Dejan Zupan

IZPITNE NALOGE IN REŠITVE NALOG S POSTOPKOM IZ PREDMETA TRDNOST NA VISOKOŠOLSKEM ŠTUDIJU GRADBENIŠTVA

Igor Planinc

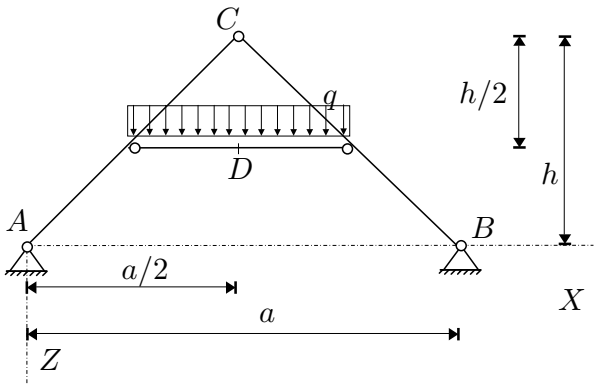
VPRAŠANJA IZ TEORIJE PRI PREDMETU TRDNOST NA VISOKOŠOLSKEM ŠTUDIJU GRADBENIŠTVA

ŠTUDIJSKO LETO: 2005/06

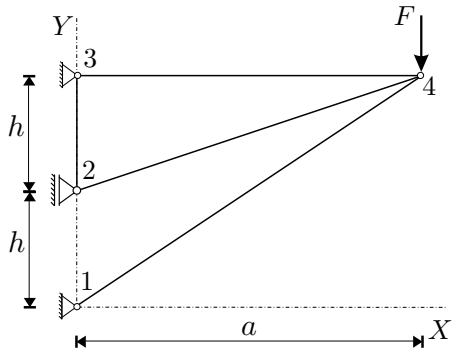
TRDNOST (VSŠ) - 1. IZPITNI ROK (25. 01. 2006)

RAČUNSKI DEL IZPITA:

1. Za konstrukcijo na sliki izračunajte notranje statične količine! Vpliva osnih in prečnih sil ni potrebno upoštevati.
(OBVEZNA NALOGA! 40%)
Podatki: $a = 3\,\mathrm{m}$, $h = 2\,\mathrm{m}$, $q = 15\,\mathrm{kN/m}$,
$E = 21000\,\mathrm{kN/cm^2}$, $J_y = 3000\,\mathrm{cm^4}$, $A_x = 40\,\mathrm{cm^2}$.



2. Za paličje na sliki določite pomike vozlišč in osne sile v palicah po metodi pomikov! (30%)
Podatki: $a = 6\,\mathrm{m}$, $h = 2\,\mathrm{m}$, $F = 15\,\mathrm{MN}$,
$E = 2 \cdot 10^5\,\mathrm{MPa}$, $A = 0.01\,\mathrm{m^2}$.



3. Deformiranje je podano s poljem pomikov $\vec{u} = 10^{-3}\left(xy, x+y, z^2\right)$.
Izračunajte:
a) tenzor majhnih deformacij;
b) vrednost tenzorja majhnih deformacij v točki $T(1,3,1)$;
c) glavne normalne deformacije in njihove smeri v točki $T(1,3,1)$.
(30%)

TEORETIČNI DEL IZPITA:

Izmed treh zastavljenih vprašanj si izberete dve, na kateri boste odgovarjali. Izbrani vprašanji jasno označite! Pišite čitljivo.

1. Pojasnite ravnotežne enačbe za delec telesa ter pripadajoče robne pogoje?

2. Opišite osnovne predpostavke pri upogibu z osno silo!

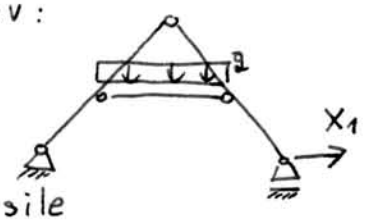
3. Napišite in komentirajte enačbe (pomen oznak) za Eulerjeve uklonske sile! Enačbo za določitev uklonske sile izpeljite za previsni nosilec (konzolo)!

TRDNOST - VSŠ 25. 1. 2006

1. NALOGA

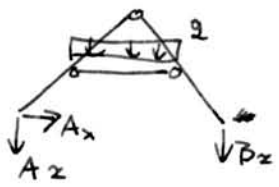
a.) $n = 4 + 3\cdot 2 - 3\cdot 3 = 1$

b.) sprostitev:



c.) notranje sile

c1.) zunanja obtežba



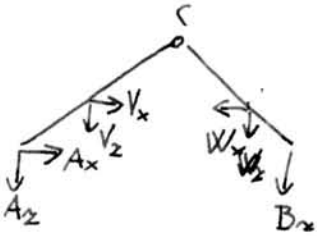
$\boxed{A_x = 0}$

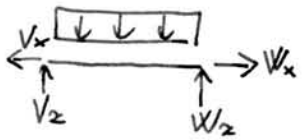
$A_z + B_z = -q\frac{a}{2}$

$-B_z\cdot a - q\frac{a}{2}\cdot\frac{a}{2} = 0$

$B_z = -q\frac{a}{4} = -11.25\ kN$

$A_z = -q\frac{a}{4} = -11.25\ kN$





$-V_x + W_x = 0$

$V_z + W_z = q\cdot\frac{a}{2}$

$W_z\cdot\frac{a}{2} - q\frac{a}{2}\cdot\frac{a}{4} = 0 \Rightarrow W_z = q\frac{a}{4}$

$V_z = q\frac{a}{4}$ $\boxed{V_z = 11.25\ kN}$

$\boxed{W_z = 11.25\ kN}$

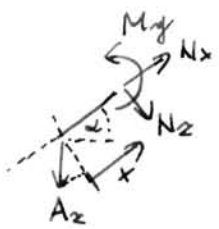
$\sum_{AC} M^C: \ A_z\cdot\frac{a}{2} + V_z\cdot\frac{a}{4} + V_x\cdot\frac{h}{2} = 0$

$V_x = \frac{-2}{h}\left(-q\frac{a^2}{8} + q\frac{a^2}{16}\right) = \frac{2a^2q}{h\cdot 16} = \frac{a^2q}{8h}$

$\boxed{V_x = 8.4\ kN}$

$\alpha = 53.13°$

polje I



$N_x = A_z\cdot\sin\alpha$

$N_z = -A_z\cdot\cos\alpha$

$M_y = -A_z\cdot\cos\alpha\cdot x$

$N_x = -9\ kN$

$N_z = 6.75\ kN$

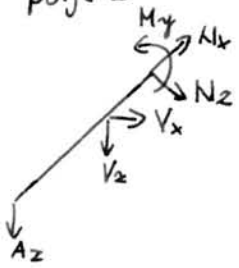
$M_y(0) = 0$

$M_y(1.25) = 8.4\ kN\,m$

$x \in [0, \ell]$

$\ell = \sqrt{1^2 + \left(\frac{3}{4}\right)^2} = 1.25$

polje II



$N_x = A_z\sin\alpha + V_z\sin\alpha - V_x\cos\alpha$

$N_x = -V_x\cos\alpha$

$N_z = -V_x\sin\alpha$

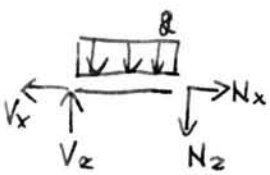
$M_y = -A_z\cos\alpha(\ell + x) - V_z\cos\alpha\cdot x - V_x\sin\alpha\cdot x$

