N_z = 5.3 - 2x \quad N_z(6) = -6.7 \text{ kN}$

$M_y = 5.3x - x^2 \quad M_y(6) = -4 \text{ kNm}$

$x = \frac{5.3}{2}$ (ekstrem) $M_y(\frac{5.3}{2}) = 7 \text{ kNm}$



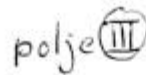
$N_x = 0$

$N_z = q \cdot x$

$M_y = -q \cdot x \cdot \frac{x}{2}$

$N_z = 2x \quad N_z(2) = 4 \text{ kN}$

$M_y = -x^2 \quad M_y(2) = -4 \text{ kNm}$



$N_x = D_z$

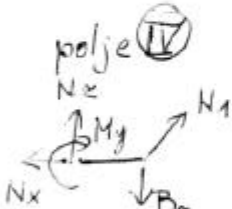
$N_z = -D_x$

$M_y = -D_x \cdot x$

$N_x = -5.3 \text{ kN}$

$N_z = 10.6 \text{ kN}$

$M_y = 10.7x \quad M_y(3) = 32$

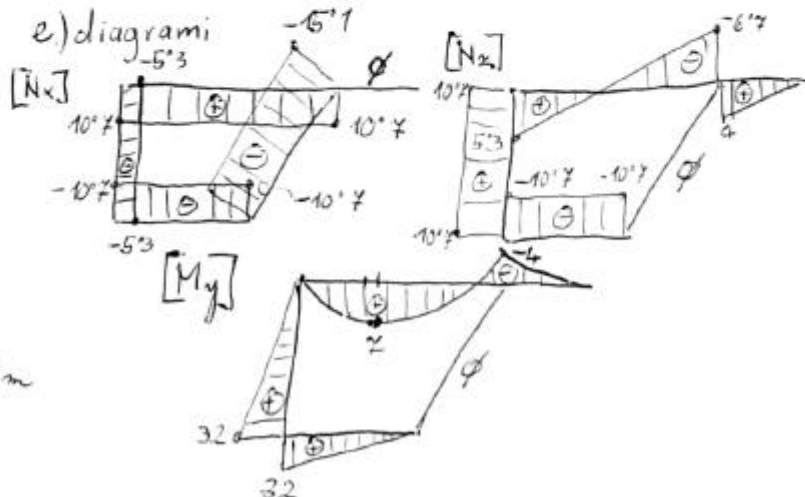


$N_x = N_1 \cdot \frac{\sqrt{2}}{2} = -10.7 \text{ kN}$

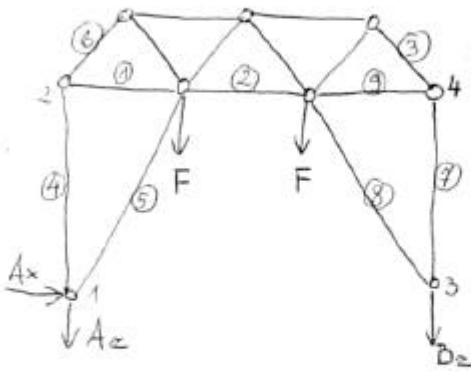
$N_z = B_z - N_1 \cdot \frac{\sqrt{2}}{2} = -10.7 \text{ kN}$

$M_y = 10.7x \quad M_y(3) = 32 \text{ kNm}$

e.) diagrami



3. NALOGA



reakcije

$$\Sigma X: A_x = 0$$

$$\Sigma Z: A_z + B_z = -2F$$

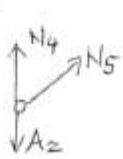
$$\Sigma M^A: -F \cdot a - F \cdot 2a - B_z \cdot 3a = 0$$

$$A_z = -10 \text{ kN} \quad B_z = -10 \text{ kN}$$

$$\begin{matrix} A_z = -F \\ B_z = -F \end{matrix}$$

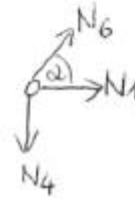
Prezovanje vozlišč

1



$$\begin{matrix} N_5 = 0 \\ N_4 = -10 \text{ kN} \end{matrix}$$

2



$$\Sigma X: N_1 + N_6 \cos \alpha = 0$$

$$\Sigma Z: N_4 - N_6 \sin \alpha = 0$$

$$N_1 = -N_6 \cot \alpha$$

$$N_1 = 5.8 \text{ kN}$$



$$\begin{matrix} N_7 = -10 \text{ kN} \\ N_8 = 0 \end{matrix}$$

4

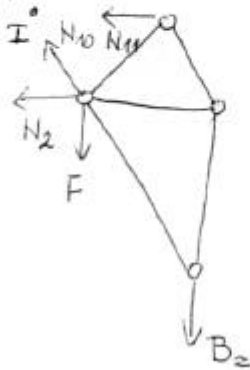


$$\Sigma Z: -N_3 \sin \alpha + N_7 = 0$$

$$N_3 = \frac{N_7}{\sin \alpha}$$

$$N_3 = -11.5 \text{ kN}$$

rez palice 2



$$\Sigma M^I: -F \frac{a}{2} - B_z \frac{3a}{2} - N_2 a \frac{\sqrt{3}}{2} = 0$$

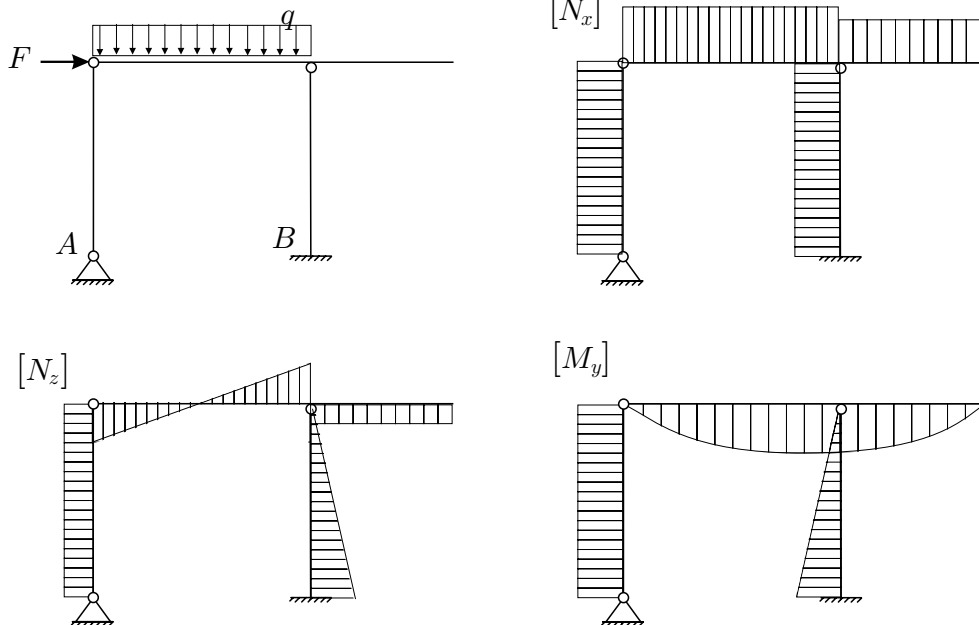
$$N_2 = \frac{1}{\sqrt{3}} (-F - 3B_z)$$

$$N_2 = 11.5 \text{ kN}$$

STATIKA (VŠŠ) - 3. IZPITNI ROK (1. 9. 2005)

