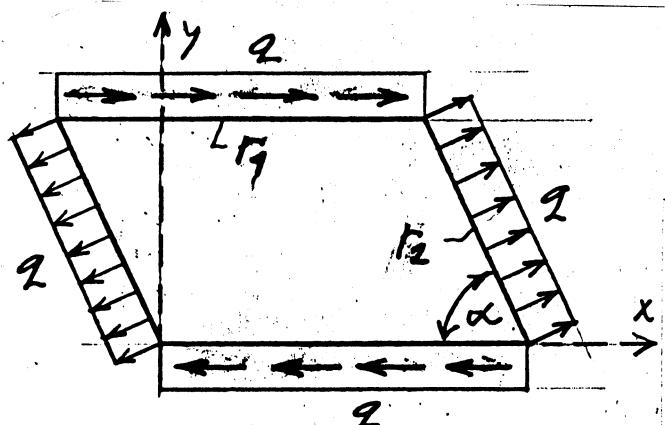


1. Na element enakomerno debele stene deluje zunanj obtežba  $q$ , kot kaže skica. Ob predpostavki, da vlada v elementu homogeno ravninsko napetostno stanje, določi kot  $\alpha$ , pri katerem je element v ravnotežju! Določi velikosti in smeri glavnih normalnih deformacij v tem primeru ter jih označi na skici! (NASVET: Zapiši ravnotežna pogoja na robovih  $r_1$  in  $r_2$ , določi  $\alpha$ , nato pa še preostale napetosti!)

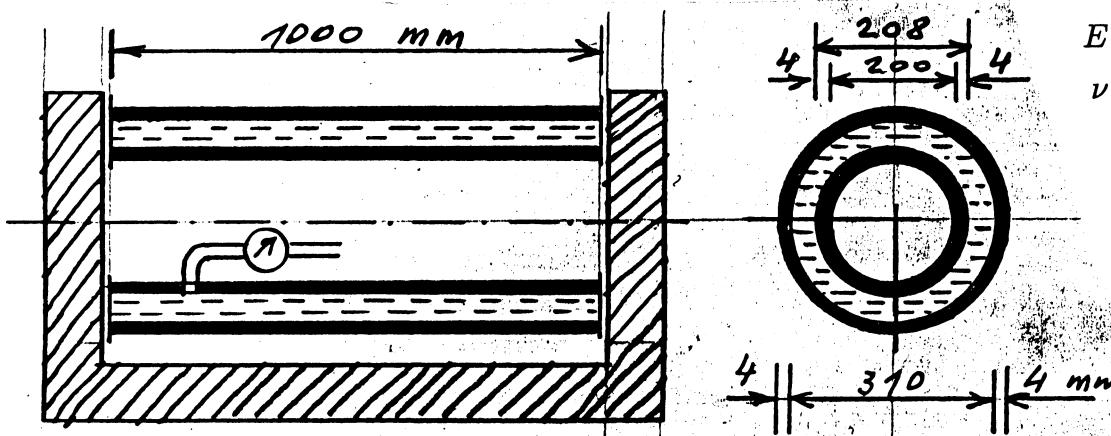


$$E = 200\,000 \text{ MPa}$$

$$\nu = 0.25$$

(35 točk)

2. V vmesni prostor med dvema bakrenima cevema z debelino stene  $\delta = 4 \text{ mm}$  načrpamo nestisljivo hladilno tekočino. Koliko tekočine porabimo, da znaš hidrostatici tlak  $p = 6 \text{ MPa}$ ? Kolikšne so tedaj normalne napetosti v tangencialni smeri v obeh ceveh? (Tesnila ob nepodajnih priključnih ploščah omogočajo neovirano deformiranje cevi. Vzdolžne normalne napetosti so zanemarljive.)