$N_x = -5\ kN$

$N_z = -6.75\ kN$

$M_y = 8.4 - 6.75\cdot x$

polje III



$\boxed{N_x = 8.4\ kN}$

$\boxed{N_z = 11.25 - 15x}$

$N_z(\ell) = -11.25$

$M_y = 11.25x - 15\frac{x^2}{2}$

$M_y(1.5) = 0$

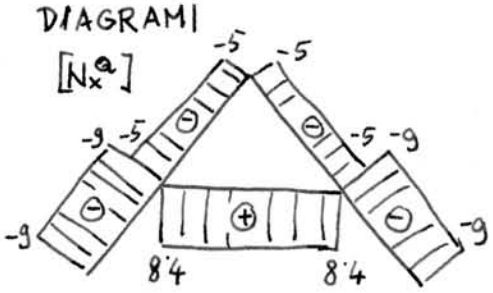
$M_y(0.75) = 4.2\ kN\,m$

poljI IV, V

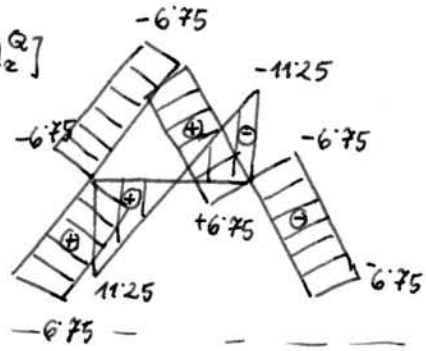
SIMETRIJA

DIAGRAMI

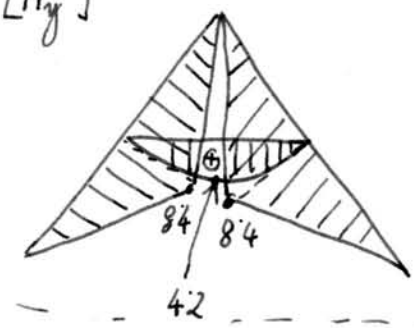
$[N_x^Q]$



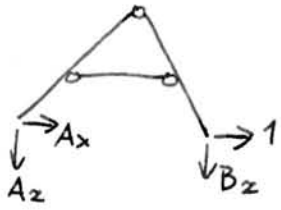
$[N_z^Q]$



$[M_y^Q]$



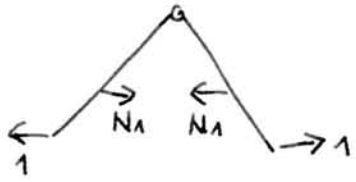
c2.) $X_1 = 1$



$\boxed{A_x = -1}$

$A_z + B_z = 0$

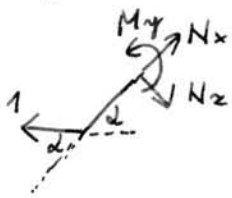
$\boxed{B_z = 0}$ $\boxed{A_z = 0}$



$\Sigma M^C: \; 1 \cdot h - N_1 \cdot \frac{h}{2} = 0$

$\boxed{N_1 = 2}$

polje I:



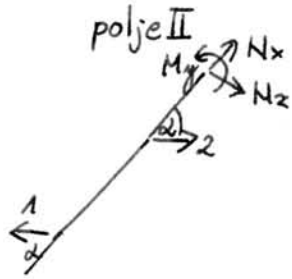
$N_x = +\cos\alpha = 0{\cdot}6$

$N_z = \sin\alpha = 0{\cdot}8$

$M_y = \sin\alpha \, x = 0{\cdot}8 \, x$

$M_y(1{\cdot}25) = 1$

polje II



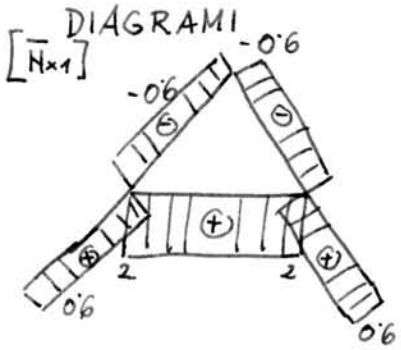
$N_x = \cos\alpha - 2 \cdot \cos\alpha = -0{\cdot}6$

$N_z = \sin\alpha - 2\sin\alpha = -0{\cdot}8$

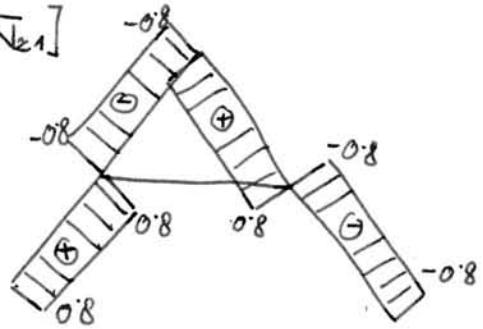
$M_y = 1 - 0{\cdot}8 \, x$

DIAGRAMI

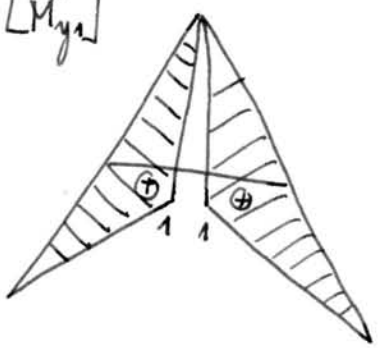
$[\bar{N}_{x1}]$



$[\bar{N}_{z1}]$



$[\bar{M}_{y1}]$



d.) določitev X_1

$$EI_y a_{11} = 4 \int_0^{\ell} \triangle^1 \triangle^1 dx = 4 \cdot 1{\cdot}25 \cdot \frac{1}{3} \cdot 1 \cdot 1 = \frac{5}{3}$$

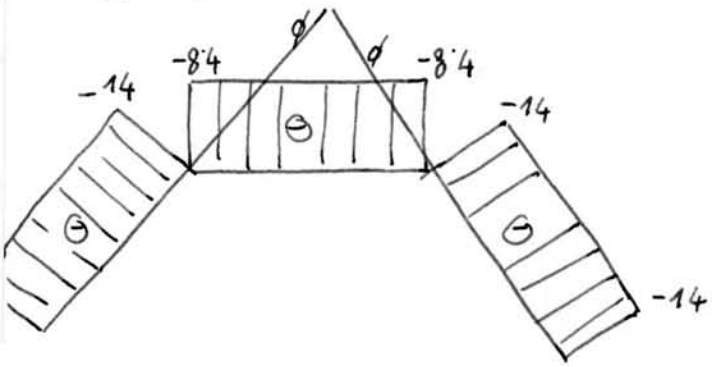
$$EI_y b_1 = 4 \cdot \int_0^{\ell} \triangle^{8{\cdot}4} \triangle^1 dx = 4 \cdot 8{\cdot}4 \cdot 1{\cdot}25 \cdot \frac{1}{3} \cdot 1 = \frac{42{\cdot}2}{3}$$

$$X_1 = -\frac{b_1}{a_{11}}$$

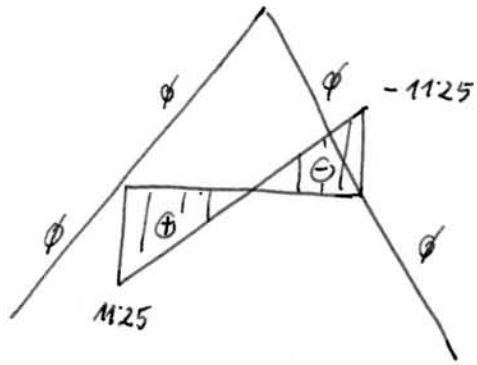
$$X_1 = -8{\cdot}4$$

e.) superpozicija

$[N_x^{nk}]$



$[N_z^{nk}]$



$[M_y^{nk}]$

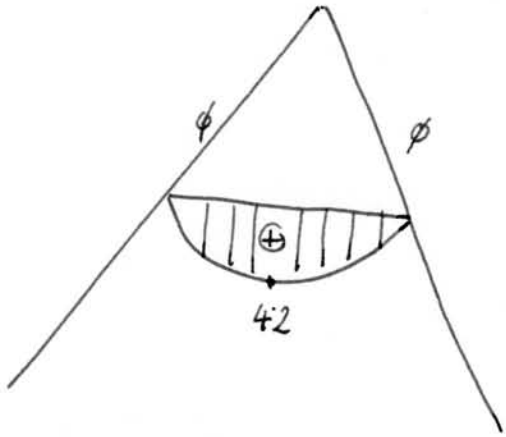


TABELA DOLŽIN, KOSINUSOV IN OSNIH TOGOSTI ZA PODANO PALIČJE

palica	vozel1	vozel2	dolzina	cos(a_ij)	cos(b_ij)	k_ij
1	1	4	7.211	0.832	0.555	277.350
2	2	3	2.000	0.000	1.000	1000.000
3	2	4	6.325	0.949	0.316	316.228
4	3	4	6.000	1.000	0.000	333.333