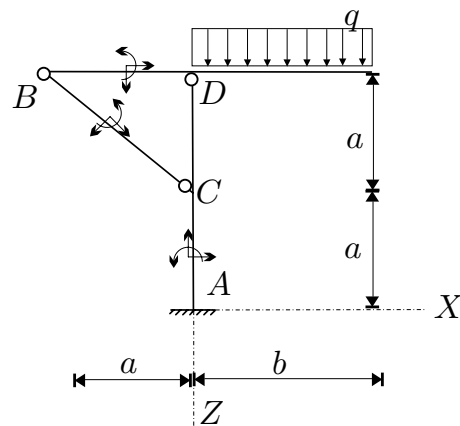
RAČUNSKI DEL IZPITA:

1. Janezek je na izpitu iz statike padel. Njegovi diagrami so polni napak. Pomagaj Janezku in poišči (BREZ RAČUNANJA) vse napake v spodnjih diagramih ! (OBVEZNA NALOGA! 25%)



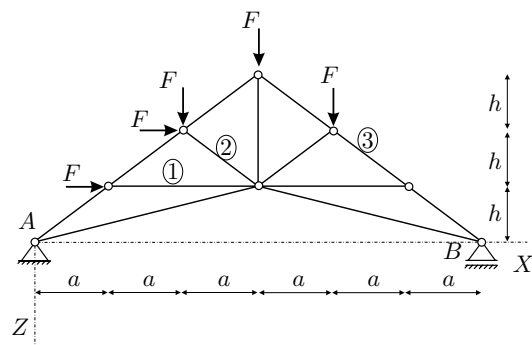
2. Za konstrukcijo na sliki izračunajte stopnjo statične nedoločenosti, reakcije in notranje statične količine (N_x, N_z, M_y)! Rezultate notranjih statičnih količin prikažite z diagrami!

Podatki: $a = 2\text{ m}$, $b = 3\text{ m}$,
 $q = 2\text{ kN/m}$.
 (OBVEZNA NALOGA! 45%)

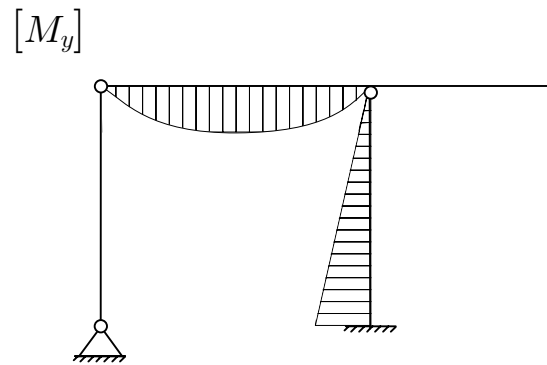
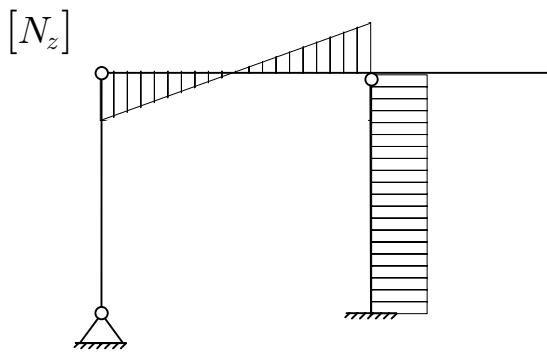
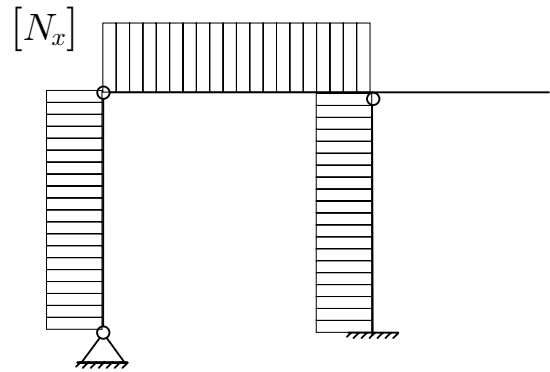
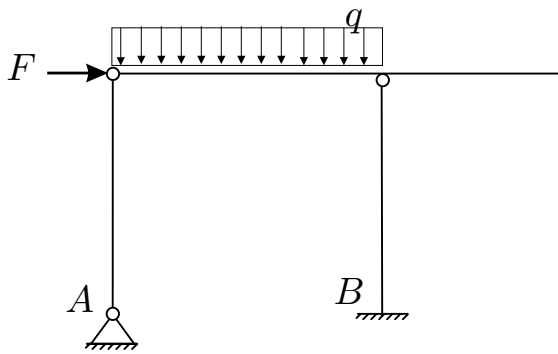


3. Za palično konstrukcijo na sliki izračunajte stopnjo statične nedoločenosti in osne sile v palicah 1, 2 in 3! (30%)

Podatki: $a = 2\text{ m}$, $h = 1.5\text{ m}$,
 $F = 5\text{ kN}$.



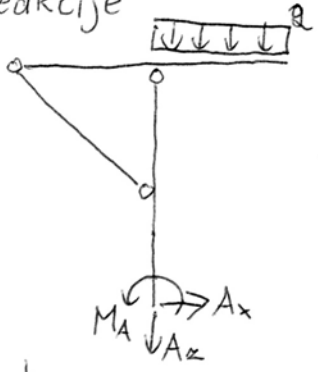
1. Naloga: PRAVILNI DIAGRAMI



2. NALOGA

a.) $\tilde{m}_{PS} = 3 \cdot 3 - 3 - 3 \cdot 2 = 0$

b.) reakcije

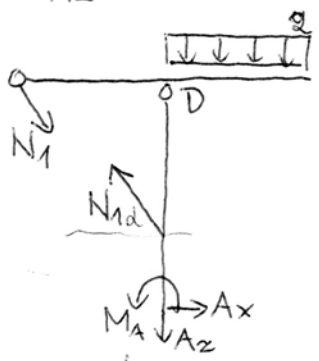


$\Sigma X: A_x = 0$

$\Sigma Z: A_z + q \cdot b = 0 \quad A_z = -6 \text{ kN}$

$\Sigma M^A: M_A - q \cdot b \cdot \frac{b}{2} = 0 \quad M_A = 9 \text{ kNm}$

c.) palica

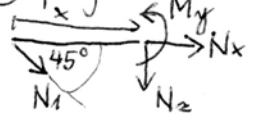


$\Sigma M_{AD}^D: M_A + A_x \cdot 2a - N_1 \cdot \frac{\sqrt{2}}{2} \cdot a = 0$

$N_1 = \frac{2}{\sqrt{2}} \frac{M_A}{a} \quad N_1 = 6.4 \text{ kN}$

d.) notranje sile

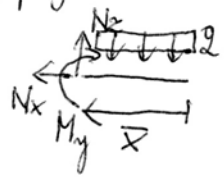
Ⓘ) polje BD



$N_x = -N_1 \frac{\sqrt{2}}{2}$
 $N_z = -N_1 \frac{\sqrt{2}}{2}$
 $M_y = -N_1 \frac{\sqrt{2}}{2} x$

$N_x = -4.5 \text{ kN} \quad N_z = -4.5 \text{ kN}$
 $M_y = -4.5 x$

Ⓜ) polje DE



$N_x = 0$
 $N_z = q \bar{x}$
 $M_y = -q \bar{x} \frac{\bar{x}}{2}$
 $N_z = 2\bar{x} \quad M_y = -\bar{x}^2$

