JCSS PROBABILISTIC MODEL CODE

PART 3: MATERIAL PROPERTIES

3.0. GENERAL PRINCIPLES

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List of symbols:

$f_x(x\|q)$	=	the variability of property x within a given lot
$f_q(q)$	=	the variability of the parameters q from lot to lot; statistical description of the production
f'(q)	=	prior distribution of q
f''(q)	=	posterior distribution of q
L(data|q)	=	likelihood function
q	=	vector of distribution parameters (e.g. mean and std. dev.)
C	=	normalising constant
d	=	decision rule

3.0.1 Introduction

The description of each material property consists of a mathematical model (e.g. elastic-plastic model, creep model, etc.) and random variables or random fields (e.g. modulus of elasticity, creep coefficient). Functional relationships between the various variables may be part of the material model (e.g. the relation between tensile stress and compressive stress for concrete).

In general, it is the response to static and time dependent mechanical loading that matters for structural design. However, also the response to physical, chemical and biological actions is important as it may affect the mechanical properties and behaviour.

It is understood that modelling is an art of reasonable simplification of reality such that the outcome is sufficiently explanatory and predictive in an engineering sense. An important aspect of an engineering models also is its operationability, i.e. the ease in handling it inapplications.

Models and values should follow from (standardised) tests, representing the actual environmental and loading conditions as good as possible. The set of tested specimen should be representative for the production of the relevant fabrication sites, cover a sufficient long period of time and may include the effect of standard quality control measures. Allowance should be made for possible differences between test circumstances and structural environment (conversion).

For the classical building materials, knowledge about the various properties is generally available from experience and from tests in the past. For new materials models and values should be obtained from an extensive and well defined testing program.

3.0.2 Material properties

Material properties are defined as the properties of material specimens of defined size and conditioning, sampled according to given rules, subjected to an agreed testing procedure, the results of which are evaluated according to specified procedures.

The main characteristics of the mechanical behaviour is described by the one dimensional σ-ε-diagram, as presented in figure 3.0.1. As an absolute minimum for structural design the

- modulus of elasticity
- the strength of the material

both for tensile and compression should be known. Other important parameters in the one-dimensional σ-ε-diagram are:

- yield stress (if present)
- limit of proportionality
- strain at rupture and strain at maximum stress

The strain at rupture is a local phenomenon and the value obtained may heavily depend on the shape and dimensions of the test specimen.

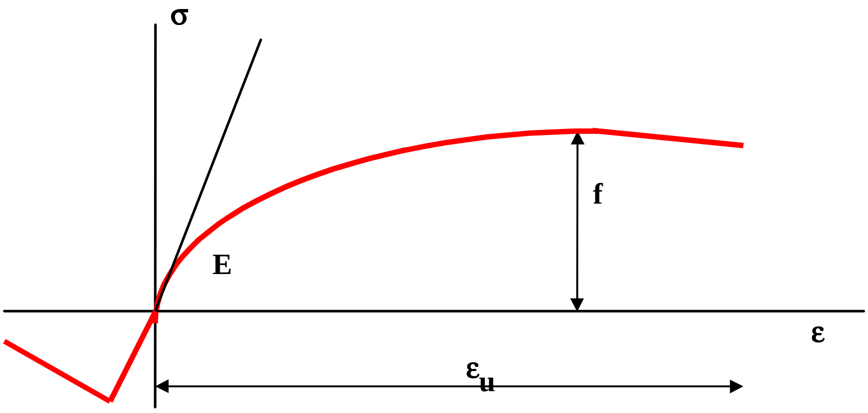

Figure 3.0.1: Stress strain relationship

Additional to the one dimensional σ-ε-diagram, information about a number of other quantities and effects is of importance, such as:

- Multi-axial stress condition
- Duration and strain rate effects
- Temperature effects
- Humidity effects
- Effects of notches and flaws
- Effects of chemical influences

In general, the various properties of one material may be correlated.

In the present version of this JCSS model code not all properties will be considered.

3.0.3 Uncertainties in material modelling

Material properties vary randomly in space: The strength in one point of a structure will not be the same as the strength in another point of the same structure or another one. This item will be further developed in the sections 3.04 and 3.05.

Additional to spatial variations of materials, the following uncertainties between measured properties of specimen and properties of the real structure should be accounted for.

1. Systematic deviations identified in laboratory testing by relating the observed structural property to the predicted property, suggesting some bias in prediction.
2. Random deviations between the observed and predicted structural property, generally suggesting some lack of completeness in the variables considered in the model.
3. Uncertainties in the relation between the material incorporated in the structural sample and the corresponding material samples.

4. Different qualities of workmanship affecting the properties of (fictitious) material samples, i.e. when modelling the material supply as a supply of material samples.
5. The effect of different qualities of workmanship when incorporating the material in actual structures, not reflected in corresponding material samples.
6. Uncertainties related to alterations in time, predictable only by laboratory testing, field observations, etc.

3.0.4 Scales of modelling variations

Material properties vary locally in space and, possibly, in time. As far as the spacial variations are concerned, it is useful to distinguish between three hierarchical levels of variation: global (macro), local (meso) and micro (see table 3.0.1).

For example, the variability of the mean and standard deviation of concrete cylinder strength per job or construction unit as shown in figure 3.0.2 is a typical form of global parameter variation. This variation primarily is the result of production technology and production strategy of the concrete producers. Parameter variations between objects are conveniently denoted as macroscale variations. The unit of that scale is in the order of a structure or a construction unit. Parameter variations may also be due to statistical uncertainties.

Given a certain parameter realisation in a system the next step is to model the local variations within the system in terms of random processes or fields. Characteristically, spatial correlations (dependencies) become negligible at distances comparable to the size of the system. This is a direct consequence of the hierarchical modelling procedure where it is natural to assume that the variation within the system is conditional on the variations between systems and the first type of variation is conditionally independent of the second. At this level one may speak of meso-scale variations. Examples are the spatial variation of soils within a given (not too large) foundation site or the number, size and spatial distribution of flaws along welding lines given a welding factory (or welding operator). The unit of this scale is in the order of the size of the structural elements and probably most conveniently measured in meters.

At the third level, the micro-level, one focuses on rapidly fluctuating variations and inhomogenities which basically are uncontrollable as they originate from physical facts such as the random distribution of spacing and size of aggregates, pores or particles in concrete, metals or other materials. The scale of these variations is measured in particle sizes, i.e. in centimeters down to the size of crystals.

The modelling process normally uses physical arguments as far as possible. Quite generally, the object is taken as an arrangement of a large number of small elements. The statistical properties of these elements usually can only be assessed qualitatively as well as their type of interaction. This, however, is sufficient to perform some basic operations such as extreme value, summation or intersection operations which describe the overall performance. The large number of elements greatly facilitates such operations because one can make use of certain limit theorems in probability theory. The advantage of using asymptotic concepts rests on the fact that the description of the element properties can then be reduced to some few essential characteristics. The central limit theorem of probability theory, asymptotic extreme value concepts, convergence theorems to the Poisson distribution, etc. will play an important role. In particular, size effects have to be taken into account at this level.

A useful concept is to introduce a reference volume of the material which in general is chosen on rather practical grounds. It most frequently corresponds to some specified test specimen volume at which material testing is carried out. This volume generally neither corresponds to the volume of the virtual strength elements introduced at the micro-scale modelling level nor to a characteristic volume for in situ strength. It needs to be related to the latter one and these operations can involve not only simple size scaling but more complicated functional relationships if the material produced is subject to further uncertain factors once put in place. Concrete is the most obvious example for the necessity of such additional consideration. Of course, scale effects may also be present at the meso-scale level of modelling.