TOGOSTNA MATRIKA PALIČJA

-192.012	-128.008	0.000	0.000	0.000	0.000	192.012	128.008
-128.008	-85.338	0.000	0.000	0.000	0.000	128.008	85.338
0.000	0.000	-284.605	-94.868	0.000	0.000	284.605	94.868
0.000	0.000	-94.868	-1031.623	0.000	1000.000	94.868	31.623
0.000	0.000	0.000	0.000	-333.333	0.000	333.333	0.000
0.000	0.000	0.000	1000.000	0.000	-1000.000	0.000	0.000
192.012	128.008	284.605	94.868	333.333	0.000	-809.950	-222.876
128.008	85.338	94.868	31.623	0.000	0.000	-222.876	-116.961

POMIKI IN REAKCIJE VOZLIŠČ DANEGA PALIČJA

vozel	u_x	u_y	R_x	R_y
1	0.00000	0.00000	20.312	13.541
2	0.00000	-0.00146	4.376	
3	0.00000	0.00000	-24.688	1.459
4	0.07406	-0.26977		

TABELA OSNIH SIL ZA PODANO PALIČJE

palica	vozel1	vozel2	N_ij
1	1	4	-24.412
2	2	3	1.459
3	2	4	-4.612
4	3	4	24.688

TRDNOST - VSŠ

3. NALOGA

a.) $\left[\frac{\partial u_j}{\partial x_i}\right] = 10^{-3}\begin{bmatrix} y & x & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 2z \end{bmatrix}$

$[\varepsilon] = 10^{-3}\begin{bmatrix} y & \frac{x+1}{2} & 0 \\ \frac{x+1}{2} & 1 & 0 \\ 0 & 0 & 2z \end{bmatrix}$

b.) $[\varepsilon]|_{(1,3,1)} = 10^{-3}\begin{bmatrix} 3 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

c.) $\det\begin{bmatrix} 3-\lambda & 1 & 0 \\ 1 & 1-\lambda & 0 \\ 0 & 0 & 2-\lambda \end{bmatrix} = (3-\lambda)(1-\lambda)(2-\lambda) - (2-\lambda) =$

$= (2-\lambda)(\lambda^2 - 4\lambda + 2)$

$\lambda_1 = 2 \qquad \lambda_{23} = \frac{4 \pm \sqrt{16-8}}{2} = 2 \pm \sqrt{2}$

$\varepsilon_{11} = (2+\sqrt{2}) \cdot 10^{-3} \qquad \varepsilon_{22} = 2 \qquad \varepsilon_{33} = (2-\sqrt{2}) \cdot 10^{-3}$

l_1: $\begin{bmatrix} 1-\sqrt{2} & 1 & 0 \\ 1 & -1-\sqrt{2} & 0 \\ 0 & 0 & -\sqrt{2} \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow$ $c = 0$

$a - (1+\sqrt{2})b = 0 \qquad a = 1$

$(1-\sqrt{2})a + b = 0 \qquad b = \sqrt{2} - 1$

$l_1 = (1, \sqrt{2}-1, 0)$

l_2: $\begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow$ $c = 1$

$a = 0, b = 0$

$l_2 = (0, 0, 1)$

l_3: $\begin{bmatrix} 1+\sqrt{2} & 1 & 0 \\ 1 & -1+\sqrt{2} & 0 \\ 0 & 0 & \sqrt{2} \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow$ $c = 0$

$(1+\sqrt{2})a + b = 0 \qquad b = 1$

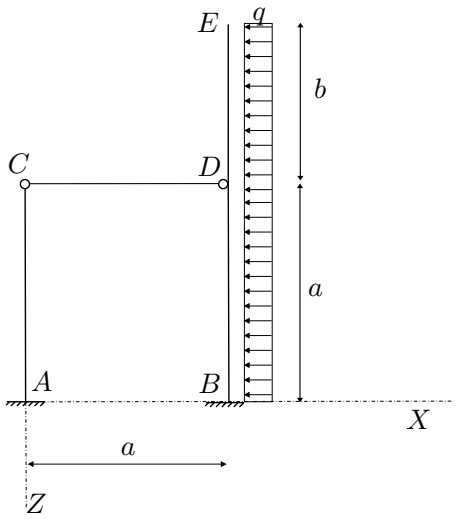
$a + (\sqrt{2}-1)b = 0 \qquad a = +1-\sqrt{2}$

$l_3 = (+1-\sqrt{2}, 1, 0)$

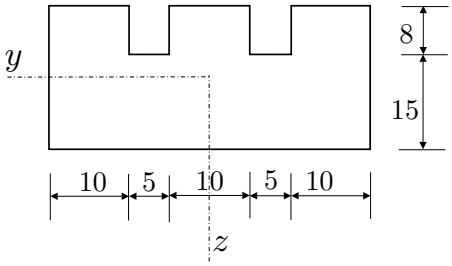
TRDNOST (VSŠ) - 2. IZPITNI ROK (03. 02. 2006)

RAČUNSKI DEL IZPITA:

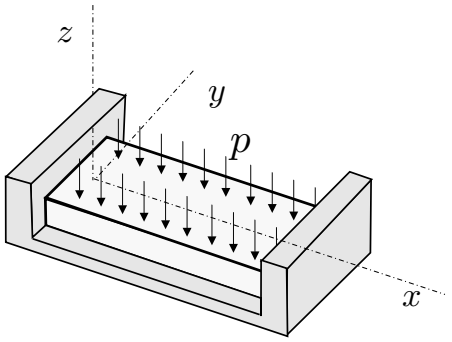
1. Za konstrukcijo na sliki izračunajte notranje statične količine in horizontalni pomik v točki E! Vpliva osnih in prečnih sil ni potrebno upoštevati.
(OBVEZNA NALOGA! 40%)
Podatki: $a = 4\,\mathrm{m}$, $b = 2\,\mathrm{m}$,
$q = 10\,\mathrm{kN/m}$,
$E = 3200\,\mathrm{kN/cm^2}$, $J_y = 90000\,\mathrm{cm^4}$,
$A_x = 1200\,\mathrm{cm^2}$.



2. Prerez na sliki je obremenjen s prečno silo $N_z = 10\,\mathrm{kN}$ in upogibnim momentom $M_y = 5\,\mathrm{kNm}$. Določite nekaj značilnih vrednosti in skicirajte diagrama osnih napetosti σ_{xx} in strižnih napetosti σ_{xy}! (30%)
Podatki za prerez so v centimetrih.