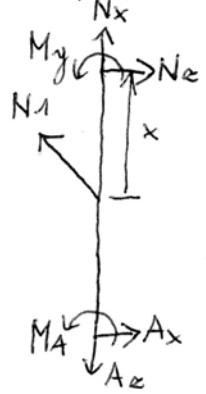
Ⓝ) polje AC



$N_x = A_z$
 $N_z = -A_x$
 $M_y = -A_x x - M_A$

$N_x = -6 \text{ kN}$
 $N_z = 0$
 $M_y = -9 \text{ kNm}$

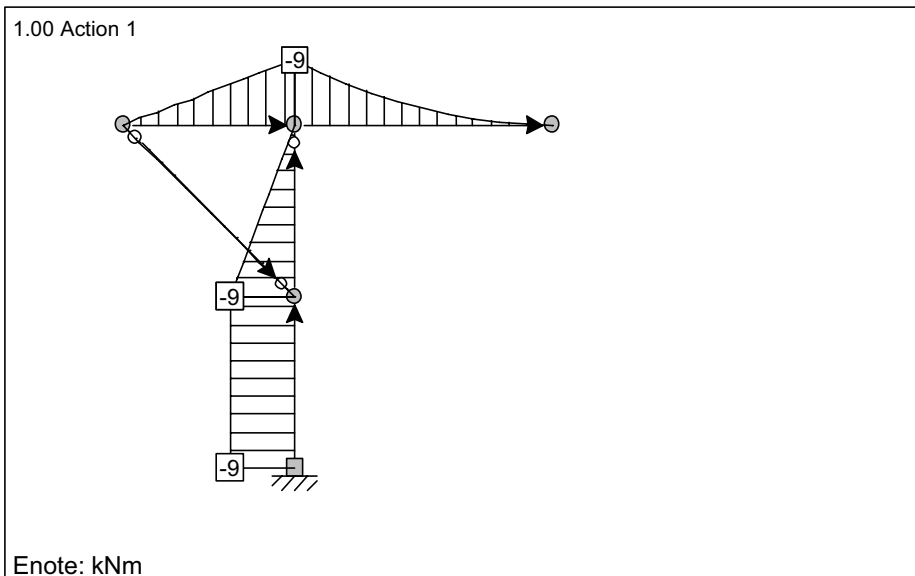
Ⓓ) polje CD



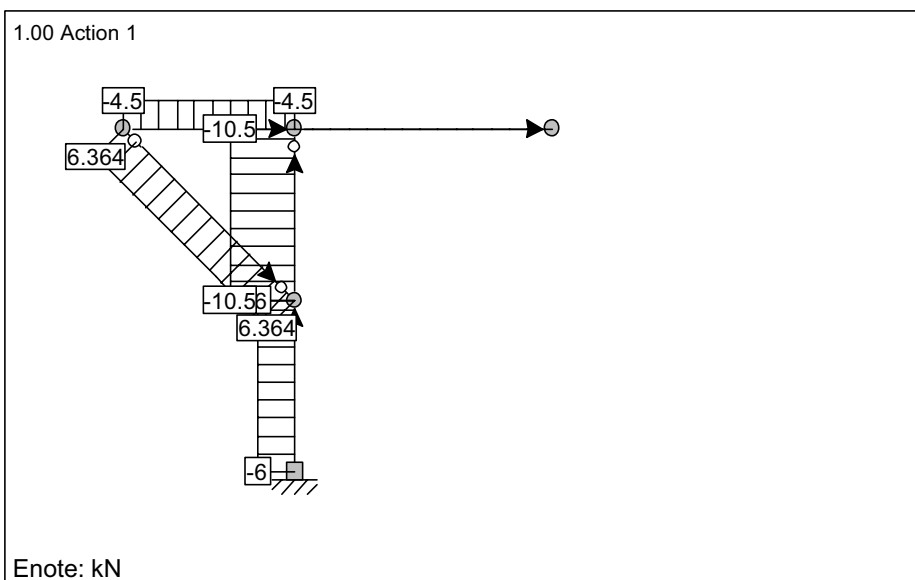
$N_x = A_z - N_1 \frac{\sqrt{2}}{2}$
 $N_z = -A_x + N_1 \frac{\sqrt{2}}{2}$
 $M_y = -M_A + N_1 \frac{\sqrt{2}}{2} x - A_x (a+x)$

$N_x = -10.5 \text{ kN}$
 $N_z = 4.5 \text{ kN}$
 $M_y = -9 + 4.5 x$

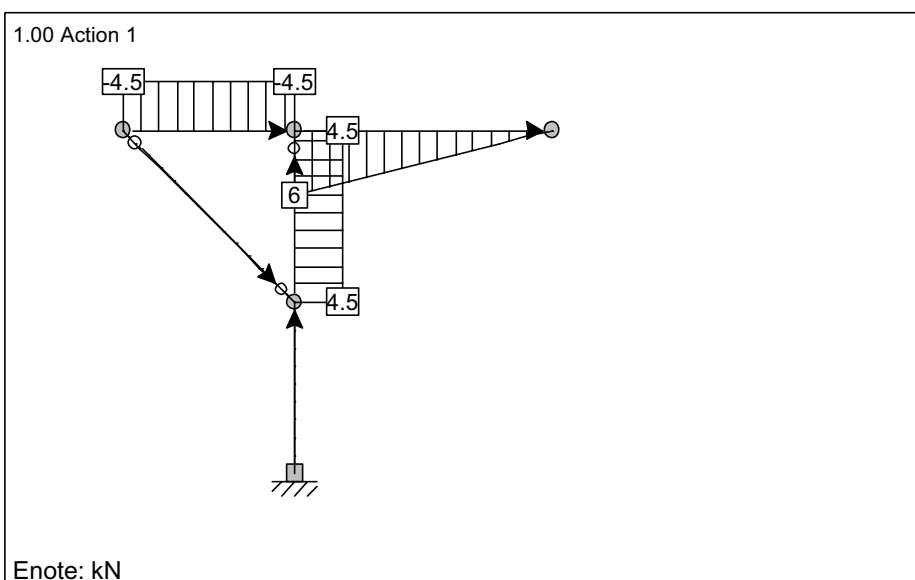
LC1: Load case 2: Upogibni moment My



LC1: Load case 2: Osna sila Fx



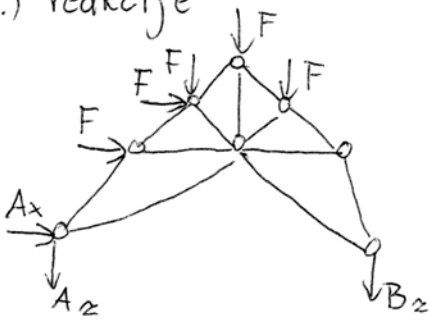
LC1: Load case 2: Preèna sila Fz



3. NALOGA

a.) $\tilde{m}_{PS} = 13 - 6 \cdot 2 - 1 = 0$

b.) reakcije



$$\sum X: A_x + 2F = 0$$

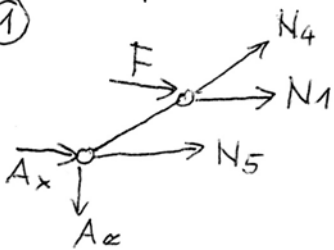
$$\sum Z: A_z + B_z + 3F = 0$$

$$\sum M^A: -B_z \cdot 6a - F \cdot 2a - F \cdot 3a - F \cdot 4a - Fh - F \cdot 2h = 0$$