The reason for this concept of modelling at several levels (steps) is the requirement for operationability not only in the probabilistic manipulations but also in sampling, estimation and quality control. This way of modelling and the considerations below are, of course, only valid under certain technical standards for production and its control. At the macro-scale level it is assumed that the production process is under control. This simply means that the outcome of production is stationary in some sense. Should a trend develop, production control corrects for it immediately or with some sufficiently small delay. Therefore, it is assumed that at least for some time interval (or spatial region) whose length (size) is to be selected carefully, approximate stationarity on the meso- and micro-scale is guaranteed. Quite frequently, the operational, mathematical models available so far even require ergodicity. Variations at the macro scale level, therefore, can be described by stationary sequences. If the sequences are or can be assumed independent, it is possible to handle macro-scale variations by the concept of random variables. Stationarity is also assumed at the lower levels. However, it may be necessary to use the theory of random processes (fields) at the lower levels, especially in order to take into account of significant effects of dependencies in time or space.

3.0.5 Hierarchical modelling

Consider a random material property X which is described by a probability density function $f_x (x \mid \mathbf{q})$, where $\mathbf{q} = (q_1, q_2, ...)$ is the statistical parameter vector, e.g. q_1 is the mean and q_2 the standard deviation. The density function $f_x (x \mid \mathbf{q})$ applies to the property of a *finite reference volume*, identical with or clearly related to the volume of the test specimen within a given unit of material. Guidance on the type of distribution may be obtained by assessing the performance of the reference volume under testing conditions in terms of some *micro system* behaviour. The performance of test specimens, regarded as a system of micro elements, can usually be interpreted by one of the following strength models

- Weakest link model
- Full plasticity model
- Daniel's bundle of threads model

When applying these models to systems with increasing number of elements, they generally lead to specific distributions for the properties of the system at the meso-scale level. The weakest link model leads to a Weibull distribution, the other two models to a normal distribution. For larger coefficients of variation the normal distribution must be replaced by a lognormal distribution in order to avoid physically impossible, negative strength values.

In the next step (see Table 3.0.1) a *unit* (a structural member) is considered as *meso-scale* (local variations). The respective unit is regarded as being constituted from a sequence of finite volumes. Hence, a property in this unit is modelled by a random sequence X_1, X_2, X_3 ... of reference volume properties. The X_i may have to be considered as correlated, with a coefficient of correlation depending on the distance Δr_{ij} and a correlation parameters ρ_o and d_c, for example:

$$\rho(\Delta r_{ij}) = \rho_o + (1-\rho_o) \exp \left[- (\Delta r_{ij} / d_c)^2 \right] \qquad (3.0.1)$$

In general $\rho_o = 0$.

In the subsequent step the complete structure or some relevant part of it is considered as a lot. A lot is defined as a set of units, produced by one producer, in a relatively short period, with no obvious changes in production circumstances and intended for one building site. In practice lots correspond to e.g.:

- the production of ready-mix concrete for a set of elements
- structural steel from one melt processed according to the same conditions
- foundation piles for a specific site

As a lot is a set of units it can also be conceived as a set of reference volumes X_i. Normally the parameters q defined before are defined on the lot level. The correlation between the X_i values within different members normally can be modelled by a single parameter

$$\rho_{ij} = \rho_0 \qquad (3.0.2)$$

Finally, at the highest *macro scale* level, we have a sequence of lots, represented by a random sequence of lot parameters (in space or in time). Here we are concerned with the estimation of the distribution of lot parameters, either from one source or several sources. The individual lots may be interpreted as random samples taken from the enlarged population or *gross supply*. The gross supply comprises all materials produced (and controlled) according to given specifications, within a country or groups of countries. The *macro-scale* model may be used if the number of producers and structures is large or differences between producers can be considered as approximately random.

Table 3.0.1: Scales of fluctuation

Scale	**Population**	**Reference name**	**Description**
macro (global)	set of structures	gross supply	**X**
meso	set of elements	lot	$\mathbf{X \mid q}$ and $\boldsymbol{\rho_o}$
meso (local)	one element	unit	$\mathbf{X \mid q}$ and $\boldsymbol{\rho(\Delta r)}$
micro	aggregate level	reference volume	type of distribution

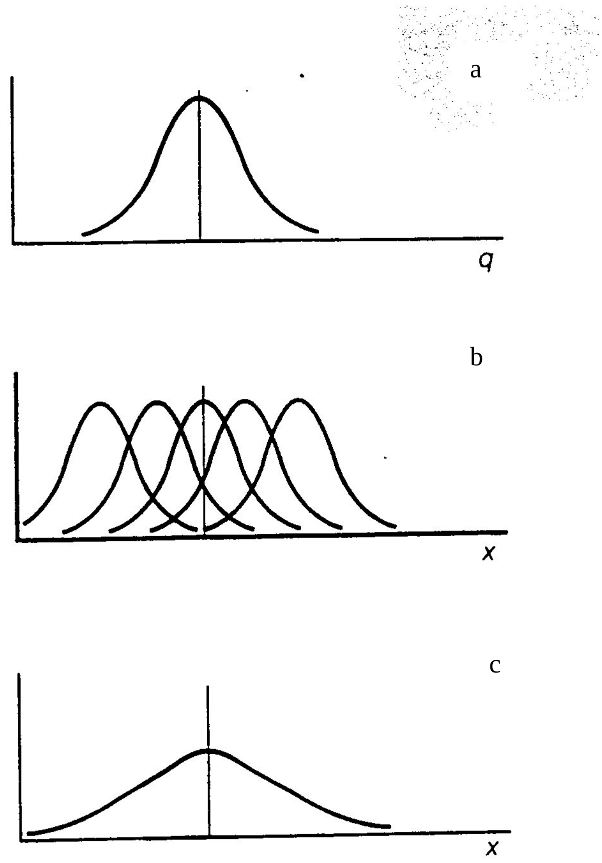

Figure a) production description $f_q(q)$

Figure b) lot description $f_x(x|q)$

Figure c) total supply $f_x(x)$

Figure 3.0.2: Description of production parameters, lots and total supply

The gross supply is described by f(q). Type and parameters should follow from a statistical survey of the fluctuations of the various lots which belong to the production under consideration. It will be assumed here that f(q) is known without statistical uncertainty. If statistical uncertainties cannot be neglected, they can be incorporated. The distribution f(q) should be monitored more or less continuously to find possible changes in production characteristics.

The probability density function (predictive density function) for an arbitrary unit (arbitrary means that the lot is not explicitly identified) can be found from the total probability theorem:

$$f_x(x) = \int f_x(x \mid q)\, f_q(q)\, dq \qquad (3.0.3)$$

The density function f(x) may be conceived as the statistical description of x within a large number of randomly selected lots. For some purposes one could also identify f(x) directly.

3.0.6 Definition of characteristic value

The characteristic value of a material with respect to a given property X is defined as the p_x – quantile in the predictive distribution, i.e.

$$x_c = F_x^{-1}(p) \qquad (3.0.4)$$

Examples for predictive distributions can be found in Annex A. Others may be found in [1], [2] and [3].

3.0.7 Quality control strategies

3.0.7.1 Types of strategies

Normally the statistical parameters of the material properties are based on general tests, taking into account standard production methods. For economic reasons it might be advantageous to have more specific forms of quality control for a particular work or a particular factory.

Quality control may be of a total (all units are tested) or a statistical nature (a sample is tested). Quality control will lead to more economical solutions, but has in general the disadvantage that the result is not available at the time of design. In those cases, the design value has to be based on the combination of the unfiltered production characteristics and the expected effect of the quality control selection rules.