3. Tanek kvader postavimo med dve togi nepomični plošči, kot kaže slika. Zgornjo ploskev obremenimo z enakomerno zvezno tlačno obtežbo $p = 5\,\mathrm{kN/cm^2}$. Ob predpostavki linearno elastičnega, izotropnega materiala določite deformacijski in napetostni tenzor! (30%)
Podatki: $\nu = 0.3$, $E = 2 \cdot 10^4\,\mathrm{kN/cm^2}$.



TEORETIČNI DEL IZPITA:
Izmed treh zastavljenih vprašanj si izberete dve, na kateri boste odgovarjali. Izbrani vprašanji jasno označite! Pišite čitljivo.

1. Zapišite definicijo deformacijskega gradienta [F] ter zvezo med dV' in dV! Zaspišite izraz za specifično spremembo prostornine!

2. Zapišite in pojasnite zvezo med normalno napetostjo in vzdolžno deformacijo pri upogibu z osno silo! Normalno napetost izrazite tudi z notranjimi silami v prečnem prerezu nosilca! Predpostavite, da sta y in z glavni vztrajnostni osi.

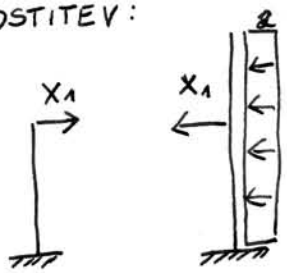
3. Napišite in komentirajte enačbe (pomen oznak) za Eulerjeve uklonske sile! Enačbo za določitev uklonske sile izpeljite za prostoležeči nosilec!

TRDNOST - VSŠ 3. 2. 2006

1. NALOGA

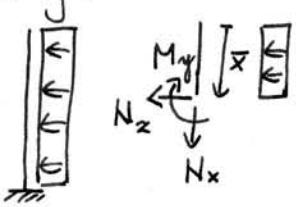
a.) $n = 3+3+2\cdot 2 - 3\cdot 3 = 1$

b.) SPROSTITEV:



c.) NOTRANJE SILE

c1.) zunanja obtežba



$N_x = 0$
$N_z = -q\cdot \bar{x}$
$M_y = q\frac{\bar{x}^2}{2}$

$N_x = 0$

$N_z(6) = -60\,kN$

$M_y(6) = 180\,kNm$ $\quad M_y(2) = 20\,kNm$

c2.) $X_1 = 1$

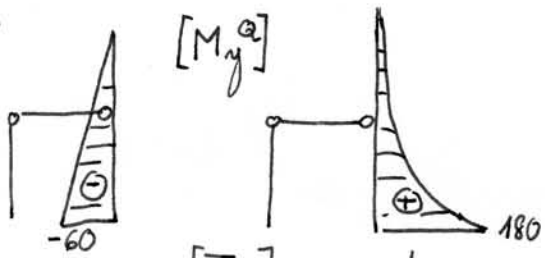
$N_x = 0$
$N_z = 1$
$M_y = -\bar{x}$

$M_y(4) = -4\,kNm$

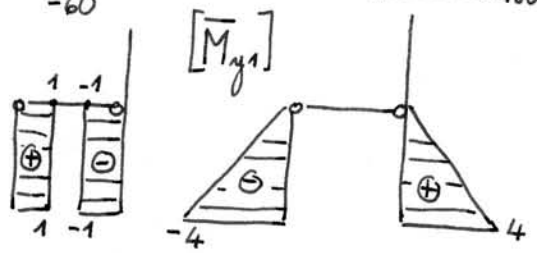
$N_x = 0$
$N_z = -1$
$M_y = \bar{x}$

DIAGRAMI:

$[N_x^{(2)}] = 0$ $\quad [N_z^{(2)}]$ $\quad [M_y^{(2)}]$



$[\bar{N}_{x1}] = 0$ $\quad [\bar{N}_{z1}]$ $\quad [\bar{M}_{y1}]$



d.) DOLOČITEV X_1

$$a_{11}\cdot EI_y = 2\cdot \int_0^a \triangle^4 \triangle^4 dx = 2\cdot 4\cdot \frac{1}{3}\cdot 44 = \frac{128}{3}$$

$$b_1\cdot EI_y = \int_0^a \square^{20} \triangle^{180} \triangle^4 dx \approx \int_0^a \square^{20} \square^{20} \triangle^4 dx + \int_0^a \triangle^{160} \triangle^4 dx$$

$$= \frac{1}{2}\cdot 4\cdot 20\cdot 4 + \frac{1}{4}\cdot 4\cdot 160\cdot 4 = 800$$ (to je le približek in to ne preveč natančen!)

$$X_1 \approx -\frac{b_1}{a_{11}} \approx -18.75$$

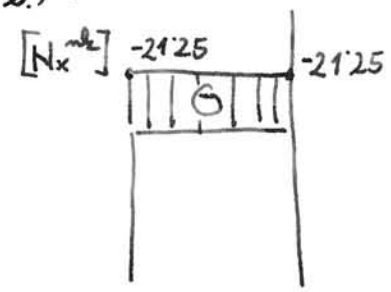
Točna določitev b_1

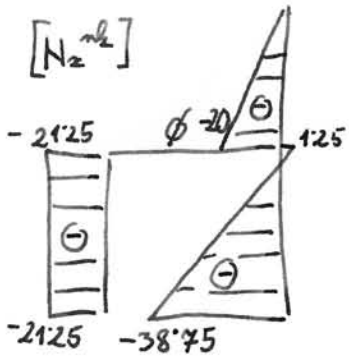
$$b_1 \cdot EI_y = \int_2^6 g\frac{\bar{x}^2}{2}\cdot(\bar{x}-2)\,d\bar{x} = \int_2^6 g\cdot\left(\frac{\bar{x}^3}{2}-\bar{x}^2\right)dx = 10\cdot\left[\frac{x^4}{8}-\frac{x^3}{3}\right]_2^6$$

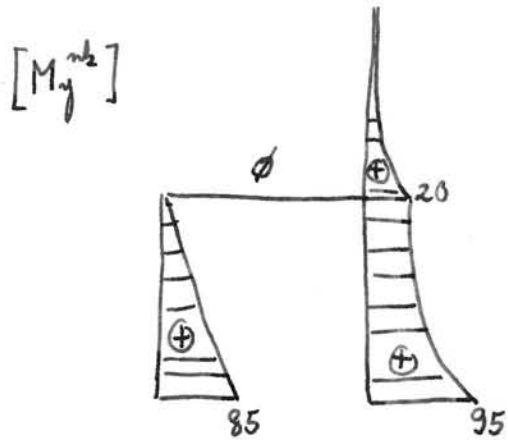
$$= 10\left(90+\frac{2}{3}\right) = 906{\cdot}7$$

$$\boxed{X_1 = -\frac{b_1}{a_{11}} = -21{\cdot}25}$$

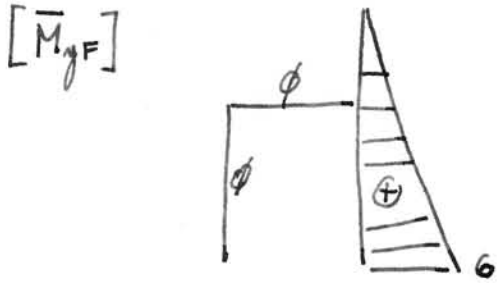
e.) SUPERPOZICIJA







f.) POMIK V E



$$w_E = \frac{1}{EI_y}\int_0^l \overset{20}{\overset{95}{\triangle}}\,{}^{6}\triangle\,dx = \frac{1}{EI_y}\int_0^l \left(\overset{180}{\triangle} + X_1\,\phi\,\triangle^{4}\right)\cdot\triangle^{6}\,dx$$