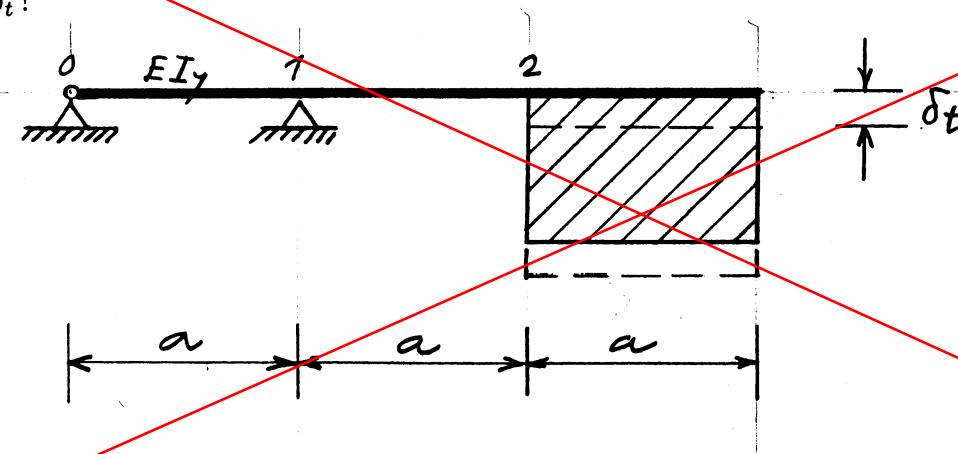


$$E = 100\,000 \text{ MPa}$$

$$\nu = 0.3$$

(35 točk)

3. Elastičen nosilec je v točki 2 togo vpet v masiven temelj. Določi reakcije, ki nastopijo v podporah 0 in 1, če se masivni temelj enakomerno posede za  $\delta_t$ ! Rezultate izrazi v odvisnosti od posedka  $\delta_t$ !

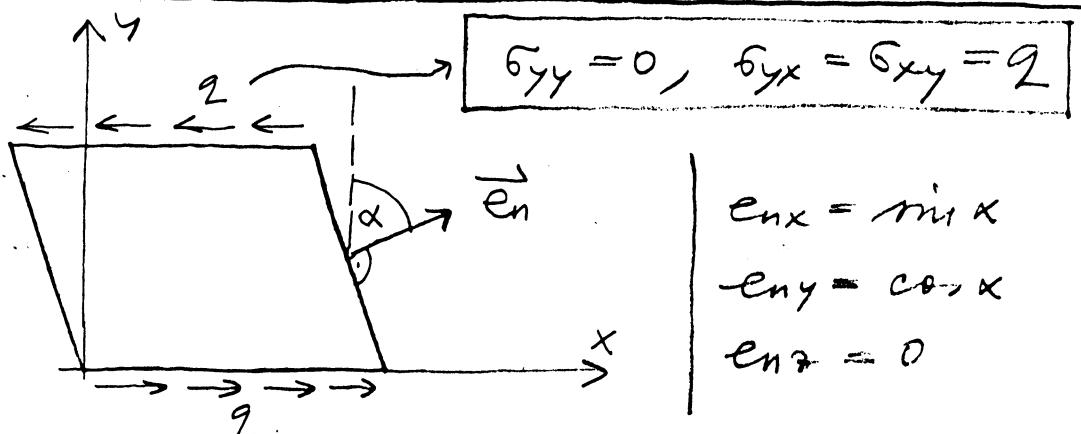


(40 točk)

MTT

IZPIT

Ad 1.)



$$\vec{e}_n = \sin \alpha \vec{e}_x + \cos \alpha \vec{e}_y \quad p_n = 2 \vec{e}_n$$

$$\vec{p}_n = 2 \sin \alpha \vec{e}_x + 2 \cos \alpha \vec{e}_y = \vec{\sigma}_x e_{nx} + \vec{\sigma}_y e_{ny}$$

$$\sigma_{nx} = 2 \sin \alpha = \sigma_{xx} e_{nx} + \sigma_{xy} e_{ny} = \sigma_{xx} \sin \alpha + 2 \cos \alpha$$

$$[\sigma_{ny} = 2 \cos \alpha = \sigma_{xy} e_{nx} + \sigma_{yy} e_{ny} = 2 \sin \alpha]$$

$$\rightarrow \cos \alpha = \sin \alpha \rightarrow \alpha = 1 \rightarrow \boxed{\alpha = 45^\circ}$$