Various quality control procedures can be activated, each one leading to a different design value. In Figure 3.0.3 an overview is presented. The easiest procedure is to perform no additional activities (option "no tests"). This means that the units, lots, production should be defined, their descriptions f(x|q) and f(q) should be established and only f(q) should be checked for long term changes in production characteristics.

If on the other hand tests are performed one may distinguish between a total (unit by unit) control and sampling on the one hand and between selection and updating on the other hand. The various options will be discussed.

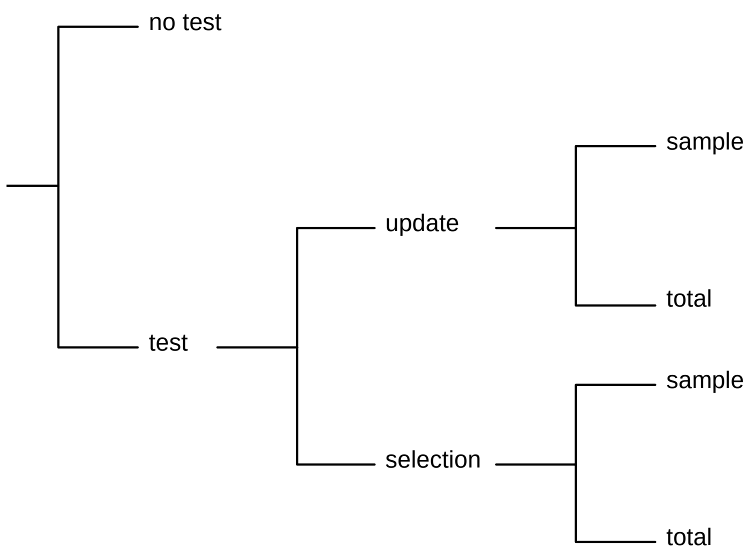

Figure 3.0.3: Strategies for Quality Control

3.0.7.2 Total testing versus Sampling

Both for updating and for selection one may test all units which go into a structure (total testing) or one may test a (random) sample only (statistical testing).

If the control is total, every produced unit is inspected. The acceptance rules imply that a unit is judged as good (accepted) or bad (not accepted). This type of control is also referred to as unit by unit control. Typically, testing all units requires a non-destructive testing technique. Therefore some kind of measurement error has to be included resulting in a smooth truncation of the distribution.

If the control is statistical only a limited member of units is tested. The procedure generally consists of the following parts:

- batching the products;
- sampling within each lot;
- testing the samples;
- statistical judgement of the results;
- decision regarding acceptance.

One normally takes a random sample. In a random sample each unit of the lot has the same probability of being sampled. Where knowledge on the inherent structure of the lot is available, this could be utilised, rendering more efficient sampling techniques, e.g.:

- sampling at weak points, when trends are known;
- sampling at specified intervals;
- stratified sampling;

The larger efficiency results in smaller sample sizes for obtaining the same filtering capability of a test. No further guidance, however, will be given here.

3.0.6.3 Updating versus selecting

Testing can be done with two purposes:

(1) to update the probability density function f(x) or f(q) of some particular lot or item (updating);
(2) to identify and reject inadequate lots or units on the basis of predefined sampling procedures and selection rules (selection).

The basic formula for the first option is given by:

$$f''(q) = C\, L(data|q))\, f'(q) \qquad (3.0.5)$$

where:

$f''(q)$	= posterior distribution of q
$f'(q)$	= prior distribution of q
L(data/q)	= likelihood of the data
q	= vector of distribution parameters (e.g. mean and std. dev.)
C	= normalising constant = $\int L(data \mid q))\, f'(q)\, dq$

For the normal case more detailed information is presented in Annex A.

The first option can only be used after production of the lot or item under consideration. This data may not be known at the time of the design (e.g. ready mix concrete). The second option, on the other hand, offers the possibility to predict the posterior f"(q) for the filtered supply for a given combination of f(x|q), f(q) and a selection rule d. In such a case the control may lead to two possible outcomes:

- the lot (or unit) is rejected : $d \notin A$

- the lot (or unit) is accepted : $d \in A$

Here d is a function of the test result of a single unit or of the combined test result of the units in a sample and A is the acceptance domain.

One may then calculate the posterior distributions for an arbitrary accepted lot:

$$f''(q \mid d \in A) = C\, P(d \in A \mid q)\, f'(q) \qquad (3.0.6)$$

Here f(q) is the distribution function for the unfiltered supply and the acceptance probability $P(d \in A \mid q)$ should be calculated from the decision rule.

The updated distribution for X can be obtained through (3.0.3) with f(q) replaced by $f''(q)$.

More information about the effect of quality cobntrol on the distribution of material properties can be found in [4].

Annex A: *Bayesian evaluation procedure for the normal and lognormal distribution – characteristic values*

If X has a normal distribution with parameters $q_1 = \mu$ and $q_2 = \sigma$ it is convenient to assume a prior distribution for μ and σ according to:

$$f'(\mu,\sigma) = k\,\sigma^{-(\nu'+\delta(n')+1)} \exp\{-\frac{1}{2\sigma^2}\{\nu' s'^2 + n'(\mu - m')^2\}\} \tag{1}$$

k = normalizing constant
δ(n') = 0 for n' = 0
δ(n') = 1 for n' > 0

This special choice enables a further analytical treatment of the various operations. The prior distribution (1) contains four parameters: m', n', s' and ν'.

Using equation (3.0.4) one may combine the prior information characterised by (1) and a test result of n observations with sample mean m and sample standard deviation s. The result is a posterior distribution for the unknown mean and standard deviation of X, which is again given by (1), but with parameters given by the following updating formula's:

$$n'' = n' + n \tag{2}$$

$$\nu'' = \nu' + \nu + \delta(n') \tag{3}$$

$$m''n'' = n'm' + nm \tag{4}$$

$$[\nu'' s''^2 + n'' m''^2] = [\nu' s'^2 + n' m'^2] + [\nu s^2 + n m^2] \tag{5}$$

Then, using equation (3.0.3) the predictive value of X can be found from:

$$X = m'' + t_{\nu''} s'' (1 + \frac{1}{n''})^{0.5} \tag{6}$$

where $t_{\nu''}$ has a central t-distribution.

In case of known standard deviation σ eq. (2) and (4) still hold for the posterior mean. The predictive value of X is

$$X = m'' + u\sigma(1 + \frac{1}{n''})^{0.5} \tag{7}$$

where u has a standard normal distribution.

The characteristic value is thus defined as

$$x_c = \begin{cases} m'' + u(p_x)\sigma(1+\frac{1}{n''})^{0.5} & \textit{for} \quad \sigma \textit{ known} \\ m'' + t_{\nu''}(p_x)s''(1+\frac{1}{n''})^{0.5} & \textit{for} \quad \sigma \textit{ unknown} \end{cases} \tag{8}$$

For $n''\nu'' \rightarrow \infty$ $x_c = m'' + u(p_x)\, s''$ in both cases with $s'' = \sigma$.

If X has a lognormal distribution, Y = ln (X) has a normal distribution. One may then use the former formula's on Y and use X = exp(Y) for results on X.