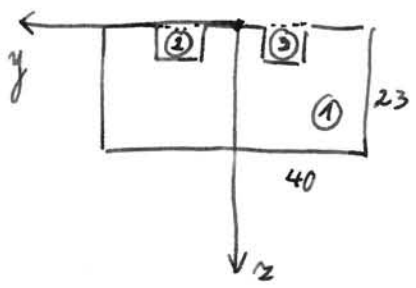
$$= \frac{1}{EI_y}\left[\int_0^l \overset{180}{\triangle}\cdot\triangle^{6}\,dx + X_1\int_0^a \triangle^{4}\,{}^{2}\square^{6}\,dx\right]$$

$$= \frac{1}{3200\cdot 9\cdot}\cdot\left(\frac{1}{4}\cdot 6\cdot 180\cdot 6 \quad -21{\cdot}25\cdot\frac{1}{6}\cdot 4\cdot 4\,(2+2\cdot 6)\right)$$

$$= 0{\cdot}0287\,m = 2{\cdot}9\,cm$$

TRDNOST - VSŠ 3.2.2006

2. NALOGA



① : $A^{①} = 920$, $y_T^{①} = 0$, $z_T^{①} = 11.5$, $I_y^{①} = 40556.\bar{6}$

② : $A^{②} = 40 = A^{③}$, $y_T^{②} = 7.5$, $y_T^{③} = -7.5$, $z_T^{②} = 4 = z_T^{③}$, $I_y^{②} = 213.\bar{3} = I_y^{③}$

$$I_y = I_y^{①} + I_y^{②} - I_y^{③} + z_T^{①2}A^{①} - z_T^{②2}A^{②} - z_T^{③2}A^{③} = 160520$$

$A = 840\,cm^2$

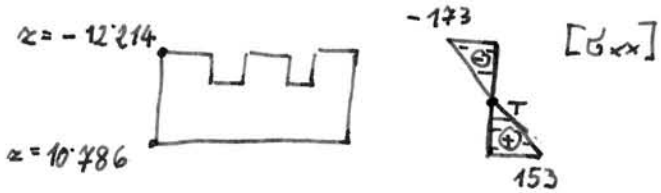
$$z_T = \frac{z^{①}A^{①} - z^{②}A^{②} - z^{③}A^{③}}{A} = 12.214\ cm$$

$$I_y^T = I_y - z^{T2}A = 35494.4\ cm^4 \quad (35200 \text{ MATHEMATICA})$$

a.) σ_{xx} : $N_x = 0$

$$\sigma_{xx} = \frac{N_x}{A_x} + \frac{M_y}{I_y} \cdot z$$

$$= \frac{5 \cdot 1000 \cdot 100\ N\,cm}{35200\ cm^4} \cdot z = 14.2 \cdot z\ [N/cm^2]$$



b.) σ_{xy} :

$$\sigma_{xy}^{*} = -\frac{1}{b^*}\frac{S_y^*}{I_y} \cdot N_x \qquad S_y^* = \int_{A^*} z\,dA$$

Ⅰ $y^* \in [-20, -10]$

$z \in [-12.2, 10.8]$

$$S_y^* = \int_{-20}^{y^*}\left(\int_{-12.2}^{10.8} z\,dz\right)dy =$$

$$= (y^*+20)\cdot\frac{1}{2}(10.8^2 - 12.2^2) = -164(y^*+20)$$

$S_y^*(-20) = 0$ $\boxed{S_y^*(-10) = -164\,cm^3}$

Ⅱ $y^* \in [-10, -5]$

$z \in [-4.2, 10.8]$

$$S_y^* = S_y^*(-10) + \int_{-10}^{y^*}\int_{-4.2}^{10.8} z\,dz\,dy =$$

$$= S_y^*(-10) + (y^*+10)\cdot\frac{1}{2}(10.8^2 - 4.2^2) =$$

$$= -164 + 49.29(y^*+10)$$

$\boxed{S_y^*(-5) = 82.45\ cm^3}$

(III) $y^* \in [-5,5]$

$z \in [-12.2, 10.8]$

$$S_y^* = S_y^*(-5) + \int_{-5}^{y^*} \int_{-12.2}^{10.8} z \, dz \, dy = 82.45 - 16.4(y^* + 5)$$

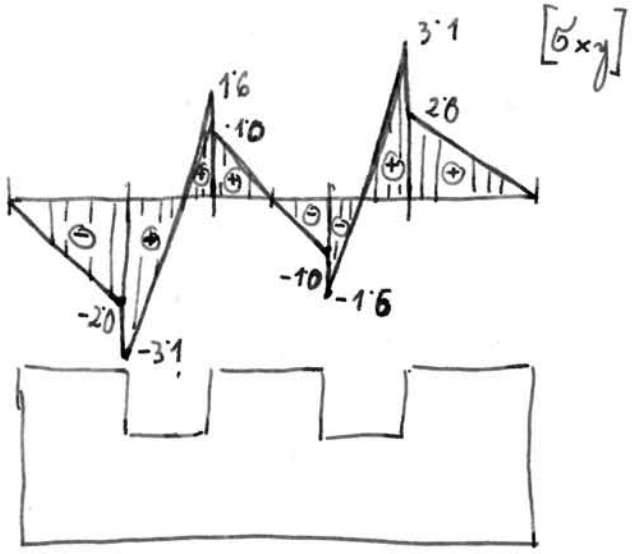
$$S_y^*(0) = 0$$

$$S_y^*(5) = -82$$

NAPREJ JE VSE SIMETRIČNO, LE PREDZNAK JE NASPROTEN

y^*	h^*	S_y^*	σ_{xy}^* [N/cm²]
-20	23	0	0
-10	23	-164	2.025
-10	15	-164	3.105
-5	15	82	-1.55
-5	23	82	-1.01
0	23	0	0
5	23	-82	1.01
5	15	-82	1.55
⋮	simetrija		⋮

$$\frac{N_z}{I_y} = \frac{10 \cdot 1000 \text{ N}}{35\,200 \text{ cm}^4} = 0.284 \frac{\text{N}}{\text{cm}^4}$$



TRDNOST - VSŠ 3.2.2006

3. NALOGA

a.) napetosti

$$\begin{bmatrix}0\\0\\-p\end{bmatrix} = [\sigma]\begin{bmatrix}0\\0\\1\end{bmatrix} \qquad \begin{bmatrix}0\\0\\0\end{bmatrix} = [\sigma]\begin{bmatrix}0\\1\\0\end{bmatrix}$$

$\sigma_{xz} = 0 \quad \sigma_{yz} = 0 \quad \sigma_{zz} = -p = -5\ kN/cm^2 \quad \sigma_{xy} = 0 \quad \sigma_{yy} = 0$

b.) deformacije

$$\varepsilon_{xx} = 0$$

c.) Hookov zakon

$$\varepsilon_{xx} = \frac{1+\nu}{E}\sigma_{xx} - \frac{\nu}{E}(\sigma_{xx} + \sigma_{yy} + \sigma_{zz})$$

$$0 = \frac{1}{E}\sigma_{xx} + \frac{\nu}{E}p$$

$$\Rightarrow \quad \sigma_{xx} = -\nu p = 1.5\ kN/cm^2$$

$$\varepsilon_{yy} = \frac{1+\nu}{E}\overset{=0}{\sigma_{yy}} - \frac{\nu}{E}sl\sigma$$

$$= -\frac{\nu}{E}(-\nu p - p) = \frac{p\nu}{E}(1+\nu) = 9.75 \cdot 10^{-5}$$

$$\varepsilon_{zz} = \frac{1+\nu}{E}\sigma_{zz} - \frac{\nu}{E}sl\sigma$$

$$= \frac{1+\nu}{E}(-p) - \frac{\nu}{E}(-\nu p - p)$$

$$= \frac{1+\nu}{E}(-p) - \frac{p\nu}{E}(1+\nu) = \frac{1+\nu}{E}(-p - p\nu) = -\frac{p}{E}(1+\nu)^2$$

$$\varepsilon_{zz} = -4.2 \cdot 10^{-4}$$

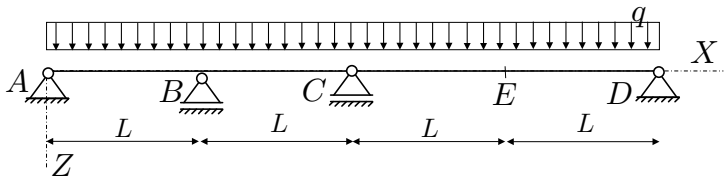
d.) tenzorji

$$[\varepsilon] = 10^{-4}\begin{bmatrix}0 & 0 & 0\\ 0 & 0.96 & 0\\ 0 & 0 & -4.2\end{bmatrix} \qquad [\sigma] = \begin{bmatrix}1.5 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -5\end{bmatrix}$$