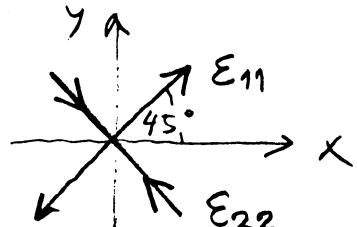
$$\sin \alpha = \cos \alpha = \frac{\sqrt{2}}{2} \rightarrow 2 \frac{\sqrt{2}}{2} = \sigma_{xx} \frac{\sqrt{2}}{2} + 2 \frac{\sqrt{2}}{2}$$

$$[\sigma_{ij}] = \begin{bmatrix} 0 & 2 & 0 \\ 2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \sigma_{11,22} = \pm \sqrt{\sigma_{xy}^2} = \pm 2$$

$\boxed{\sigma_{xx} = 0}$

$\boxed{\sigma_{11} = 2, \sigma_{22} = -2}$

$$\text{if } 2\alpha_0 = \frac{2\sigma_{xy}}{\sigma_{xx} - \sigma_{yy}} = \infty \rightarrow 2\alpha_0 = 90^\circ \rightarrow \boxed{\alpha_0 = 45^\circ}$$



$$\boxed{I_7 = 0}$$

$$\boxed{\epsilon_{11} = \frac{1+\nu}{E} 2}$$

$$\boxed{\epsilon_{22} = -\frac{1+\nu}{E} 2}$$

$$\boxed{\epsilon_{33} = 0}$$

$$\text{Ad 2)} \quad D_1 = 208 \text{ mm} \quad , \quad D_2 = 310 \text{ mm}$$

$$V = \frac{\pi l}{4} (D_2^2 - D_1^2) \quad \dots \quad V' = \frac{\pi l}{4} (D_2'^2 - D_1'^2)$$

Notmengfa cew:  $\sigma_{11}^1 = -\frac{p D_1}{2\delta}$ ,  $\sigma_{rr}^1 = -p$

$$\varepsilon_{11}^1 = \frac{1}{E} (\sigma_{11}^1 - \nu \sigma_{rr}^1) = -\frac{p}{E} \left( \frac{D_1}{2\delta} - \nu \right)$$

$$\varepsilon_{11}^1 = -\frac{p}{E} \left( \frac{D_1}{2\delta} - \nu \right) \rightarrow \boxed{\varepsilon_{11}^1 = -0,00154}$$

Zumangfa cew:  $\sigma_{11}^2 = \frac{p D_2}{2\delta}$ ,  $\sigma_{rr}^2 = -p$

$$\varepsilon_{11}^2 = \frac{1}{E} (\sigma_{11}^2 - \nu \sigma_{rr}^2) = \frac{p}{E} \left( \frac{D_2}{2\delta} - \nu \right)$$

$$\boxed{\varepsilon_{11}^2 = 0,00231}$$

$$D_1' = D_1 (1 + \varepsilon_{11}^1) = 208 \cdot (1 - 0,00154)$$

$$\boxed{D_1' = 207,679 \text{ mm}}$$

$$D_2' = D_2 (1 + \varepsilon_{11}^2) = 310 \cdot (1 + 0,0023)$$

$$\boxed{D_2' = 310,715 \text{ mm}}$$

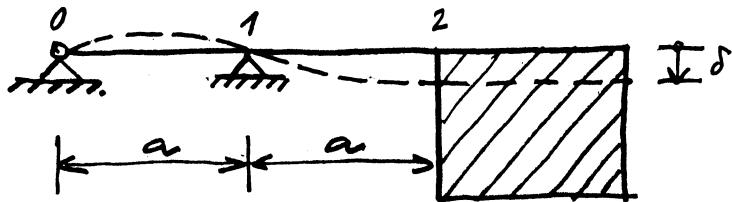
$$V' = \frac{\pi \cdot 1000}{4} (310,715^2 - 207,679^2) = 41.950.661 \text{ mm}^3$$