Annex B: Bayesian evaluation procedure for regression – characteristic value

If only indirect measurements for the quantity of interest are possible and a linear regression model $y = a_0 + a_1 x$ is suitable the predictive value of y has also a t-distribution given by

$$y = a_0 + a_1 x + t_\nu\, s \left(1 + \frac{1}{n} + \frac{(\bar{x} - x_0)^2}{\sum_{i=1}^{n} (x_i - \bar{x})^2} \right)^{1/2}$$

where

$$a_0 = \bar{y} - a_1 \bar{x}$$

$$a_1 = \frac{\sum_{i=1}^{n} x_i y_i - n\overline{xy}}{\sum_{i=1}^{n} x_i^2 - n\bar{x}^2}$$

$$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$$

$$\bar{y} = \frac{1}{n} \sum_{i=1}^{n} y_i$$

$$s^2 = \frac{1}{n-2} \sum_{i=1}^{n} \left(y_i - a_0 - a_1 x_i \right)^2$$

$$\nu = n - 2$$

The characteristic value corresponding to the quantile p is

$$y_c = a_0 + a_1 x + T^{-1}(p, \nu)\, s \left(1 + \frac{1}{n} + \frac{(\bar{x} - x_0)^2}{\sum_{i=1}^{n} (x_i - \bar{x})^2} \right)^{1/2}$$

For example, for S-N curves, it is y = ln(N), x = ln(Δσ), a_1 = -m und a_0 = lna. The characteristic value of N for given ln($\Delta\sigma_E$) = x_0 is $N_c = \exp[y_c]$.

References

[1] Aitchison, J., Dunsmore, I.R., Statistical Prediction Analysis, Cambridge University Press, Cambridge, 1975

[2] Raiffa, H., Schlaifer, R., Applied Statistical Decision Theory, MIT Press, Cambridge, 1968

[3] Engelund, S., Rackwitz, R., On Predictive Distribution Functions for the Three Asymptotic Extreme Value Distributions, Structural Safety, Vol. 11, 1992, pp. 255-258

[4] Kersken-Bradley, M., Rackwitz, R., Stochastic Modeling of Material Properties and Quality Control, JCSS Working Document, IABSE-publication, March 1991

JCSS PROBABILISTIC MODEL CODE
PART 3: RESISTANCE MODELS

3.1 CONCRETE PROPERTIES

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3.1.1 Basic Properties
3.1.2 Stress-strain-relationship
3.1.3 The probabilistic model
3.1.4 Distribution for Y_{kj}
3.1.5 Distribution for f_{co}

List of symbols:

f_{co} = basic concrete compression strength
M_j = the logarithmic mean at strength job j
Σ_j = the logarithmic strength standard deviation at job j
$Y_{1,j}$ = a log-normal variable representing additional variations due to the special placing, curing and hardening conditions of in situ concrete at job j
U_{ij} = a standard normal variable
λ = lognormal variable with mean 0.96 and coefficient of variation 0.005; generally it suffices to take λ deterministically
$\alpha(t,\tau)$ = is a deterministic function which takes into account the concrete age at the loading time t and the duration of loading τ
$\varphi(t,\tau)$ = is the creep coefficient.
β_d = total load and depends from the type of the structure
E_c = modulus of elasticity
f_c = in situ strength
ε_e = strain at yielding
ε_u = ultimate strain

3.1.1 Basic Properties

The reference property of concrete is the compressive strength f_{co} of standard test specimens (cylinder of 300 mm height and 150 mm diameter) tested according to standard conditions and at a standard age of 28 days (see ISO/DIS 2736 and ISO 3893). Other concrete properties are related to the reference strength of concrete according to:

In situ compressive strength: $f_c = \alpha(t,\tau)\ f_{co}^{\lambda}$ [MPa] (1)

Tensile strength: $f_{ct} = 0.3\ f_c^{2/3}$ [MPa] (2)

Modulus of elasticity: $E_c = 10.5\, f_c^{1/3}\,(\frac{1}{1+\beta_d\,\varphi(t,\tau)})$ [GPa] (3)

Ultimate compression strain: $\varepsilon_u = 6.10^{-3}\ f_c^{-1/6}\,(1+\beta_d\,\varphi(t,\tau))$ [m/m] (4)

λ is a factor taking into account the systematic variation of in situ compressive strength and strength of standard tests (see 3.1.3)

$\alpha(t,\tau)$ is a deterministic function which takes into account the concrete age at the loading time t [days] and the duration of loading τ [days]. The function is given by:

$\alpha(t,\tau) = \alpha_1(\tau)\ \alpha_2(t)$ (5a)

$\alpha_1(\tau) = \alpha_3(\infty) + [1-\alpha_3(\infty)]\exp[-a_\tau\,\tau]$ with $\alpha_3(\infty) \approx 0.8$ and $a_\tau = 0.04$.

$\alpha_2(t) = a + b\ 1n(t)$ (5b)

In most applications $\alpha_1(\tau) = 0.8$ can be used. The coefficients a and b in $\alpha_2(t)$ depend on the type of cement and the climatical environment; under normal conditions a = 0.6 and b = 0.12.

$\varphi(t,\tau)$ is the creep coefficient according to some modern code assumed to be deterministic.

β_d is the ratio of the permanent load to the total load and depends on the type of the structure; generally β_d is between 0.6 and 0.8.

3.1.2 Stress-strain-relationship

For concrete under compression the following simplified stress-strain relationship holds:

$\sigma = E_c\,\varepsilon$ for $\varepsilon < \varepsilon_e$ (6)
$\sigma = f_c$ for $\varepsilon_e < \varepsilon < \varepsilon_u$ (7)
$\varepsilon_e = f_c/E_c$ (8)

For calculations where the form of the stress-strain relationships is important the following relationship should be used:

$$\sigma = f_c \left[1 - \left[1 - \frac{\varepsilon}{\varepsilon_s} \right]^k \right] \tag{9}$$

$$\varepsilon_s = 0.0011\, f_c^{\ 1/6} \tag{10}$$

$$k = \frac{E_c \varepsilon_s}{f_c} \tag{11}$$

The relationship holds for $0 < \varepsilon < \varepsilon_s$.

3.1.3 The probabilistic model

The strength of concrete at a particular point i in a given structure j as a function of standard strength f_{c0} is given as:

$$f_{c,ij} = \alpha(t,\tau)\, (f_{co,ij})^{\lambda}\, Y_{1,j} \tag{12}$$

$$f_{co,ij} = \exp((U_{ij}\, \Sigma_j + M_j\,)) \tag{13}$$

in which

$f_{c0,ij}$ = log-normal variable, independent of $Y_{1,j}$, with distribution parameters M_j and Σ_j
M_j = the logarithmic mean at job j
Σ_j = the logarithmic standard deviation at job j
$Y_{1,j}$ = a log-normal variable representing additional variations due to the special placing, curing and hardening conditions of in situ concrete at job j
U_{ij} = a standard normal variable representing the variability within one structure
λ = lognormal variable with mean 0.96 and coefficient of variation 0.005; generally it suffices to take λ deterministically

The variable $Y_{1,j}$ can also be taken as a spatially varying random field whose mean value function takes account of systematic influences in space.

Correspondingly, for the other three basic properties:

$$f_{ct,ij} = 0.3 \quad f_{c,ij}^{2/3} Y_{2,j} \tag{14}$$

$$E_{c,ij} = 10.5 f_{c,ij}^{1/3} Y_{3,j} (1+\beta_d \varphi(t,\tau))^{-1} \tag{15}$$

$$\varepsilon_{u,ij} = 6\,10^{-3} f_{c,ij}^{-1/6} Y_{4,j} (1+\beta_d \varphi(t,\tau)) \tag{16}$$

where the variables $Y_{2,j}$ to $Y_{4,j}$ mainly reflect variations due to factors not well accounted for by concrete compressive strength (e.g., gravel type and size, chemical composition of cement and other ingredients, climatical conditions).