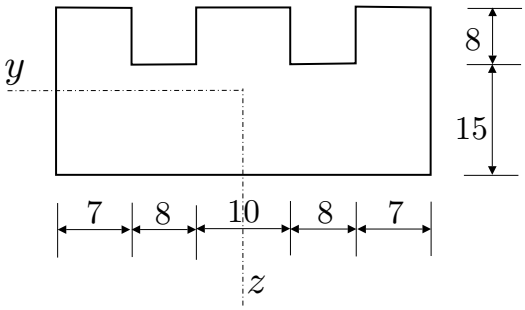
TRDNOST (VSŠ) - 1. IZREDNI IZPITNI ROK (17. 03. 2006)

RAČUNSKI DEL IZPITA:

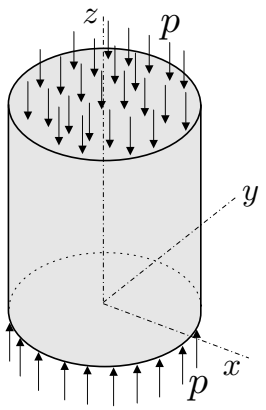
1. Za konstrukcijo na sliki izračunajte notranje statične količine! Upoštevajte tudi vpliv osnih sil! **(OBVEZNA NALOGA! 40%)**
Podatki: $L = 3\,\mathrm{m}$, $q = 10\,\mathrm{kN/m}$,
$E = 21000\,\mathrm{kN/cm^2}$, $J_y = 2800\,\mathrm{cm^4}$, $A_x = 30\,\mathrm{cm^2}$.



2. Prerez na sliki je obremenjen z upogibnim momentom $M_y = 25\,\mathrm{kNm}$. Določite nekaj značilnih vrednosti in skicirajte diagram strižnih napetosti σ_{xz}! (30%)
Podatki za prerez so v centimetrih.



3. Valj iz linearno elastičnega, izotropnega materiala je obremenjen z enakomerno zvezno obtežbo, kot kaže slika. Določi velikosti obtežbe p s katero moramo obremeniti valj , da bo specifična sprememba volumna enaka $\epsilon_V = -0.002$! (30%)
Podatki: $\nu = 0.3$, $E = 2.1 \cdot 10^4\,\mathrm{kN/cm^2}$.



TEORETIČNI DEL IZPITA:

Izmed treh zastavljenih vprašanj si izberete dve, na kateri boste odgovarjali. Izbrani vprašanji jasno označite! Pišite čitljivo.

1. Kdaj je napetostno stanje v delcu telesa definirano?

2. Opišite osnovne predpostavke pri upogibu z osno silo in enakomerni torziji!

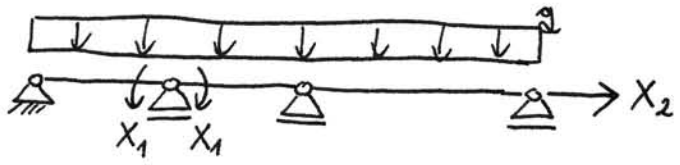
3. Napišite in komentirajte enačbe (pomen oznak) za Eulerjeve uklonske sile! Enačbo za določitev uklonske sile izpeljite za previsni nosilec!

TRDNOST - VSŠ

1. NALOGA

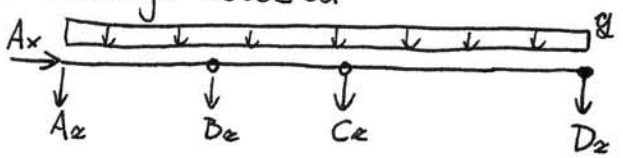
a.) $n = 2 \cdot 2 + 2 \cdot 1 + 2 - 2 \cdot 3 = 2$ (2x statično nedoločena)

b.) SPROSTITEV:



c.) NOTRANJE SILE SPROŠČENE KONSTRUKCIJE

c1.) zunanja obtežba



$A_x = 0$

$A_z + C_z + B_z + D_z = -g \cdot 4L$

$A_z \cdot L + g \cdot L \cdot \frac{L}{2} = 0$ $\Rightarrow$ $\boxed{A_z = -g \cdot \frac{L}{2}}$

$D_z \cdot 2L + g \cdot 2L \cdot L = 0$ $\Rightarrow$ $\boxed{D_z = -gL}$

$A_z \cdot 2\cancel{L} + B_z \cdot \cancel{L} + g \cdot 2L \cdot \cancel{L} = 0$

$\boxed{B_z = -gL}$

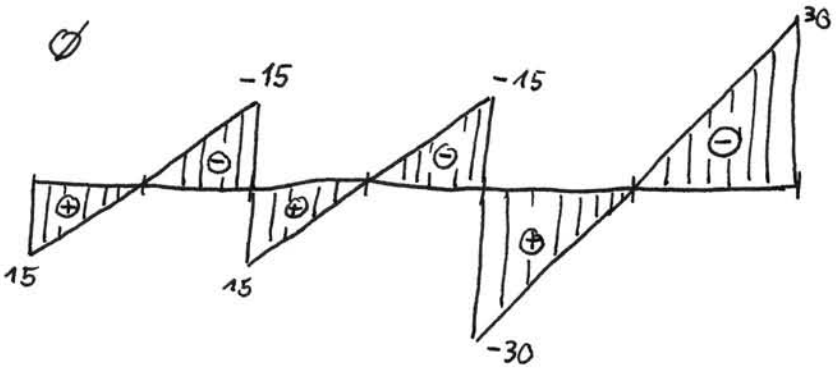
$\boxed{C_z = -3g\frac{L}{2}}$

$\boxed{\begin{aligned} A_z &= -15\ kN \\ B_z &= -30\ kN \\ C_z &= -45\ kN \\ D_z &= -30\ kN \end{aligned}}$

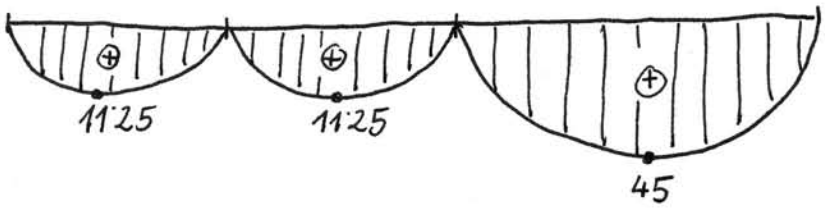
- OSNE SILE SO NIČ
- PREČNE SO LINEARNE, NIČ NA SREDI VSAKEGA POLJA
- MOMENTI SO KVADRATNE PARABOLE
  EKSTREMI SO NA SREDI POLJ IN ZNAŠAJO $\frac{gL^2}{8}$