$B_z = -9.375 \text{ kN}$
$A_z = -5.625 \text{ kN}$
$A_x = -10 \text{ kN}$

c.) sile v palicah

①

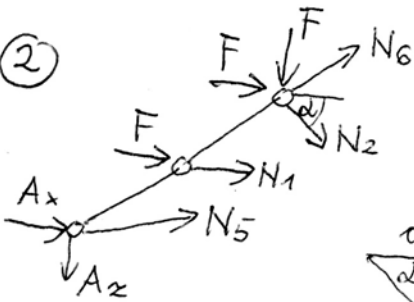


$$\sum M^A: -F \cdot h - N_1 \cdot h = 0$$

$$N_1 = -F$$

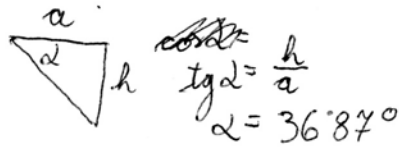
$N_1 = -5 \text{ kN}$

②



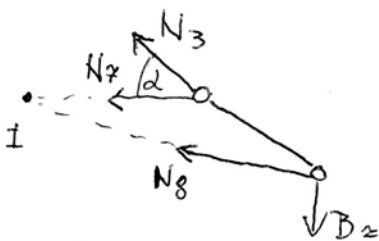
$$\sum M^A: -F \cdot h - N_1 \cdot h - F \cdot 2h - F \cdot 2a$$

$$-N_2 \cdot \cos \alpha \cdot 2h - N_2 \cdot \sin \alpha \cdot 2a = 0$$



$N_2 = -7.3 \text{ kN}$

③



$$\sum M^I: -B_z \cdot 3a + N_3 \cdot \sin \alpha \cdot 2a = 0$$

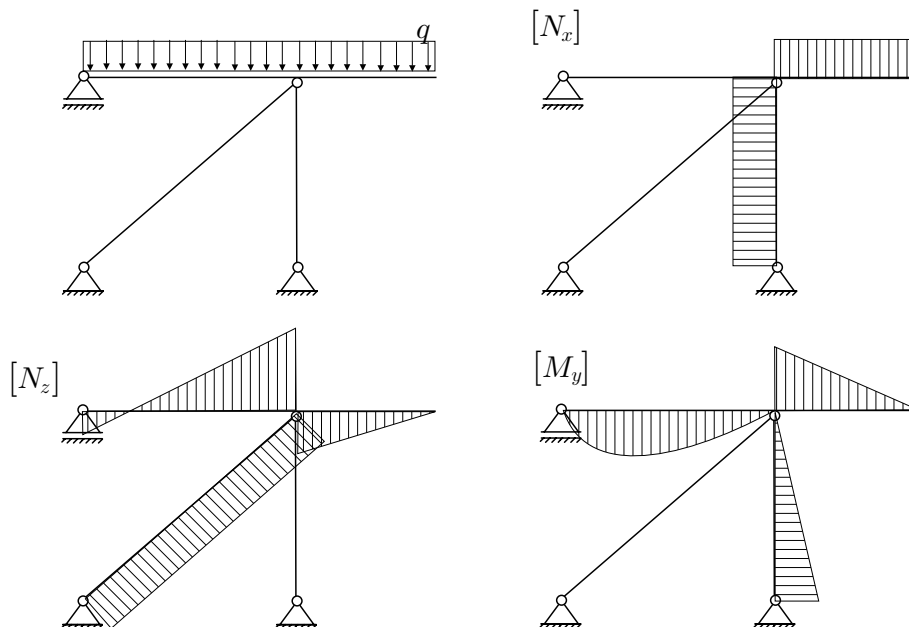
$$N_3 = \frac{3}{2} \frac{B_z}{\sin \alpha}$$

$N_3 = -23.4 \text{ kN}$

STATIKA (VSŠ) - 4. IZPITNI ROK (15. 9. 2005)

RAČUNSKI DEL IZPITA:

1. Janezek je na izpitu iz statike padel. Njegovi diagrami so polni napak. Pomagaj Janezku in poišči (BREZ RAČUNANJA) vse napake v spodnjih diagramih ! (OBVEZNA NALOGA! 25%)

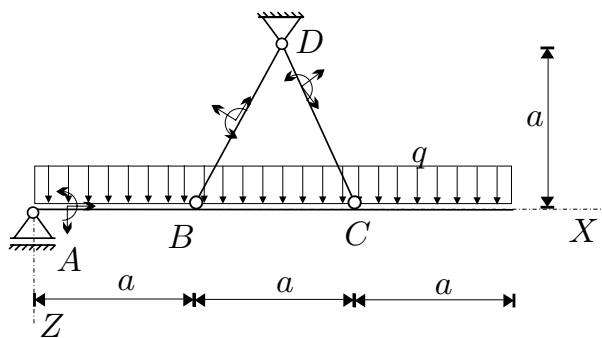


2. Za konstrukcijo na sliki izračunajte stopnjo statične nedoločenosti, reakcije in notranje statične količine (N_x, N_z, M_y)! Rezultate notranjih statičnih količin prikažite z diagrami! (OBVEZNA NALOGA! 45%)

Podatki: $a = 3\text{ m}$,

$q = 5\text{ kN/m}$.

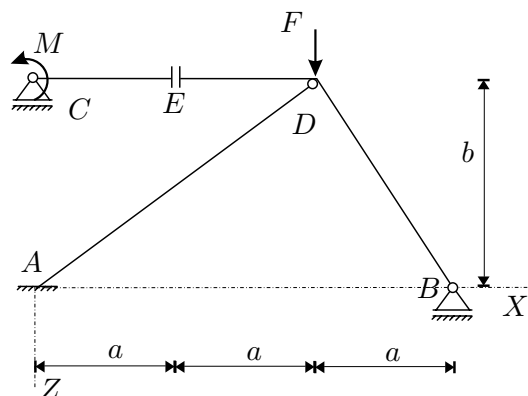
(OBVEZNA NALOGA! 45%)



3. Za konstrukcijo na sliki izračunajte reakcije in sile v vezeh! (30%)

Podatki: $a = 2\text{ m}$, $b = 3\text{ m}$,

$F = 10\text{ kN}$, $M = 2\text{ kNm}$.