The variables U_{ij} and U_{kj} within one member are correlated by:

$$\rho(U_{ij}, U_{kj}) = \rho + (1-\rho)\exp\left\{\frac{(r_{ij} - r_{kj})^2}{{d_c}^2}\right\} \tag{17}$$

where d_c = 5 m and ρ = 0.5. For different jobs U_{ij} and U_{kj} are uncorrelated.

3.1.4 Distributions of Y_{kj}

Unless direct measurements are available, the parameters of the variables $Y_{k,j}$ can be taken from Table 3.1.1. The variables are distributed according to the log-normal distibution. The variability of the variables $Y_{k,j}$ can further be split into a part depending only on the job under consideration and a part representing spatial variability.

If direct measurements are available, the parameters in Table 3.1.1 are taken as parameters of an equivalent prior sample with size n' = 10 (see Part 1 for the details of updating).

Variable	Distribution type	Mean	Coefficient of variation	Related to
$Y_{1,j}$	LN	1.0	0.06	compression
$Y_{2,j}$	LN	1.0	0.30	tension
$Y_{3,j}$	LN	1.0	0.15	E-modulus
$Y_{4,j}$	LN	1.0	0.15	ultimate strain

Table 3.1.1: Data for parameters Y_i

3.1.5 Distribution for f_{co}

The distribution of $x_{ij} = \ln(f_{co,ij})$ is normal provided that its parameters M and Σ obtained from an ideal infinite sample. In general it must be assumed that concrete production varies from production unit, site, construction period, etc. and that sample sizes are limited. Therefore, the parameters M and Σ must also be treated as random variables. Then, x_{ij} has a student distribution according to:

$$F_x(x) = F_{t_{\nu''}}\left[\frac{\ln(x/m'')}{s''}(1+\frac{1}{n''})^{-0.5}\right]$$

where $F_{t\nu''}$ is the Student distribution for ν″ degrees of freedom. $f_{co,ij}$ can be represented as

$$f_{co,ij} = \exp(m'' + t_{\nu''} s''(1+\frac{1}{n''})^{0.5}$$

The values of m”, n”, s” and ν” depend on the amount of specific information. Table 3.1.2 gives the values if no specific information is available (prior information).

Table 3.1.2: Prior parameters for concrete strength distribution (f_{co} in MPa) [1, 2]

Concrete type	Concrete grade	Parameters			
		m’	n’	s’	ν’
Ready mixed	C15	3.40	3.0	0.14	10
	C25	3.65	3.0	0.12	10
	C35	3.85	3.0	0.09	10
	C45	3.98	3.0	0.07	10
	C55	-	-	-	-
Pre-cast elements	C15	-	-	-	-
	C25	3.80	3.0	0.09	10
	C35	3.95	3.0	0.08	10
	C45	4.08	4.0	0.07	10
	C55	4.15	4.0	0.05	10

The prior parameters may depend on the geographical area and the technology with which concrete is produced.

If $n''\nu'' > 10$, a good approximation of the concrete strength distribution is the log-normal distribution with mean m'' and standard deviation $s''\sqrt{\frac{n''}{n''-1}\frac{\nu''}{\nu''-2}}$.

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3.* Static Properties of Structural Steel (Rolled Sections)

Properties Considered

The following properties of structural steel are dealt with herein:

f_y	= yield strength	[MPa]
f_u	= ultimate tensile strength	[MPa]
E	= modulus of elasticity	[MPa]
ν	= Poisson's ratio	
ε_u	= ultimate strain	

A probabilistic model is proposed for the random vector $\mathbf{X} = (f_y, f_u, E, \nu, \varepsilon_u)$ to be used for any particular steel grade, which may be defined in terms of nominal values verified by standard mill tests (e.g. following the procedures of EN 10025 for sampling and selection of test pieces and the requirements of EN 10002-1 for testing) or in terms of minimum (hereinafter referred to as code specified) values given in material specifications (e.g. EN 10025: 1990).

Only distinct points or parts of the full stress-strain curve are considered, thus the proposed model can be used in applications where this type of information is compatible with the parameters of the mechanical model used for strength analysis.

In applications where strain-hardening (and in particular the extent of the yield plateau and the initial strain-hardening) are important (e.g. inelastic local buckling) a more detailed model, which describes the full stress-strain behaviour, may be warranted. Several deterministic models exist in the literature which would allow a probabilistic model to be developed. The parameters of the model chosen to describe the full stress-strain curve should be selected in a way that does not invalidate the statistics given below for the key points of the stress-strain diagram.

In certain cases, where an absence of a yield phenomenon is noted, the values given for the yield strength may be used instead for the 0.2% proof strength. However, it should be emphasised that most of the data examined refers to steels exhibiting a yield phenomenon, hence this is only a tentative proposal.

Probabilistic Models and Range of Applicability

Mean values and coefficients of variation for the above vector are given in Table A whereas the correlation matrix is given in Table B. A multi-variate log-normal distribution is recommended. The values given are valid for static loading.

The values in Table A may be used for steel grades and qualities given in EN 10025: 1990, which have code specified yield strength of up to 380 MPa. Some studies suggest that it is the standard deviation of the yield strength, rather than its coefficient of variation (CoV), that remains constant, whilst others point to the converse.

A practice which creates problems with sample homogeneity, and hence with consistency of estimated statistical properties, is downgrading of material, i.e. re-classifying higher grade steel to a lower grade if it fails to meet the code specified values for the higher grade on the basis of quality control tests. This practice produces bi-modal distributions and is clearly seen in some of the histograms reported in the studies referenced below. Higher mean values but also significantly higher CoV's than those given in Table A are to be expected in such cases.

The values given in Tables A and B should not be used for ultra high strength steels (e.g. with code specified f_y = 690 MPa) without verification. In any case, ultra high strength carbon steel (and stainless steel) grades are characterised by a non-linear uniaxial stress-strain response, usually modelled through the Ramberg-Osgood expression. Practically no statistical data have been found for the three parameters describing the Ramberg-Osgood law (initial modulus, 0.2% proof stress and non-linearity index).

The CoV values refer to total steel production and are based primarily on European studies from 1970 onwards. In US and Canada higher CoV's have been used (on average, about 50% higher). The main references on which these estimates are based are given below.

The estimates for ultimate strain, ε_u, are very sensitive to test instrumentation and rate of loading up to the point of failure. Both significantly higher and lower CoV's have, on occasions, been reported.

Within-batch COVs can be taken as one fourth of the values given in Table A but within-batch variability for the modulus of elasticity, E, and Poisson's ratio, ν, may be neglected. Variations along the length of a rolled section are normally small and can be neglected.

If direct measurements are available, the numbers in Table A should be used as prior statistics with a relatively large equivalent sample size (e.g. $n' \approx 50$).

For applications involving seismic loads, a random variable called 'yield ratio', denoted by r and defined as the ratio of yield to ultimate strength, is often of interest. The statistical properties of this ratio can be derived from those given in Tables A and B for the two basic random variables. Given the positive correlation between f_y and f_u , it follows that there is also a positive correlation between r and f_y. It can also be shown that the CoV for r lies between the CoV's for f_y and f_u.

