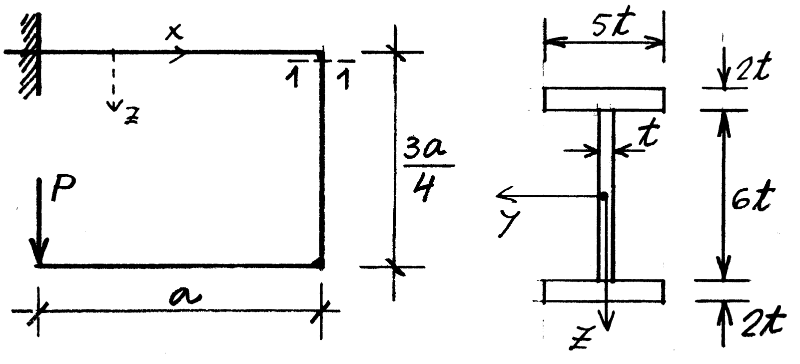
1. Nosilec I–prereza z lomljeno osjo je v prostem krajišču obtežen s točkovno silo $P = 19\,kN$.
   a. Dimenzioniraj prikazani nosilec, če je dovoljena normalna napetost $[\sigma] = 16.5\,kN/cm^2$, dovoljena strižna napetost pa $[\tau] = 9.5\,kN/cm^2$.
   b. Določi značilne vrednosti in nariši diagrame normalnih in strižnih napetosti v prerezu 1–1!
   c. S *Castigliano*-vim izrekom določi navpični pomik prijemališča sile P!

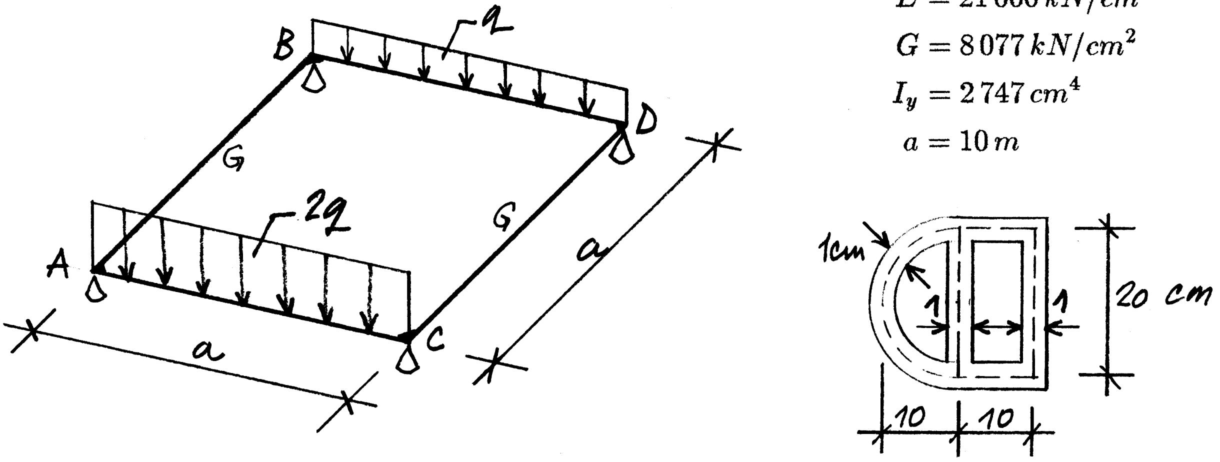


$E = 21\,000\,kN/cm^2$

$a = 2.0\,m$

$A_x \gg A_p$

2. Na skici je prikazan prečni prerez veznih nosilcev AB in CD. Določi dovoljeno enakomerno zvezno obtežbo q, če je dopustna strižna napetost $[\tau] = 10\,kN/cm^2$ in nastopa v elementih prikazane konstrukcije čista torzija!