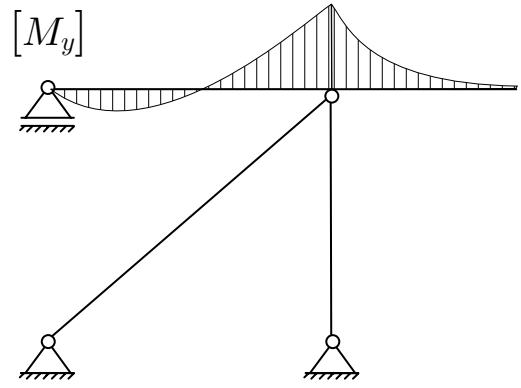
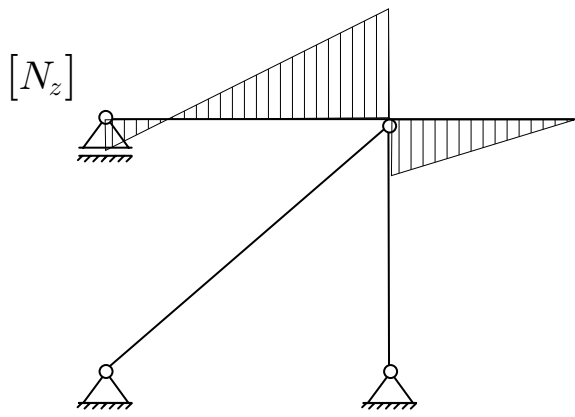
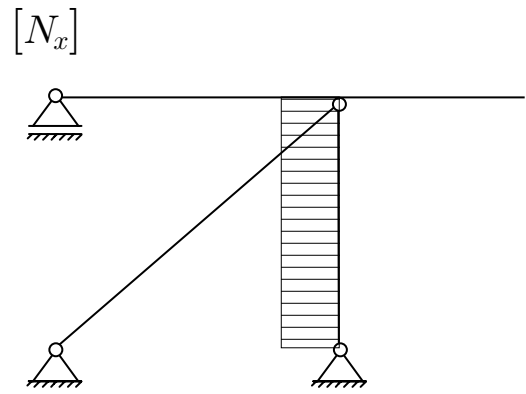
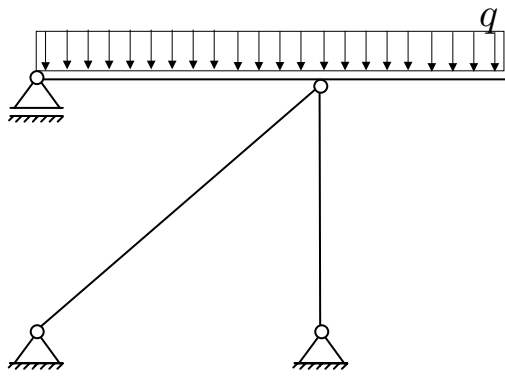


TEORETIČNI DEL IZPITA:

Izmed treh zastavljenih vprašanj si izberete dve, na kateri boste odgovarjali. Izbrani vprašanji jasno označite! Pišite čitljivo.

1. Ravnotežni par sil in dvojica sil!
2. Kdaj govorimo o statično določenih oziroma statično nedoločenih linijskih konstrukcijah? Odgovor konkretizirajte z značilnimi primeri!
3. Ravnotežni pogoji za linijski element z ravno osjo (izpeljava diferencialnih enačb)! Ravnotežne pogoje izpeljite za raven ravninski nosilec, ki je obtežen samo z linijsko obtežbo prečno na os nosilca! Koliko je ravnotežnih enačb, ki niso identično zadoščene?

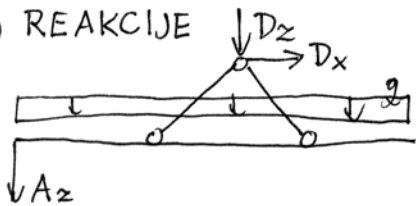
1. Naloga: PRAVILNI DIAGRAMI



2. NALOGA

a.) $\tilde{m}_{ps} = 3 \cdot 3 - 2 - 1 - 3 \cdot 2 = 0$

b.) REAKCIJE



$\Sigma X: D_x = 0$

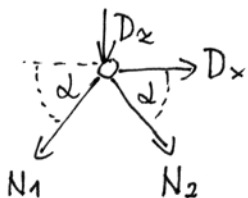
$\Sigma Z: A_z + D_z + g \cdot 3a = 0$

$\Sigma M^A: -g \cdot 3a \cdot \frac{3a}{2} - D_z \cdot \frac{3a}{2} = 0$

$A_z = 0$

$D_z = -45 \text{ kN}$

c.) SILE V PALICAH



$\Sigma X: -N_1 \cos \alpha + N_2 \cos \alpha + D_x = 0 \Rightarrow N_1 = N_2$

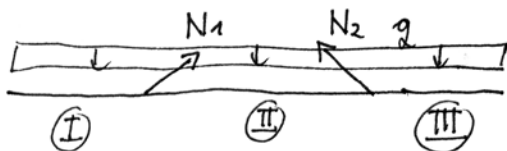
$\Sigma Z: D_z + N_1 \sin \alpha + N_2 \sin \alpha = 0$

$N_1 = -\frac{D_z}{2 \sin \alpha}$

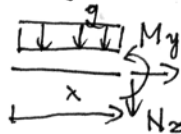
$N_1 = N_2 = 25.2 \text{ kN}$

$\tan \alpha = \frac{3}{1.5} = 2$

d.) NOTRANJE SILE



polje I



$\Sigma x: N_x = 0$

$\Sigma z: N_z = -g \cdot x$

$\Sigma M: M_y = -g \cdot x \cdot \frac{x}{2}$

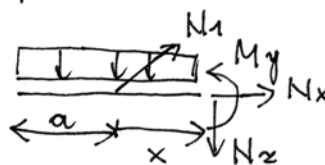
$N_z = -5x$
 $M_y = -2.5x^2$

$N_z(3) = -15 \text{ kN}$

$M_y(3) = -22.5 \text{ kNm}$

polje II

$x \in [0, 3]$



$\Sigma x: N_x = -N_1 \cos \alpha$

$\Sigma z: N_z = -g(a+x) + N_1 \sin \alpha$

$\Sigma M: M_y = -g(a+x) \cdot \frac{a+x}{2} + N_1 \sin \alpha \cdot x$

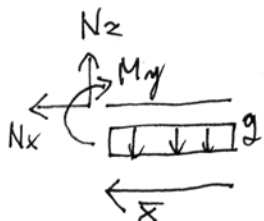
$N_x = -11.25 \text{ kN}$ $N_z = 7.5 - 5x$

$M_y = -22.5 + 7.5x - 2.5x^2$

$x = 1.5$ (ekstrem)