Table A: Mean and COV values

Property	Mean Value, E[.]	COV, v
f_y	$f_{ysp} \,.\, \alpha.\, exp\,(-u\,.\,v) - C$	0.07
f_u	$B.\, E[f_u]$	0.04
E	E_{sp}	0.03
ν	ν_{sp}	0.03
ε_u	ε_{usp}	0.06

Table B: Correlation Matrix

	f_y	f_u	E	ν	ε_u

f_y	1	0.75	0	0	-0.45
f_u		1	0	0	-0.60
E			1	0	0
ν				1	0
ε_u					1

Definitions and Remarks

- the suffix ($_{sp}$) is used for the code specified or nominal value for the variable considered
- α is spatial position factor (α=1.05 for webs of hot rolled sections and α=1 otherwise)
- u is a factor related to the fractile of the distribution used in describing the distance between the code specified or nominal value and the mean value; u is found to be in the range of -1.5 to -2.0 for steel produced in accordance with the relevant EN standards; if nominal values are used for f_{ysp} the value of u needs to be appropriately selected.
- C is a constant reducing the yield strength as obtained from usual mill tests to the static yield strength; a value of 20 MPa is recommended but attention should be given to the rate of loading used in the tensile tests.
- B = 1.5 for structural carbon steel
  = 1.4 for low alloy steel
  = 1.1 for quenched and tempered steel

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3.2 STATIC PROPERTIES OF REINFORCING STEEL

Table of contents:

3.2.1 Basic Model
3.2.2 The probabilistic model
3.2.3 Effect of Prior Investigations and Statistical Quality Control
3.2.4 Strength of Bundles of Bars

List of symbols:

f_y = basic yield stress
σ = standard deviation

3.2.1 Basic Model

Reinforcing steel generally is classified and produced according to grades, for example S300, S400 and S500, the numbers denoting a specified (minimum) yield stress limit. The basic mechanical property is the static yield strength f_y defined at strain 0.2‰. The stress-strain curve for hot rolled steels can be approximated by a bi-linear relationship up to strains of 1% to 2%. The (initial) modulus of elasticity can be taken as constant E_a=205[Gpa]. The stress-strain relationship for cold worked steel can also be represented by a bi-linear law but more realistically by a continuous curve for which several convenient analytical forms exist.

3.2.2 Probabilistic Model

The yield stress, denoted by X_1, can be taken as the sum of three independent Gaussian variables

$$X_1(d) = X_{11} + X_{12} + X_{13} \qquad \text{[MPa]} \qquad (1)$$

where $X_{11} \sim N(\mu_{11}(d), \sigma_{11})$ represents the variations in the global mean of different mills, $X_{12} \sim N(0, \sigma_{12})$ he variations in a mill from batch(melt) to bath and $X_{13} \sim N(0, \sigma_{13})$ the variations within a melt. D is the nominal bar diameter in [mm]. For high standard steel production the following values have been found: σ_{11}=19 [MPa], σ_{12} =22 [MPa], σ_{13}=8 [MPa] resulting in an overall standard deviation σ_1 of about 30 [MPa]. The mean $\mu_{11} = \mu_1$ is under controlled conditions Sxxx + 2 σ_1. Strength fluctuations along bars are negligible. The value of $\mu_1(d)$ is defined as the overall mean from the entire production given a particular bar diameter.

$$\mu_1(d) = \mu_1(0.87 + 0.13\exp[-0.08d])^{-1} \qquad \text{[MPa]} \qquad (2)$$

Statistical parameters of some other relevant properties are given in the following table:

Quantity	Mean	σ	C.o.V.	ρ_{ij}			
Bar area [mm²]	Nom. Area	-	0.02	1.00	0.50	0.35	0
Yield stress [MPa]	S_{nom} + 2σ	30	-		1.00	0.85	-0.50
Ultimate strength [MPa]	-	40	-	sym		1.00	-0.55
δ_{10} [%]	-	-	0.09				1.00

For these quantities a normal distribution can be adopted.

3.2.3 Effect of Prior Investigations and Statistical Quality Control

Tests of the lot of reinforcing steel to be used can considerably diminish steel variations, if the lot is known to belong to the production of a specific mill and if it originates from the same batch. Very few direct tests are necessary. Acceptance control for a given lot can be very efficient to eliminate bad quality lots.

3.2.4 Strength of Bundles of Bars

The yield forces of bundles of bars under static loading is the sum of the yield forces of each contributing bar (full plasticity model). In general, it can be assumed that all reinforcing steel used at a job originates from a single (but unknown) mill. The correlation coefficient between yield forces of individual bars of the same diameter can then be taken as 0.9. The correlation coefficient between yield forces of bars of different diameter and between the yield forces in different cross-sections in different beams in a structure can be taken as 0.4. Along structural members the correlation is unity within distances of roughly 10m (representative for bar length) and vanishes outside.

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3.9 MODEL UNCERTAINTIES

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3.9.1 General
3.9.2 Types of models for structural analysis
3.9.3 Recommendations for practice

List of Symbols

Y = response of the structure according to the model
Y' = real response of the structure
$f(\ )$ = model function
$f'(\ )$ = model function including model uncertainties
X_i = basic variable
θ_i = model uncertainty

3.9 MODEL UNCERTAINTIES

3.9.1 General

In order to calculate the response of a structure with certain (random) properties under certain (random) actions use is made of models (see Part I, section 5). In general such a model can be described as a functional relation of the type:

$$Y = f\,(X_1,X_2,\ldots X_n) \qquad (3.9.1)$$

Y = response of the structure
$f(\ )$ = model function
X_i = basic variables (actions and structural properteis)

The model function f (..) is usually not complete and exact, so that the outcome Y cannot be predicted without error, even if the values of all random basic variables are known. The real outcome *Y'* of the experiment can formally be written down as:

$$Y' = f'(X_1\ldots X_n\ ,\ \theta_1\ldots\theta_2) \qquad (3.9.2)$$

The variables θ_i are referred to as parameters which contain the model uncertainties and are treated as random variables. The model uncertainties account for:

- random effects that are neglected in the models
- simplifications in the mathematical relations

Ideally model uncertainties should be obtained from a set of representative laboratory experiments and measurements on real structures where all values of X_i are measured or controlled. In those case a model uncertainty has the nature of an intrinsic uncertainty. If the number of measurements is small the statistical uncertainty may be large. Additional there may be uncertainty due to measurement errors both in the X_i and in the Y. Bayesian regressian analysis usually is the appropriate tool to deal with the above situation.

In many cases, however, a good and consistent set of experiments is lacking and statistical properties for model uncertainties are purely based on engineering judgement. Sometimes a comparison between various models may help to defend certain propositions.

The most common way of introducing the model uncertainty into the calculation model is as follows:

$$Y' = \theta_1\, f\,(X_1\ldots X_n\,) \qquad (3.9.3)$$

or

$$Y' = \theta_1 + f\,(X_1\ldots X_n\,) \qquad (3.9.4)$$

or a combination of both. The first definition is clarified in Figure 3.9.1

It should be kept in mind that this way the statistical properties of the model uncertainties depend on the exact definition of the model output. A theoretical elegant way to avoid these definition dependency is to link model uncertainties directly to the basic variables, that is to introduce $X'i = \theta_1 X_i$..

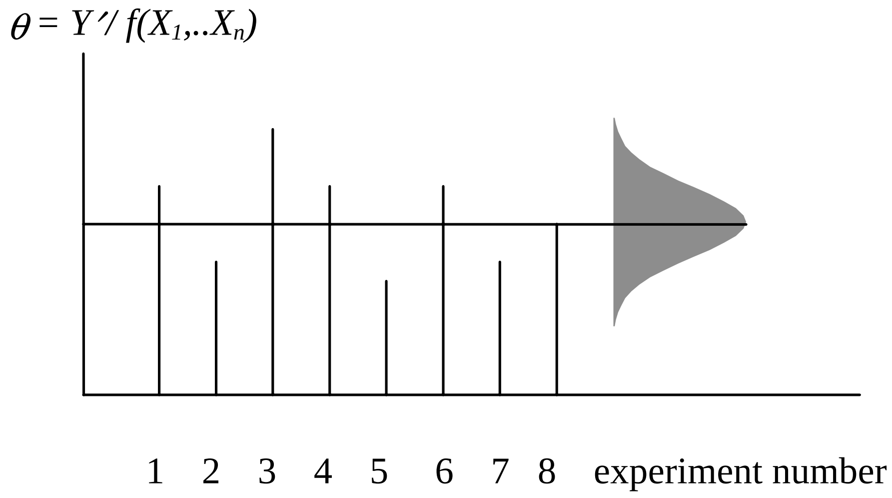

Figure 3.9.1: estimation of model uncertainty statistics on a number of tests following definition 3.9.3

3.9.2 Types of models for structural analysis

Model uncertainties can be subdivided into:

-load calculations models
-load effect calculation models
-local stiffness and resistance models

For the model uncertainties in the load models reference is made to Part 2.