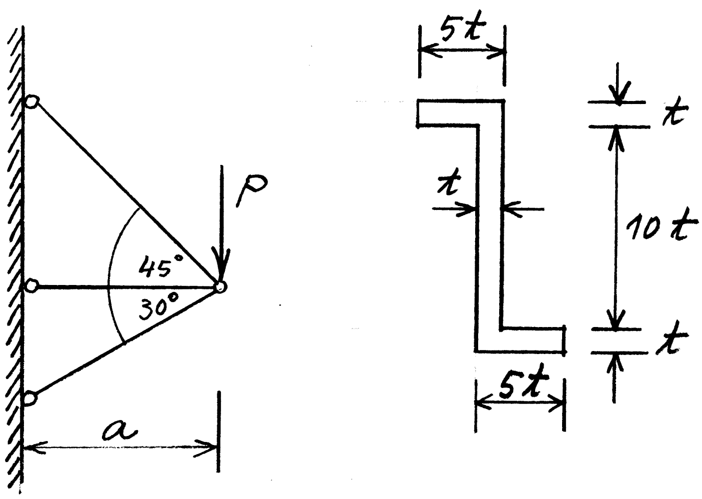
$E = 21\,000\,kN/cm^2$

$G = 8\,077\,kN/cm^2$

$I_y = 2\,747\,cm^4$

$a = 10\,m$

3. Vse palice prikazane konstrukcije imajo enak prečni prerez (Z–profil). Določi dimenzijo t tako, da bo varnost glede na lokalni uklon enaka 2!

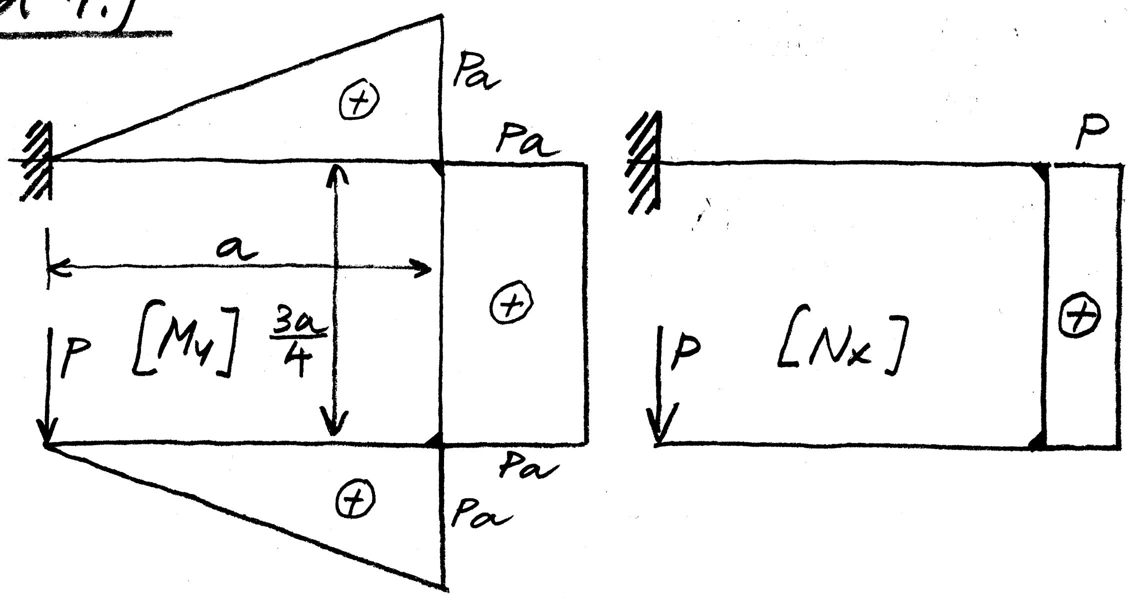


$P = 40\,kN$

$a = 3\,m$

$E = 20\,000\,kN/cm^2$

Ad 1.)



a) $M_{y\,max} = P \cdot a = 19 \cdot 200 = 3800$ kNcm

$N_{max} = P = 19$ kN

$$I_y = \frac{1}{12}\left[5t\cdot(10t)^3 - 4t\cdot(6t)^3\right] \rightarrow \boxed{I_y = 344{,}67\,t^4}$$

$$W_y = \frac{I_y}{5t} \longrightarrow \boxed{W_y = 68{,}93\,t^3}$$

$$A_x = 2\times 5t\cdot 2t + 6t\cdot t \rightarrow \boxed{A_x = 26\,t^2}$$

$$\sigma_{xx\,max} = \frac{M_{y\,max}}{W_y} + \frac{N_{x\,max}}{A_x} = \frac{3800}{68{,}93\,t^3} + \frac{19}{26\,t^2}$$