$$\boxed{V' = 41,951 \text{ litrov}}$$

$$\sigma_{11}^n = -\frac{6 \cdot 208}{2 \cdot 4} \rightarrow \boxed{\sigma_{11}^n = -756 \text{ MPa}}$$

$$\sigma_{11}^z = \frac{6 \cdot 310}{2 \cdot 4} \rightarrow \boxed{\sigma_{11}^z = 232,5 \text{ MPa}}$$

Ad 3.



$$V_0 = \frac{M}{a} - P$$

$$V_1 = 2P - \frac{M}{a}$$

$$M_y = V_0 x + V_1 (x-a) = \left(\frac{M}{a} - P\right)x + \left(2P - \frac{M}{a}\right)(x-a)$$

$$M_y = -P(x - 2(x-a)) + \frac{M}{a}(x - (x-a)) = -EI_y w''$$

$$EI_y w'' = P(x - 2(x-a)) - \frac{M}{a}(x - (x-a))$$

$$EI_y w' = \frac{P}{2}(x^2 - 2(x-a)^2) - \frac{M}{2a}(x^2 - (x-a)^2) + C_1$$

$$EI_y w = \frac{P}{6}(x^3 - 2(x-a)^3) - \frac{M}{6a}(x^3 - (x-a)^3) + C_1 x + C_2$$

$$x=0 \dots w=0 \rightarrow C_2 = 0$$

$$x=a \dots w=0 \rightarrow \frac{P}{6}a^3 - \frac{M}{6a}a^3 + C_1 a = 0$$

$$C_1 = -\frac{Pa^2}{6} + \frac{Ma}{6}$$

$$\omega_y = -\frac{1}{EI_y} \left[ \frac{P}{2}(x^2 - 2(x-a)^2) - \frac{M}{2a}(x^2 - (x-a)^2) - \frac{Pa^2}{6} + \frac{Ma}{6} \right]$$

$$\boxed{\omega_y = \frac{P}{6EI_y} (a^2 - 3x^2 + 6(x-a)^2) - \frac{M}{6aEI_y} (a^2 - 3x^2 + 3(x-a)^2)}$$

$$\boxed{w = \frac{P}{6EI_y} (x^3 - 2(x-a)^3 - a^2 x) - \frac{M}{6aEI_y} (x^3 - (x-a)^3 - a^2 x)}$$

$$x=2a \dots w_y = 0$$

$$w_y(2a) = \frac{P}{6EI_y} (a^2 - 12a^2 + 6a^2) -$$

$$\frac{M}{6aEI_y} (a^2 - 12a^2 + 3a^2)$$

$$w_y(2a) = -\frac{5Pa^2}{6EI_y} + \frac{4Ma}{3EI_y} = 0 \rightarrow M = P \cdot \frac{5a}{8}$$

$$w(2a) = \frac{P}{6EI_y} (8a^3 - 2a^3 - 2a^3) - \frac{M}{6aEI_y} (8a^3 - a^3 - 2a^3)$$

$$w(2a) = P \frac{2a^3}{3EI_y} - M \frac{5a^2}{6EI_y}$$

$$w(2a) = P \frac{2a^3}{3EI_y} - P \frac{5a}{8} \cdot \frac{5a^2}{6EI_y}$$

$$w(2a) = P \frac{7a^3}{48EI_y} \rightarrow w(2a) = \delta$$

$$P = \delta \frac{48EI_y}{7a^3} \rightarrow M = \delta \frac{30EI_y}{7a^2}$$

$$V_0 = -\delta \frac{18EI_y}{7a^3}$$

$$V_1 = \delta \frac{66EI_y}{7a^3}$$