The load effect calculation models have to do with the linear or nonlinear calculation of stresses, axial forces, shear forces and bending and torsional moments in the various structural elements. The model uncertainties are usually the result of negligence of for example 3D-effects, inhomogenities, interactions, boundary effects, simplification of connection behaviour, imperfections and so on. The scatter of the model uncertainty will also depend on the type of structure (frame, plates, shell, solids, etc).

The local models are used to define the behaviour of an element, a typical cross section or even of the material in a single point. One may think in this respect of the visco-elastic model, the elastic plastic model, the yield condition (Von Mises, Tresca, Coulomb), the hardening and softening behaviour, the thermal properties and so on.

3.9.3. Recommendations for practice

Models may be of a numerical , analytical or empirical nature. In the recommended values in Table 3.9.1 a more or less standard structural Finite Element Model has been kept in mind.

The model uncertainties are assumed to be partly correlated throughout the structure: on one point of the structure the circumstances will usually be different from another point which makes it unlikely that a full correlation exists. For that reason the Table 3.9.1 also includes an estimate for the degree of correlation between various points or critical cross sections in one structure.

Table 3.9.1 Recommended probabilistic models for Model Uncertainties

	Model type	Distr	mean	CoV	correlation
	load effect calculation				
	moments in frames	LN	1.0	0.1	
	axial forces in frames	LN	1.0	0.05	
	shear forces in frames	LN	1.0	0.1	
	moments in plates	LN	1.0	0.2	
	forces in plates	LN	1.0	0.1	
	stresses in 2D solids	N	0.0	0.05	
	stresses in 3D solids	N	0.0	0.05	
	resistance models steel (static)				
	bending moment capacity (1)	LN	1.0	0.05	
	shear capacity	LN	1.0	0.05	
	welded connectio capacity	LN	1.15	0.15	
	bolted connection capacity	LN	1.25	0.15	
	resistance models concrete (static)				
	bending moment capacity (1)	LN	1.2	0.15	
	buckling	LN	1.4	0.25	
	shear capacity	LN	1.0	0.1	
	connection capacity				

(1) including the effects of normal and shear forces.

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3.10 DIMENSIONS

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3.10.1 External dimensions of concrete components
3.10.2 Concrete cover
3.10.3 Differences between concrete columns, slabs and beams
3.10.4 Cross-section dimensions of hot rolled steel products
3.10.5 Theoretical models
3.10.6 Correlations
3.10.7 References

3.10 DIMENSIONS

3.10.1 External dimensions of concrete components

In the following the only time independent effects are considered. Dimensional deviations of a dimension X is described by statistical characteristics of its deviations Y from the nominal value X_{nom}:

$$Y = X - X_{nom} \tag{1}$$

Concerning external (perimeter) dimensions of reinforced concrete cross-section of horizontal members (beams, plates), available data are quite extensive, although not convincing. The following general remarks follow from recent analysis of large samples of measurements [1,2,3,4]. It has been observed that the following aspects do not significantly affect dimensional deviations of reinforced concrete cross-section:
- the type of the elements (reinforced, prestressed),
- the shape of the cross/section (rectangular, I, T, L),
- the class of concrete (strength of concrete),
- dimension orientation (depth, width),
- position of the cross-section (mid-span, support).

It has been found [4] that external dimensions of concrete cross-sections are only slightly dependent on the mode of production (precast, cast in situ).

When precast and cast in situ elements are taken together [2] then for the mean and standard deviation (the normal distribution seems to be satisfactory) of Y may be expected within the limits:

$$0 \le \mu_y = 0.003\, X_{nom} \le 3 \text{ mm} \tag{2}$$

$$\sigma_y = 4 \text{ mm} + 0.006\, X_{nom} \le 10 \text{ mm} \tag{3}$$

These formulae are valid for the nominal value X_{nom}, up to about 1000 mm (no significant dependence is observed beyond this size). Note that recent European document [6] on execution of concrete structures specifies is in a good agreement with the above mentioned data. The maximum permitted deviation ± 19 mm (corresponding to about σ_y = 12 mm is specified given for X_{nom} = 1000 mm

3.10.2 Concrete cover

Top Steel

According to the data reported in [1] the average concrete cover to the top steel of beams and slabs in systematically greater than the nominal value (by about 10 mm), the standard deviation is also around 10 mm (within an interval from 5 to 15 mm). Reasonable average formulae (with a great uncertainty) for the cover to beam and plate to steel may be written in an approximate form as:

$$5 \text{ mm} \le \mu_y \le 15 \text{ mm} \tag{4}$$

$$5 \text{ mm} \leq \sigma_y \leq 15 \text{ mm} \quad (5)$$

Bottom steel

Even more scattered and less conclusive are data indicated [1] for cover to bottom steel of beams and slabs. Depending on type of spacers (and perhaps on many other production conditions) data reported in [1] indicate that the mean μ_y may be expected within an extremely broad range from -20 mm to +20 mm, while the standard deviation seems to be relatively small, around 5 mm only, thus

$$-20 \text{ mm} \leq \mu_y \leq 20 \text{ mm} \quad (6)$$

$$\sigma_y \cong 5 \text{ mm} \quad (7)$$

Effective depth

Obviously, the above relations are providing only gross estimates and particular values must be chosen taking into account other specific conditions. Nevertheless, they are in a reasonable agreement with observations concerning effective depth of the cross-section (the depth and concrete cover could be highly correlated). If no further information is available, it is indicated in [2] that the characteristics may be assessed by:

$$\mu_y \cong 10 \text{ mm} \quad (8)$$

$$\sigma_y \cong 10 \text{ mm} \quad (9)$$

Further experimental measurements (related to specified production procedure) with a special emphasis on internal dimensions of horizontal, as well as vertical elements are obviously needed.

3.10.3 Differences between concrete columns, slabs and beams

Concerning external dimensions no significant differences have been found between columns, slabs and beams [4]. There are, however, some differences in concrete cover of these elements. Table 2 shows characteristics of concrete cover based on data reported in [1] collected in UK.

Table 1. Characteristics of concrete cover.

Concrete cover	Mean μ_Y [mm]	Standard deviation σ_Y [mm]
in column - [1] (two samples)	1; 3	0,2; 7
in wall - [1] (one sample-241 obs.)	1	12
of slab bottom steel - [1] and {UK}	–8 to 5 {7 to 23}	6 to 15 {3 to 4}
of slab top steel - [1] and {UK}	– 13 to 11 {5 to 16}	11 to17 {6 to 16}
of beam bottom steel in UK	– 17* to 3	2 to 5
of beam top steel in UK	1 to 12	8 to14

* Note: The negative mean of deviations was observed when using plastic spacers.