$$\sigma_{xx\,max} = \frac{55{,}126}{t^3} + \frac{0{,}731}{t^2} = [\sigma] = 16{,}5$$

$$16{,}5\,t^3 - 0{,}731\,t - 55{,}126 = 0$$

$$\phi(t) = 16{,}5\,t^3 - 0{,}731\,t - 55{,}126 = 0$$
$$\phi'(t) = 49{,}5\,t^2 - 0{,}731$$

$$t_{i+1} = t_i - \frac{\phi(t_i)}{\phi'(t_i)}$$

$t_0 = 1$ cm ; $t_1 = 1{,}807$ cm , $t_2 = 1{,}553$ cm ,
$t_3 = 1{,}506$ cm , $t_4 = 1{,}505$ cm

izberem : $\boxed{t = 1{,}51 \text{ cm}}$

$\boxed{I_y = 1792 \text{ cm}^4}$, $\boxed{A_x = 59{,}3 \text{ cm}^2}$

$\boxed{W_y = 237{,}3 \text{ cm}^3}$

$$\sigma_{xx}^{n} = \frac{3800}{237{,}3} + \frac{19}{59{,}3} = 16{,}33 \text{ kN/cm}^2$$

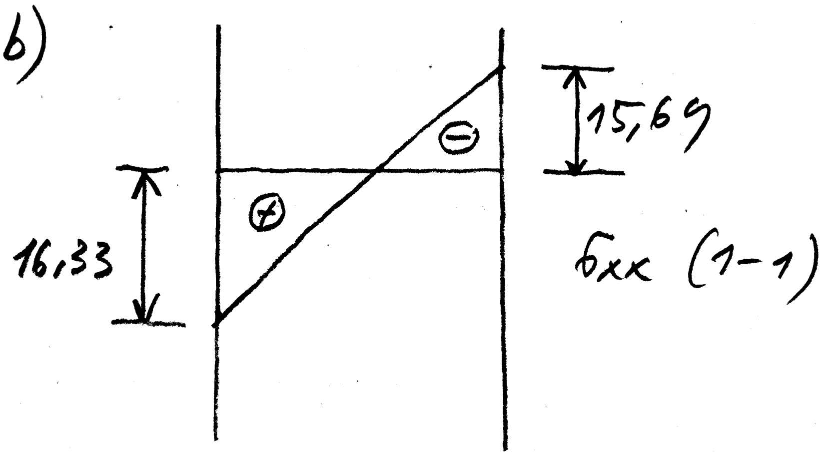
$$\sigma_{xx}^{z} = \frac{-3800}{237{,}3} + \frac{19}{59{,}3} = -15{,}69 \text{ kN/cm}^2$$

$$S_y^*(z=-3t) = 10t^2 \cdot 4t = 40t^3 = 137{,}7 \text{ cm}^3$$

$$S_y^*(z=0) = 40t^3 + 3t^2 \cdot 1{,}5t = 44{,}5t^3 = 153{,}2 \text{ cm}^3$$

$N_z(1-1) = 0$! $\sigma_{xz}(1-1) = 0$

b)



c) $$D = \frac{1}{2}\int_L \frac{M_y^2}{EI_y}\,dx = \frac{1}{2EI_y}\left\{2\cdot\frac{Pa^2}{2}\cdot\frac{2}{3}Pa + Pa\,\frac{3a}{4}\cdot Pa\right\}$$

$$D = \frac{1}{2EI_y}\cdot\frac{17P^2a^3}{12} \longrightarrow \boxed{D = \frac{17P^2a^3}{24EI_y}}$$

$$w_P = \frac{\partial D}{\partial P} = 2\cdot\frac{17Pa^3}{24EI_y}$$

$$\boxed{w_P = P\,\frac{17a^3}{12EI_y}}$$

$$\boxed{w_P = 0{,}3\ \text{cm}}$$

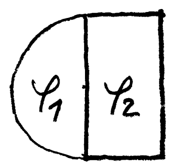
Ad 2.)