$M_y(1.5) = -16.8 \text{ kNm}$

polje III



$\Sigma x: N_x = 0$

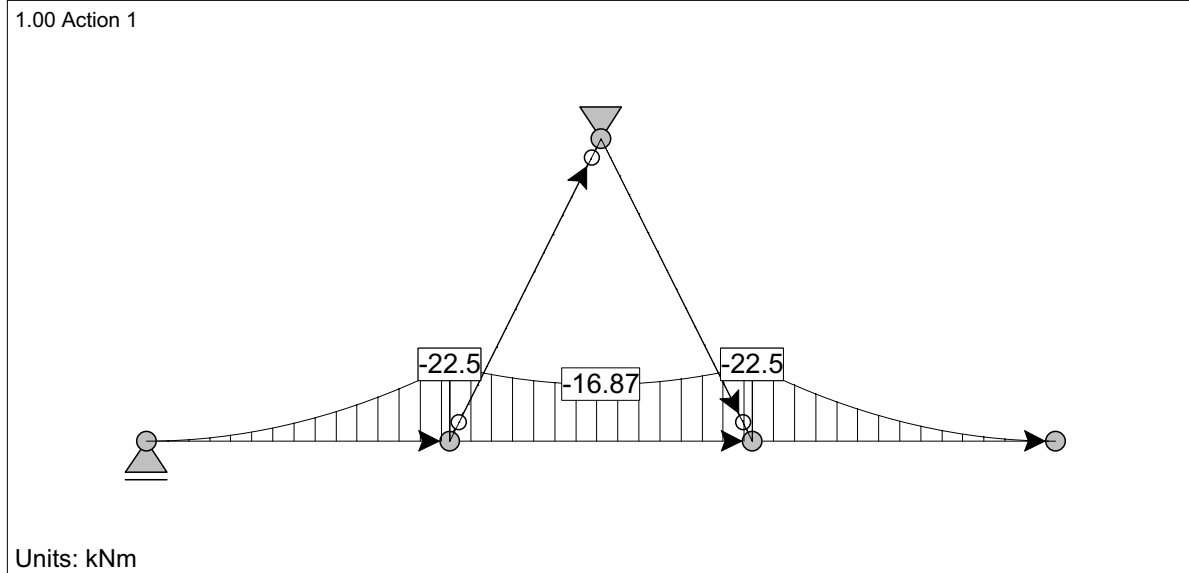
$\Sigma z: N_z = g \cdot x$

$\Sigma M: M_y = -g \cdot x \cdot \frac{x}{2}$

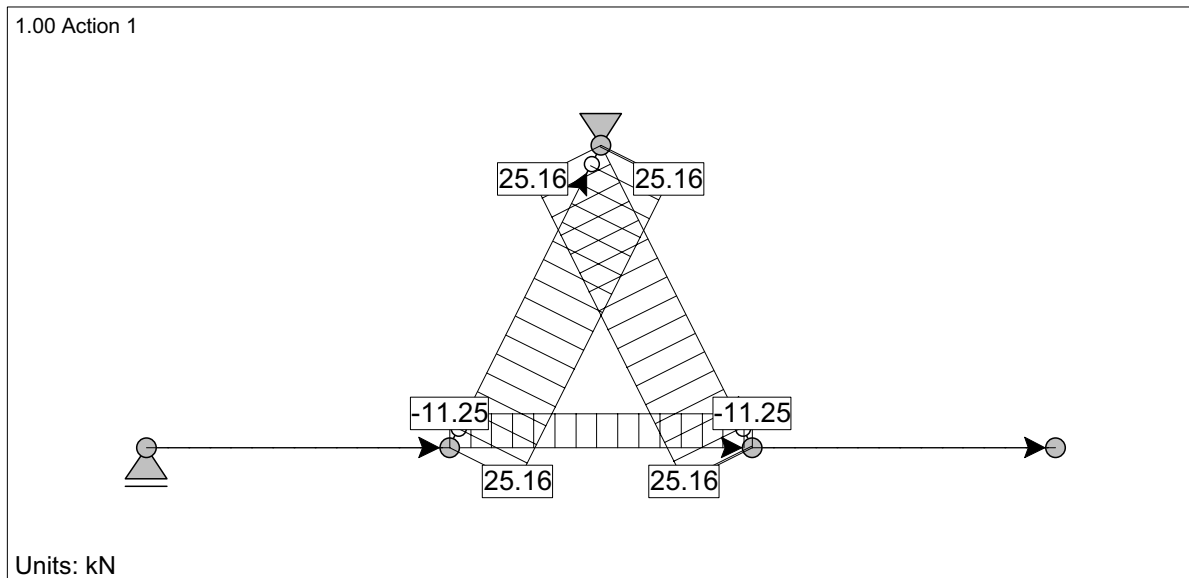
$N_z = 5x$

$M_y = -2.5x^2$

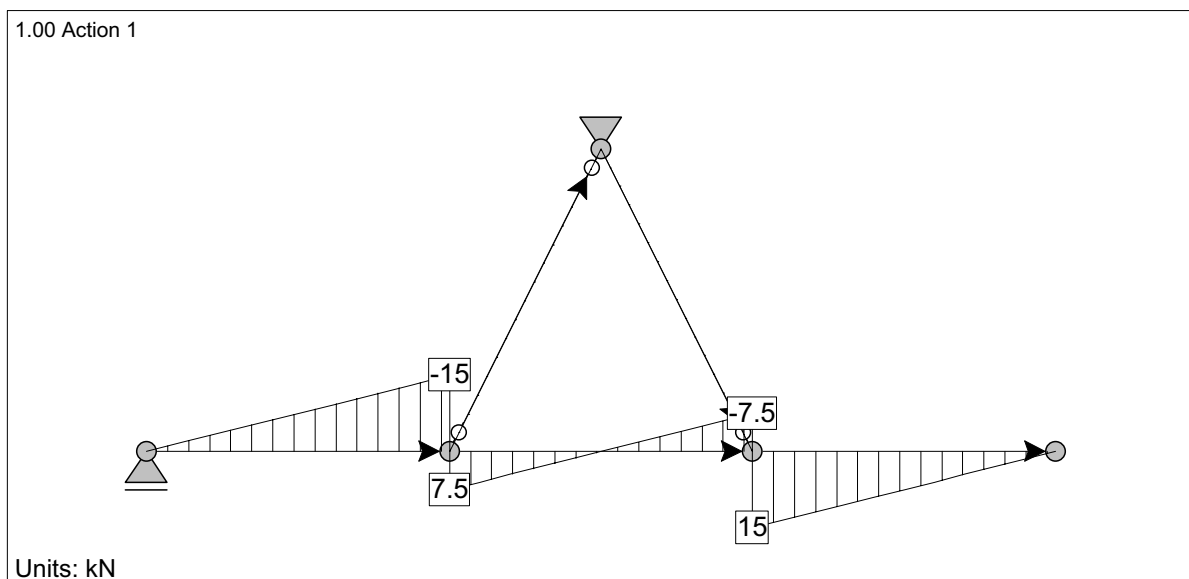
LC1: Load case 2: Bending Moments M_y



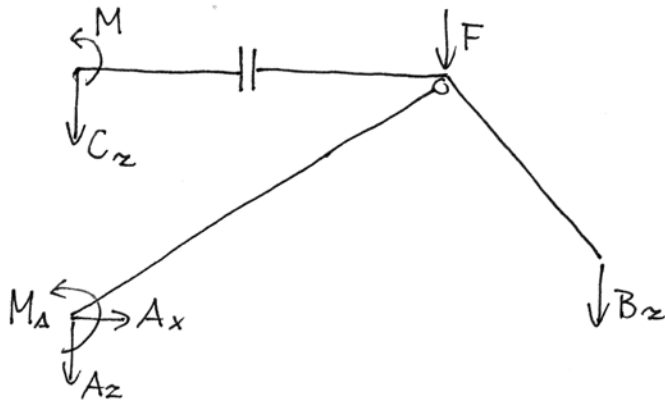
LC1: Load case 2: Axial Forces F_x



LC1: Load case 2: Shear Forces F_z



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$$\Sigma X: A_x = 0$$

$$\Sigma Z: A_z + B_z + C_z + F = 0$$

$$\Sigma M^A: M_A - B_z \cdot 3a - F \cdot 2a + M = 0$$

DODATNI ENAČBI

$$\Sigma Z: C_z = 0$$

CE

$$\Sigma M^D: M_A + A_z \cdot 2a = 0 \Rightarrow -2a A_z + F \cdot 3a + A_z \cdot 3a - F \cdot 2a + M = 0$$