[Nx] ∅

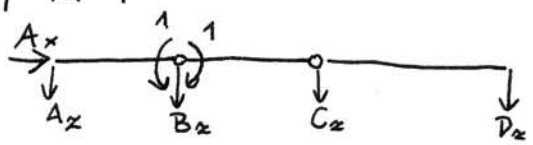
[Nz]



[My]



c2.) $X_1 = 1$



$A_x = 0$

$A_z + B_z + C_z + D_z = 0$

$A_z \cdot L + 1 = 0 \quad \boxed{A_z = -\frac{1}{L}}$

$\boxed{D_z = 0}$

$C_z \cdot L + 1 = 0 \quad \boxed{C_z = -\frac{1}{L}}$

$\boxed{B_z = \frac{2}{L}}$

$\boxed{A_z = -\frac{1}{3}}$

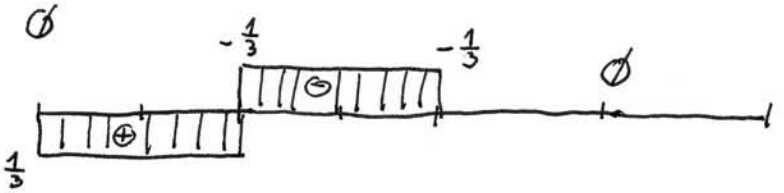
$\boxed{B_z = \frac{2}{3}}$

$\boxed{D_z = -\frac{1}{3}}$

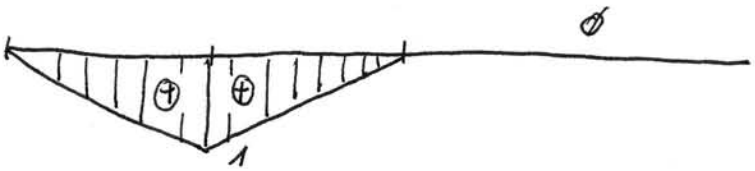
- KONSTANTNE PREČNE SILE
- LINEARNI MOMENTI

$[N_x]$ Ø

$[N_z]$



$[M_y]$

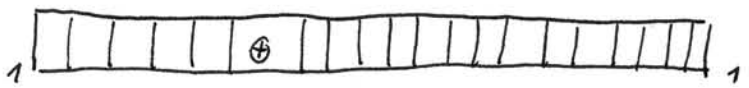


c3.) $X_2 = 1$

$A_x = -1$

$A_z = B_z = C_z = D_z = 0$

$[N_x]$



$[N_z]$ Ø

$[M_y]$ Ø

d.) DOLOČITEV X_1 IN X_2

$$a_{11} = \frac{1}{EI_y} \cdot 2 \int_0^L 1 \cdot 1 \, dx = \frac{2L}{3EI_y}$$

$$a_{22} = \frac{1}{EA_x} \cdot \int_0^{4L} 1 \, dx = \frac{4L}{EA_x}$$

$$a_{12} = 0$$

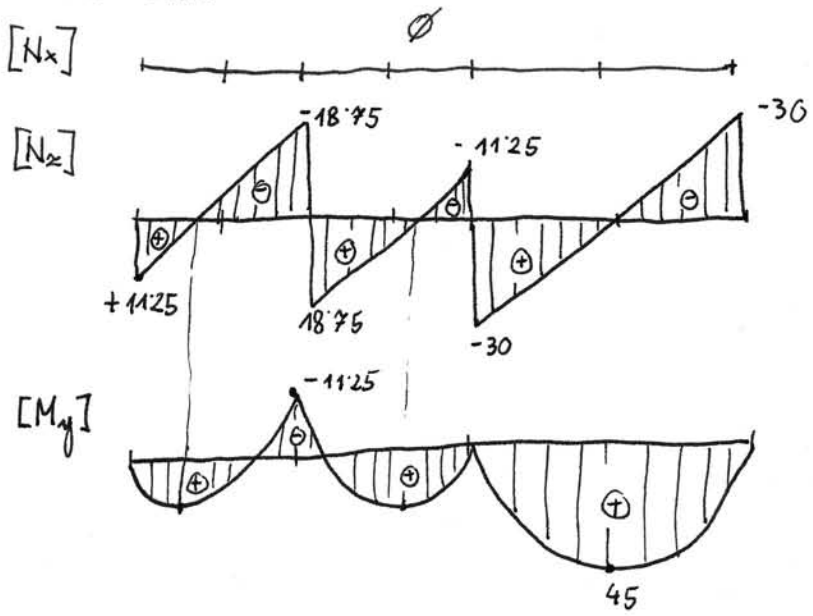
$$b_1 = \frac{1}{EI_y} \cdot 2 \int_0^L 11{:}25 \cdot 1 \, dx = \frac{2L \cdot 11{:}25}{3EI_y}$$

$$b_2 = 0$$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = - \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \Rightarrow \quad X_2 = 0$$

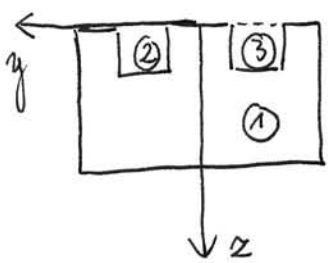
$$X_1 = -\frac{2\cancel{L} \cdot 11{:}25 \; 3\cancel{EI_y}}{3\cancel{EI_y} \cdot 2\cancel{L}} = -11{:}25$$

e.) SUPERPOZICIJA



TRDNOST - VSŠ

2. NALOGA



① : $A^{①} = 920\ cm^2$
$y_T^{①} = 0\ cm$
$z_T^{①} = 11.5\ cm$
$I_y^{①} = 40556.\overline{6}\ cm^4$

(2,3) : $A^{②} = A^{③} = 64\ cm^2$
$y_T^{②} = 9\ cm \quad y_T^{③} = -9\ cm$
$z_T^{②} = z_T^{③} = 4\ cm$
$I_y^{②} = I_y^{③} = 314.\overline{3}\ cm^4$

$$A = A^{①} - A^{②} - A^{③} = 792\ cm^2$$

$$z_T = \frac{z^{①}A^{①} - z^{②}A^{②} - z^{③}A^{③}}{A} = 12.712\ cm \quad (12.712121)$$

$$I_y = I_y^{①} - I_y^{②} - I_y^{③} + z^{①2}A^{①} - z^{②2}A^{②} - z^{③2}A^{③} = 159550\ cm^4$$

$$I_y^T = I_y - z_T^2 A = 31564\ cm^4$$

$$\sigma_{xz}^* = -\frac{1}{b^*}\frac{S_{oy}^*}{I_y}N_z \qquad \sigma_{xz}^* = -\frac{1}{b^*}\frac{S_y^*}{I_y}N_z$$

a.) $z^* \in [-12.7, -4.7] \quad x \in [-12.7, z^*]$
$y \in [-20, -13] \cup [-5, 5] \cup [13, 20]$