$$A_1 = \frac{1}{2}\pi\cdot 10^2 \longrightarrow A_1 = 157{,}1\ \text{cm}^2$$

$$A_2 = 10\cdot 20 \longrightarrow A_2 = 200\ \text{cm}^2$$

$$a_{11} = \frac{1}{1}(20 + 10\pi) = 51{,}4$$

$$a_{22} = \frac{1}{1}(2\cdot 10 + 2\cdot 20) = 60$$



$$a_{12} = a_{21} = -\frac{1}{1}\cdot 20 = -20$$

$$\begin{bmatrix} 51{,}4 & -20 \\ -20 & 60 \end{bmatrix}\cdot\begin{bmatrix}\varphi_1 \\ \varphi_2\end{bmatrix} = \begin{bmatrix}157{,}1\cdot 2 \\ 200\cdot 2\end{bmatrix} \longrightarrow \boxed{\begin{matrix}\varphi_1 = 10 \\ \varphi_2 = 10\end{matrix}}$$

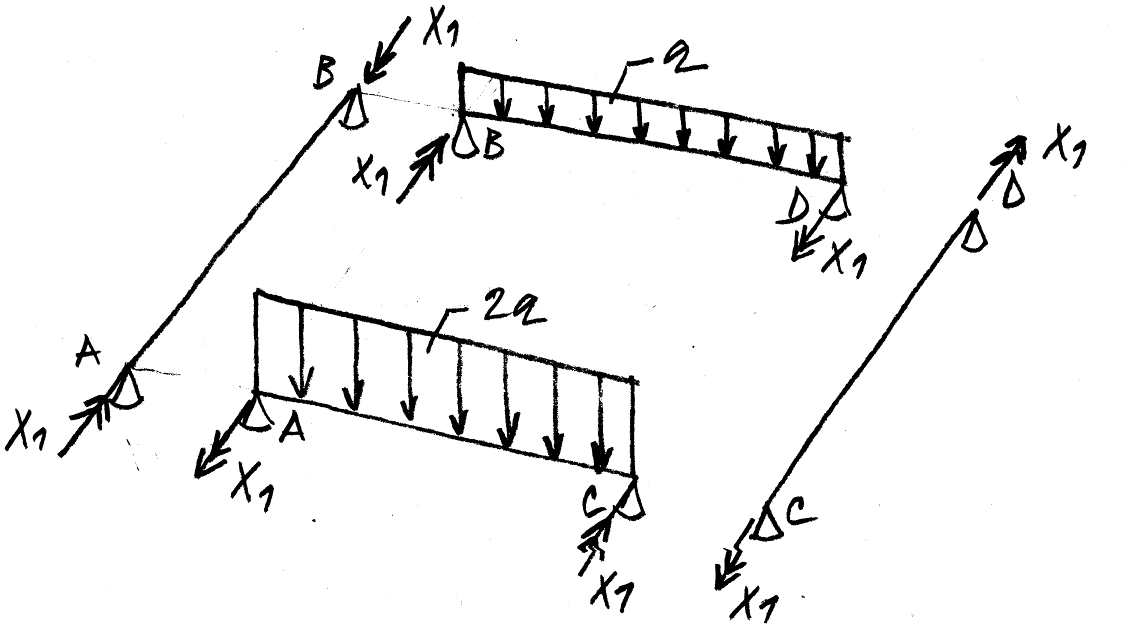
$$I_x = 2(\varphi_1 A_1 + \varphi_2 A_2) = 2\cdot 10\,(157{,}1 + 200)$$