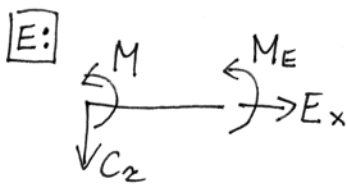
AD

$$A_z = -11 \text{ kN}$$

$$B_z = +1 \text{ kN}$$

$$M_A = +44 \text{ kNm}$$

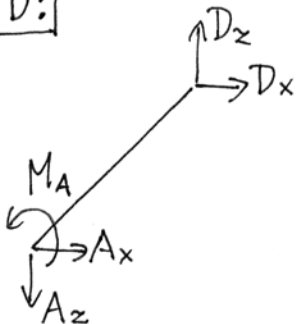
b.) VEZI



$$E_x = 0$$

$$M_E = -M - C_z \cdot a \quad M_E = -2 \text{ kNm}$$

D:



$$\Sigma X: D_x = -A_x \quad D_x = 0$$

$$\Sigma Z: D_z = A_z \quad D_z = 11 \text{ kN}$$

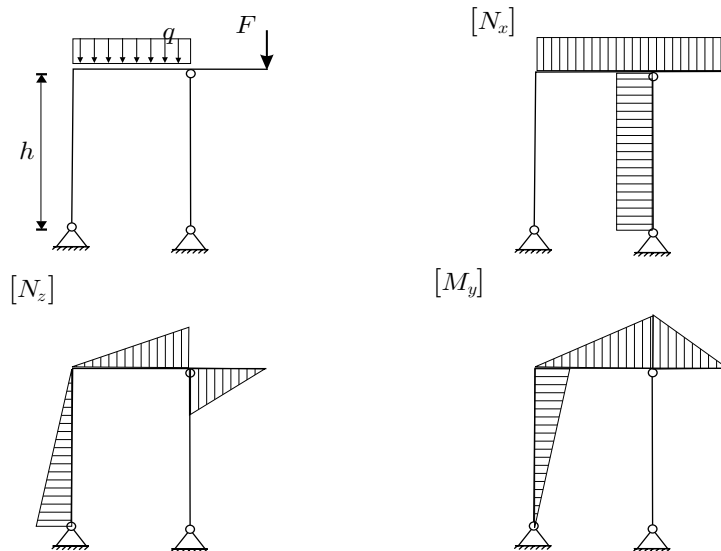
KONTROLA

$$\Sigma M^A: M_A - D_z \cdot 2a = 0$$

STATIKA (VSŠ) - 2. IZREDNI IZPITNI ROK (7. 12. 2005)

RAČUNSKI DEL IZPITA:

1. Janezek je na izpitu iz statike padel. Njegovi diagrami so polni napak. Pomagaj Janezku in poišči (BREZ RAČUNANJA) vse napake v spodnjih diagramih! (OBVEZNA NALOGA! 25%)

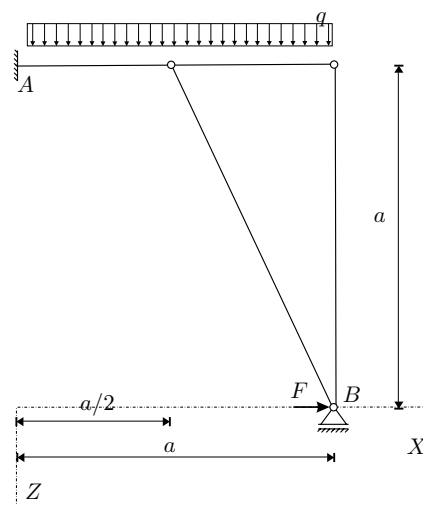


2. Za konstrukcijo na sliki izračunajte stopnjo statične nedoločenosti, reakcije in notranje statične količine (N_x, N_z, M_y)! Rezultate notranjih statičnih količin prikažite z diagrami! (OBVEZNA NALOGA! 45%)

Podatki: $a = 3 \text{ m}$,

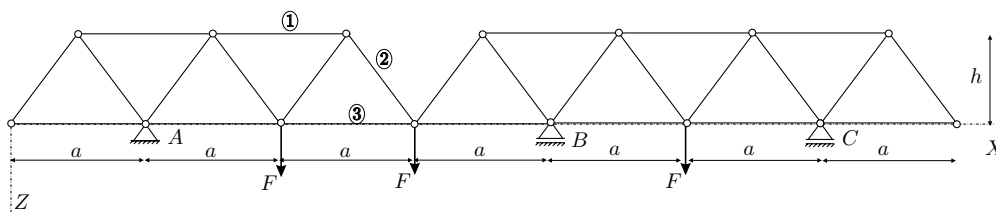
$q = 10 \text{ kN/m}$, $F = 20 \text{ kN}$.

(OBVEZNA NALOGA! 45%)



3. Za palično konstrukcijo na sliki določite (BREZ RAČUNANJA) palice, v katerih je osna sila 0, izračunajte stopnjo statične nedoločenosti in osne sile v palicah 1, 2 in 3! (30%)

Podatki: $a = 3 \text{ m}$, $h = 2 \text{ m}$, $F = 10 \text{ kN}$.

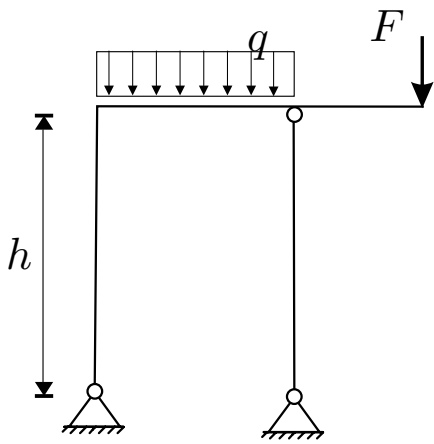


TEORETIČNI DEL IZPITA:

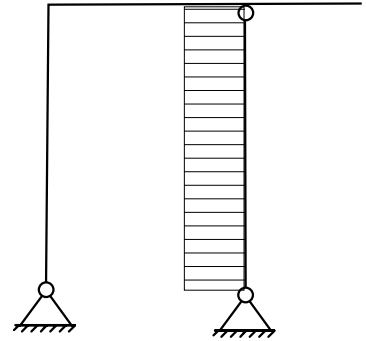
Izmed treh zastavljenih vprašanj si izberete dve, na kateri boste odgovarjali. Izbrani vprašanja jasno označite! Pišite čitljivo.

1. Definicija števila prostostnih stopenj (ilustracija z značilnimi primeri: gibanja delca po ravnini, gibanje N delcev po ravnini, gibanje togega telesa v prostoru, gibanje N togih teles, ki imajo skupno poljubno vez)!
2. Izpeljite ravnotežne pogoje za sile, ki delujejo na sistem delcev s togimi vezmi! Pokaži, da sta izpeljana ravnotežna pogoja potrebna in zadostna pogoja za ravnotežje sistema sil na sistemu sil s togimi vezmi!
3. Račun osnih sil v ravninskem paličju! (Opišite vse metode in jih ilustrirajte s primeri! Zapišite samo ustrezne ravnotežne enačbe, brez računa!)