According to the data in Table 1 the following characteristics of concrete cover may be considered as a first approximations (intervals indicated for the mean and standard deviation represent a reasonable bonds which are dependent on particular conditions and quality of production):

- column and wall:

$$\mu_Y = 0 \text{ to } 5 \text{ mm} \tag{10}$$

$$\sigma_Y = 5 \text{ to } 10 \text{ mm} \tag{11}$$

- slab bottom steel:

$$\mu_Y = 0 \text{ to } 10 \text{ mm} \tag{12}$$

$$\sigma_Y = 5 \text{ to } 10 \text{ mm} \tag{13}$$

- beam bottom steel:

$$\mu_Y = -10 \text{ to } 0 \text{ mm} \tag{14}$$

$$\sigma_Y = 5 \text{ to } 10 \text{ mm} \tag{15}$$

- slab and beams top steel:

$$\mu_Y = 0 \text{ to } 10 \text{ mm} \tag{16}$$

$$\sigma_Y = 10 \text{ to } 15 \text{ mm} \tag{17}$$

Obviously, these values represent only very gross estimates of basic statistical characteristics of concrete cover and particular values should be chosen in accordance with relevant production conditions. Further experimental measurements (related to given production procedure) with a specific emphasis on internal dimensions of horizontal, as well as vertical elements are obviously needed.

Note that recent European document [6] on execution of concrete structures specifies is in a good agreement with the above mentioned data. The minimum permitted deviation of concrete cover is -10 mm (corresponding to $\sigma_y = 6$), the maximum permitted deviation is from 10 mm up to 20 mm (corresponding to about σ_y from 6 to 13 mm).

3.10.4 Cross-section dimensions of hot rolled steel products

In Czech Republic [5] some data on dimensional deviations of cross-sections of rolled products (profile I, L, T) are collected and evaluated at present. Preliminary results obtained for profile I (IPE 80 to 200) indicate that the mean and standard deviation of Y of the basic dimensions (height, width and thickness) is less than 1 mm, while the coefficient of skewness is negligible, thus the indicative values of the deviation characteristics are

$$-1{,}0 \text{ mm} \le \mu_Y \le +1{,}0 \text{ mm} \tag{18}$$

$$\sigma_Y \le 1{,}0 \text{ mm} \tag{19}$$

For cross-section area and modulus it has been found that independently on the profile height the mean of both quantities differ from their nominal values insignificantly (the differences are practically zero) and the coefficients of variations for cross-section area are about 3,2 %, for cross-section modulus about 4,0 %. The normal distribution seems to be fully satisfactory model for all geometrical properties.

3.10.5 Theoretical models

Several theoretical models were considered in previous studies [1] and [2]. It appears that unless further data are available, normal distribution provides a good general model for external dimensions of both reinforced concrete and steel elements and also for effective depth of reinforced concrete cross-section.

However, concrete cover to reinforcement in concrete cross-sections of various concrete elements is a special random variable, which may hardly be described by a normal distribution. In this case different types of one or two side-limited distribution should be considered.

Taking into account various combination of the coefficient of variation $w = \sigma/\mu$, and skewness α (the subscripts are omitted here), the following commonly used distributions could be considered:

- for all w and α beta distribution with general lower and upper bound a and b, denoted Beta(μ;σ,a;b),
- for all w and α>0 shifted lognormal distribution with lower bound a, denoted sLN(μ;σ; a),
- for all $\alpha < 2w$ beta distribution with the lower a at zero (a = 0), and a general upper bound b, which is denoted Beta(μ;σ;0;b),
- for $\alpha = 3w+w^3$ lognormal distribution with the lower bound a at zero (a = 0),
- for $\alpha = 2w$ gamma distribution (which has the lower bound a at zero (a = 0) by definition), denoted Gamma(μ;σ).

3.10.6 Correlations

It has been found [4] that external dimensions of concrete cross-sections are only slightly dependent on the mode of production (precast, cast in situ). No significant correlation (the correlation coefficients being around 0,12) has been found between vertical and horizontal dimensions. No data are available concerning correlation of internal (concrete cover) and external dimensions even though the depth and concrete cover of some elements could be highly correlated. There may be a strong auto-correlation along the element; correlation distance may be assessed as multiple (say from 3 to 5) of the cross section height or as a part of the span (say 1/4 to 1/2).

3.10.7 References

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3.11 EXCENTRICITIES

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3.11.2 Basic model
3.11.3 Probability modelling
3.11.4 References

List of symbols:

$\rho(i,j)$	= coefficient of correlation for two columns i and j
e	= average eccentricities
f	= central eccentricity due to curvature
ϕ	= out of plumbness
μ	= mean
σ	= standard deviation

3.11.1 Introduction

The bearing capacity of slender elements depends to some extend on the difference between the actual and theoretical lining, the so called excentricity. In this section we will present the models for the excentrities of columns in braced and unbraced frameworks.

3.11.2 Basic model

In the analysis three types of eccentricities can be distinguished (see figure 3.11.1)

- the average excentricity
- the initial curvature
- the out of plumbness ϕ

For the braced frame the out of plumpness is only relevant for the bracing system, but not for the column under consideration; for the unbraced frame especially the out of plumpness is usually dominant over the end point eccentricity and the curvature.

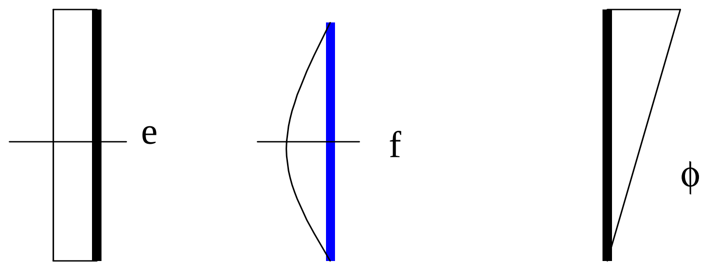

Figure 3.11.1: The three basic excentiricties e, f and ϕ

3.11.3 Probabilistic models

Distribution type, mean and scatter

The probabilistic model for the three basic parameters are presented in Table 3.11.1. For all three cases it is assumed that the distribution is symmetrical around zero and that small eccentricities are more likely than large ones, although large ones are more dangerous. Note that in special cases non-symmetrical cross sections may have $\mu(f) \neq 0$ due to the fabrication process.

In many cases only the absolute values of the excentricities are important. From the table it can be derived that these absolute values are distributed with a truncated normal distribution, the truncation point being the mean of the untruncated distribution. The absolute value has a mean of about 0.80 times the standard deviation of the untruncated distribution; the coefficient of variation is 0.75.

X	description	type	μ	σ
e	average excentricity	normal	0 m	L/1000
f	out of straightness	normal	0 m	L/1000
ϕ	the out of plumbness	normal	0 rad	0.0015 rad

Table 3.11.1 statistical properties for excentricities (for steel and concrete columns)

All eccentricity parameters e, f and ϕ shall be regarded as independent variables.

Time and spatial dependency

In general eccentricities may be treated as being time independent. An exception might be timber where in particular the initial curvature may depend on the moisture content.

For the spatial fluctuation the dependency between various columns in one building is important. In this code the average eccentricity e as well as the out of straightness f will be considered as being uncorrelated for all members. For ϕ the following correlation pattern is recommended:

$\rho(\phi_i,\phi_j) = 0{,}5$ for two columns on the same floor
$\rho(\phi_i,\phi_j) = 0$ for columns on different floors

In this model some possible negative correlation between columns in vertical direction, resulting from (over) corrections for out of plumbness on lower storeys is not considered. This is a conservative assumption.

Note on applications

The limit state function for a simple slender column, clamped at the bottom and free at the top, may be presented as:

$$Z = M_p - \frac{P_E}{P_E - P} P\phi h$$

M_p = plastic moment
P = vertical load
P_E = Euler buckling load
h = height of the column

3.11.4. References

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