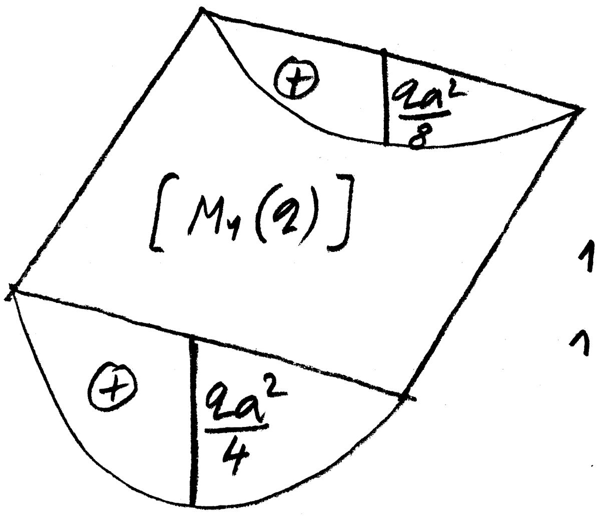
$$\boxed{I_x = 7142\ \text{cm}^4}$$

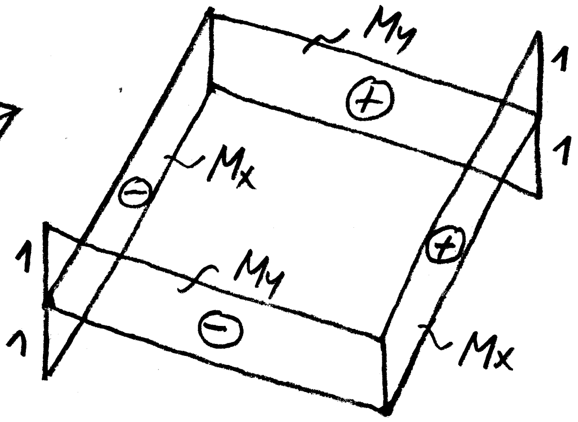
$$GI_x = 8077 \cdot 7142 = 57.685.934 \text{ kNcm}^2$$

$$EI_y = 21000 \cdot 2747 = 57.687.000 \text{ kNcm}^2$$

$$\boxed{EI_y \cong GI_x}$$







$M_x, M_y \ (X_1 = 1)$

$$a_{11} = \frac{2a}{EI_y} + \frac{2a}{GI_x} \longrightarrow a_{11} = \frac{4a}{EI_y}$$

$$b_1 = \frac{1}{EI_y}\left(-\frac{qa^2}{4} \cdot \frac{2a}{3} + \frac{qa^2}{8} \cdot \frac{2a}{3}\right) \rightarrow b_1 = -\frac{qa^3}{12EI_y}$$

$$a_{11} X_1 + b_1 = 0 \longrightarrow 4a X_1 = \frac{qa^3}{12}$$

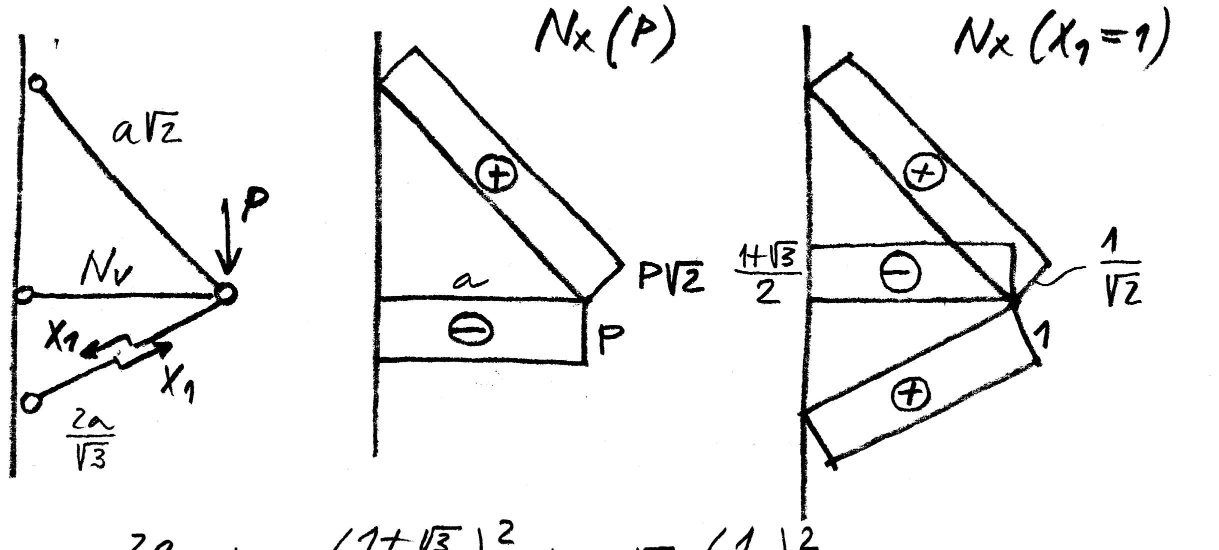
$$\boxed{X_1 = \frac{qa^2}{48}} \qquad |M_x(AB)| = X_1 = \frac{q \cdot 1000^2}{12}$$

$$\tau_{max} = \frac{M_x}{I_x} \cdot \left(\frac{\partial \varphi}{\partial \eta}\right)_{max} \leq [\tau]$$

$$M_x = [\tau] \cdot I_x \cdot \frac{1}{\left(\frac{\partial \varphi}{\partial \eta}\right)_{max}} = 10 \cdot 7142 \cdot \frac{1}{10} = 7142 \text{ kNcm}$$

$$7142 = \frac{q \cdot 1000^2}{12} \longrightarrow \boxed{\begin{array}{l} [q] = 0{,}086 \text{ kN/cm} \\ [q] = 8{,}6 \text{ kN/m} \end{array}}$$