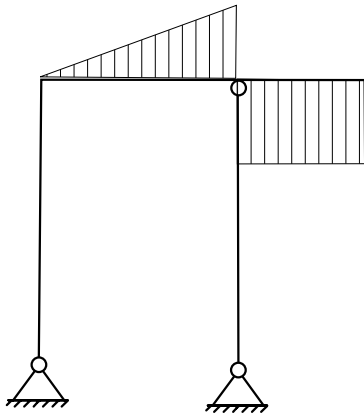
1. Naloga: PRAVILNI DIAGRAMI



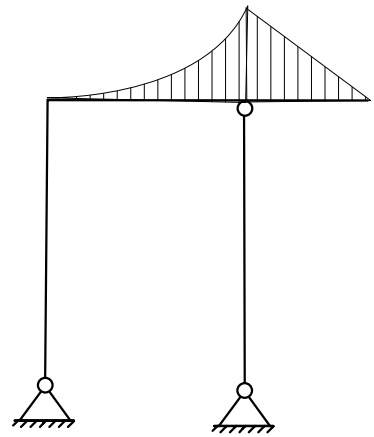
$[N_x]$



$[N_z]$



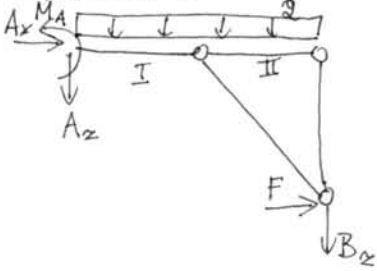
$[M_y]$



2. NALOGA

a.) $\tilde{m}_{ps} = 4 \cdot 3 - 3 - 1 - 2 \cdot 2 - 2 \cdot 2 = 0$

b.) REAKCIJE

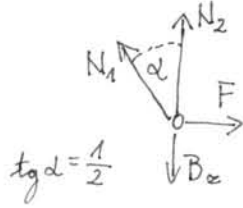


$\sum X: A_x + F = 0 \Rightarrow A_x = -F$

$\sum Z: A_z + B_z + g \cdot a = 0$

$\sum M_A: M_A - g \cdot a \cdot \frac{a}{2} + F \cdot a - B_z \cdot a = 0$

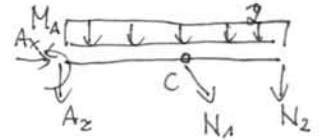
DODATNE ENAČBE: IZREŽEMO PALICI



$\sum x: N_1 \cdot \sin \alpha = F$

$\sum z: N_1 \cdot \cos \alpha + N_2 = B_z$

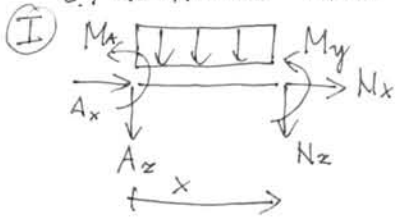
- $A_x = -20 \text{ kN}$
- $B_z = 32.5 \text{ kN}$
- $A_z = -62.5 \text{ kN}$
- $M_A = 82.5 \text{ kNm}$



$\sum M_C: -N_2 \cdot \frac{a}{2} - g \cdot \frac{a}{2} \cdot \frac{a}{4} = 0$

$N_2 = -g \cdot \frac{a}{4} \Rightarrow N_2 = -7.5 \text{ kN}$
 $N_1 = \frac{F}{\sin \alpha} \Rightarrow N_1 = 44.7 \text{ kN}$

c.) NOTRANJE SILE

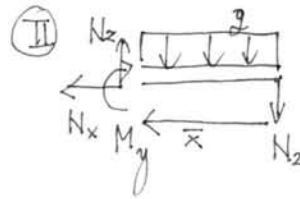


$N_x = 20 \text{ kN}$

$N_z = -A_z - g \cdot x$

$M_{yI} = -M_A - A_z \cdot x - g \cdot x \cdot \frac{x}{2}$

$N_z = 62.5 - 10x$
 $M_{yI} = -82.5 + 62.5x - 5x^2$



$N_x = 0$

$N_z = N_2 + g \cdot x$

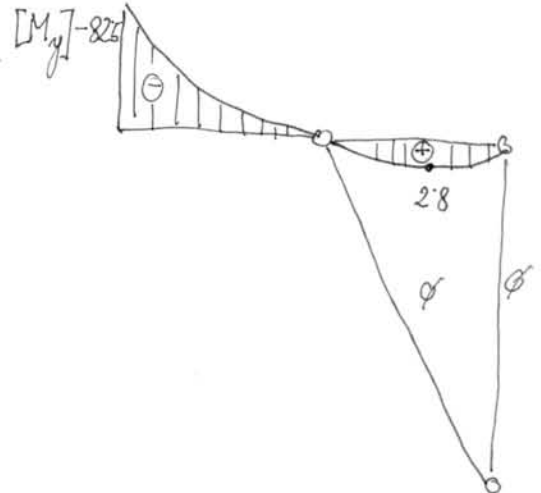
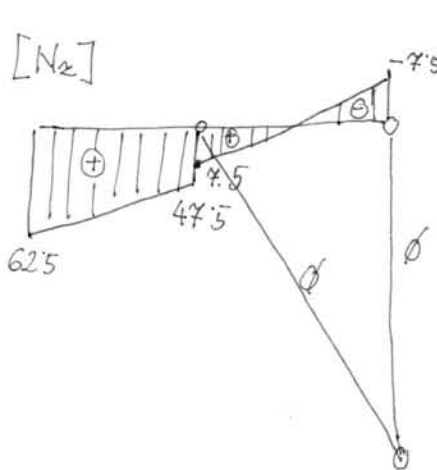
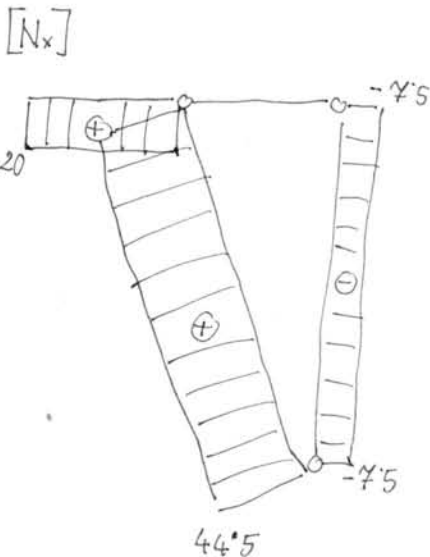
$M_{yII} = -N_2 \cdot x - g \cdot \frac{x^2}{2}$

$N_z = -7.5 + 10x$
 $M_{yII} = 7.5x - 5x^2$

$N_z(1.5) = 15 - 7.5 = 7.5 \text{ kN}$

$M_{yII}(0.75) = 2.8 \text{ kNm (ekstrem)}$

d.) DIAGRAMI



STATIKA - VŠŠ 7.12.2005

3. NALOŽA

a.) $\tilde{m}_{P3} = 2 \cdot 12 + 1 + 1 - 26 = 0$

b.) REAKCIJE

$\Sigma X: A_x = 0$

$\Sigma Z: A_z + B_z + C_z + 3F = 0$

$\Sigma M^A: -F \cdot \alpha - F \cdot 2\alpha - B_z \cdot 3\alpha - F \cdot 4\alpha - C_z \cdot 5\alpha = 0$

dotatna enačba

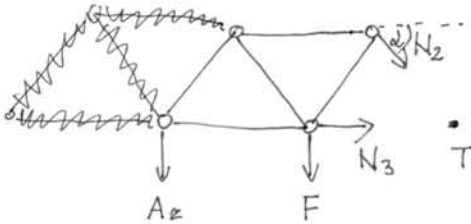
$\Sigma M^T: F \cdot \alpha + A_z \cdot 2\alpha = 0$

$B_z + C_z = -25$

$3B_z + 5C_z = -70$

$A_z = -\frac{F}{2} \quad A_z = -5 \text{ kN}$

B_z in C_z
sploh ne
potrebujemo!



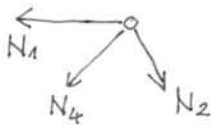
$\Sigma X: N_3 + N_2 \cos \alpha = 0$

$\Sigma Z: N_2 \sin \alpha + F + A_z = 0$

$N_3 = \frac{F}{2} \cdot \text{ctg} \alpha = 3.75 \text{ kN}$
 $N_2 = -\frac{F}{2 \sin \alpha} = -6.25 \text{ kN}$

α
 $\text{tg} \alpha = \frac{2}{1.5}$

izrežemo vozlišče



$\Sigma X: -N_1 - N_4 \cos \alpha + N_2 \cos \alpha = 0$

$\Sigma Z: N_4 \sin \alpha + N_2 \sin \alpha = 0 \Rightarrow N_4 = -N_2$

$N_1 = 2N_2 \cos \alpha$

$N_1 = -7.5 \text{ kN}$