Ad 3.)



$$a_{11} = \frac{2a}{\sqrt{3}} + a\left(\frac{1+\sqrt{3}}{2}\right)^2 + a\sqrt{2}\cdot\left(\frac{1}{\sqrt{2}}\right)^2$$

$$a_{11} = a\left[\frac{2}{\sqrt{3}} + \left(\frac{1+\sqrt{3}}{2}\right)^2 + \frac{\sqrt{2}}{2}\right] \rightarrow a_{11} = 4{,}7278\, a$$

$$b_1 = a\sqrt{2} \cdot P\sqrt{2} \cdot \frac{1}{\sqrt{2}} + Pa\frac{1+\sqrt{3}}{2}$$

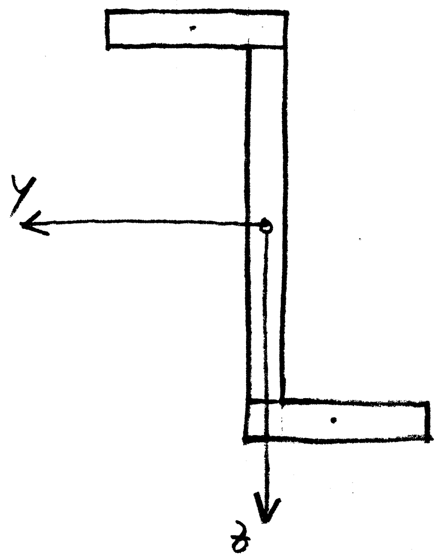
$$b_1 = a\left[\sqrt{2} + \frac{1+\sqrt{3}}{2}\right]P \rightarrow b_1 = Pa \cdot 2{,}7802$$

$$a_{11}X_1 + b_1 = 0 \longrightarrow 4{,}7278\, a\, X_1 = -2{,}7802\, Pa$$

$$\boxed{X_1 = -0{,}588\ P}$$

$$N_V = -P + 0{,}588\, P \cdot \frac{1+\sqrt{3}}{2} \longrightarrow \boxed{N_V = -0{,}197\ P}$$

$$\boxed{N^t_{max} = 0{,}588\, P = 23{,}522\ kN}$$



$$I_y = \frac{1}{12}\left[5t\cdot(12t)^3 - 4t\cdot(10t)^3\right]$$

$$I_y = 386{,}67\, t^4$$

$$I_z = \frac{1}{12}\left[2\cdot t\cdot(5t)^3 + 10t\cdot t^3\right] +$$

$$+2\cdot 5t^2\cdot(2t)^2 \rightarrow I_z = 61{,}67\, t^4$$

$$I_{yz} = -\left[-5t^2 \cdot 2t \cdot 5{,}5t \cdot 2\right]$$

$$I_{yz} = 110\, t^4$$

$$I_{1,2} = \frac{I_y + I_z}{2} \pm \sqrt{\left(\frac{I_y - I_z}{2}\right)^2 + I_{yz}^2}$$

$$I_{1,2} = \left\{\frac{386{,}67 + 61{,}67}{2} \pm \sqrt{\left(\frac{386{,}67 - 61{,}67}{2}\right)^2 + 110^2}\right\} t^4$$

$$I_1 = 420{,}40\, t^4 \qquad \boxed{I_2 = I_{min} = 27{,}94\, t^4}$$

$$N_{kr} = \frac{\pi^2 E I_{min}}{l_u^2} = 2 N^t_{max} \qquad l_u = \frac{2a}{\sqrt{3}}$$

$$I_{min} = 2 N^t_{max} \frac{4a^2}{3 \cdot \pi^2 E}$$

$$27{,}94\, t^4 = 2 \cdot 23{,}522 \frac{4 \cdot 300^2}{3 \pi^2 \cdot 20000}$$

$$t^4 = 1{,}0236 \text{ cm}^4$$

$$\boxed{t \doteq 1{,}00 \text{ cm}}$$