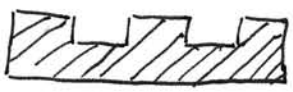
$$S_y(z^*) = \int_{A^*} z\,dA = \int_{-20}^{-13}\int_{-12.7}^{z^*} z\,dx\,dy + \int_{-5}^{5}\int_{-12.7}^{z^*} z\,dx\,dy + \int_{13}^{20}\int_{-12.7}^{z^*} z\,dx\,dy$$

$$= 7\cdot\frac{z^2}{2}\Big|_{-12.7}^{z^*} + 10\cdot\frac{z^2}{2}\Big|_{-12.7}^{z^*} + 7\cdot\frac{z^2}{2}\Big|_{-12.7}^{z^*}$$

$$= 24\cdot\frac{1}{2}(z^{*2} - 12.7^2) = 12(z^{*2} - 12.7^2)$$

$$S_y(-12.7) = 0 \qquad S_y(-4.7) = -1672.7\ cm^4$$

b.) $z^* \in [-4.7, 10.3]$



$$S_y(z^*) = S_y(-4.7) + \int_{-20}^{20}\int_{-4.7}^{z^*} z\,dx\,dy$$

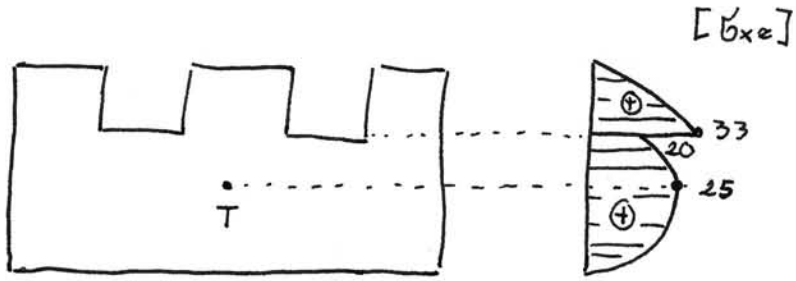
$$= S_y(-4.7) + 20\cdot(z^{*2} - 4.7^2)$$

$$= -2116.8 + 20z^{*2}$$

$$S_y(0) = -2116.8\ cm^4 \quad (\text{ekstrem}) \qquad S_y(10.3) = 0$$

z^*	b^*	S_y^*	τ_{xz}^* [N/cm²]
-12·7	24	0	0
-4·7	24	-1672·7	33
-4·7	40	-1672·7	20
0	40	-2117	25
10·3	40	0	0

$$\frac{N_x}{I_y} = \frac{15000\,N}{31564\,cm^4} = 0{\cdot}47\ N/cm^4$$



TRDNOST - VSŠ 17. 3. 2005

3. NALOGA

a.) ROBNI POGOJI

smer x: $\begin{bmatrix}0\\0\\0\end{bmatrix} = \begin{bmatrix}\sigma\end{bmatrix}\begin{bmatrix}1\\0\\0\end{bmatrix}$ smer y: $\begin{bmatrix}0\\0\\0\end{bmatrix} = \begin{bmatrix}\sigma\end{bmatrix}\begin{bmatrix}0\\1\\0\end{bmatrix}$

$\sigma_{xx}=0 \quad \sigma_{xy}=0 \quad \sigma_{xz}=0$ $\sigma_{xy}=0 \quad \sigma_{yy}=0 \quad \sigma_{yz}=0$

smer z: $\begin{bmatrix}0\\0\\-p\end{bmatrix} = \begin{bmatrix}\sigma\end{bmatrix}\begin{bmatrix}0\\0\\1\end{bmatrix}$

$\sigma_{xz}=0 \quad \sigma_{yz}=0 \quad \boxed{\sigma_{zz}=-p}$

b.) HOOKOV ZAKON

$$\varepsilon_{xx} = \frac{1+\nu}{E}\cancel{\sigma_{xx}} - \frac{\nu}{E}(\cancel{\sigma_{xx}} + \cancel{\sigma_{yy}} + \cancel{\sigma_{zz}}) \quad \sigma_{zz}$$

$$\varepsilon_{xx} = -\frac{\nu}{E}\cdot\sigma_{zz} \Rightarrow \boxed{\varepsilon_{xx} = \frac{\nu p}{E}}$$

$$\varepsilon_{yy} = \frac{1+\nu}{E}\cancel{\sigma_{yy}} - \frac{\nu}{E}(\cancel{\sigma_{xx}} + \cancel{\sigma_{yy}} + \sigma_{zz})$$

$$\varepsilon_{yy} = -\frac{\nu}{E}\sigma_{zz} \Rightarrow \boxed{\varepsilon_{yy} = \frac{\nu p}{E}}$$

$$\varepsilon_{zz} = \frac{1+\nu}{E}\sigma_{zz} - \frac{\nu}{E}(\cancel{\sigma_{xx}} + \cancel{\sigma_{yy}} + \sigma_{zz})$$

$$\varepsilon_{zz} = \frac{1}{E}\sigma_{zz} \Rightarrow \boxed{\varepsilon_{zz} = -\frac{p}{E}}$$

c.) SPEC. SPREMEMBA VOLUMNA

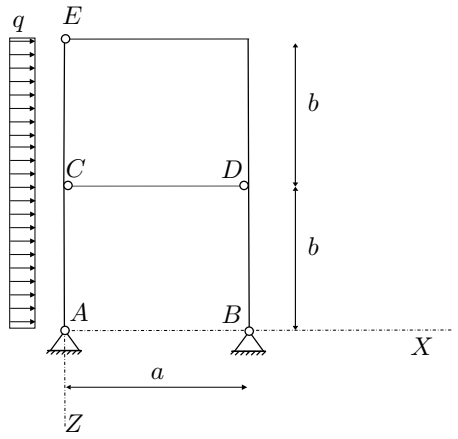
$$\varepsilon_v = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz} = \frac{1}{E}(\nu p + \nu p - p) = \frac{p}{E}(2\nu - 1)$$

$$p = \frac{E\cdot\varepsilon_v}{2\nu - 1} = 105 \ \mathrm{kN/cm^2}$$

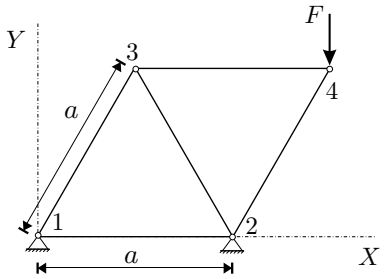
TRDNOST (VSŠ) - 3. IZPITNI ROK (26. 06. 2006)

RAČUNSKI DEL IZPITA:

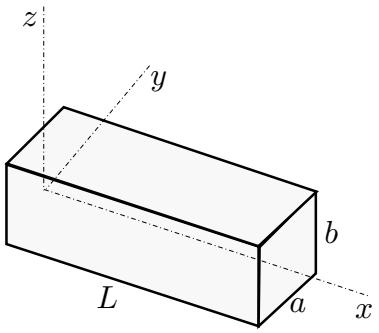
1. Za konstrukcijo na sliki izračunajte notranje statične količine! Vpliva osnih in prečnih sil ni potrebno upoštevati.
(OBVEZNA NALOGA! 40%)
Podatki: $a = 4\,\mathrm{m}$, $b = 3.5\,\mathrm{m}$,
$q = 10\,\mathrm{kN/m}$,
$E = 3400\,\mathrm{kN/cm^2}$, $J_y = 90000\,\mathrm{cm^4}$,
$A_x = 1200\,\mathrm{cm^2}$.



2. Za paličje na sliki določite pomike vozlišč in osne sile v palicah po metodi pomikov! (30%)
Podatki: $a = 2\,\mathrm{m}$, $F = 10\,\mathrm{MN}$,
$E = 2 \cdot 10^5\,\mathrm{MPa}$, $A = 0.01\,\mathrm{m^2}$.



3. Kvader na sliki ima stranice dolžin $L = 100$ cm, $a = 10$ cm in $b = 5$ cm. Izmerjene spremembe dolžin stranic znašajo $\Delta L = 5$ cm, $\Delta a = 0.4$ cm in $\Delta b = 0.2$ cm. Telesna diagonala se je podaljšala za 5.05 cm, kota med osema x in y ter med y in z pa se nista spremenila. Določite tenzor majhnih deformacij! (30%)



TEORETIČNI DEL IZPITA:

Izmed treh zastavljenih vprašanj si izberete dve, na kateri boste odgovarjali. Izbrani vprašanji jasno označite! Pišite čitljivo.

1. Opišite tenzor deformacij! Pojasnite geometrijski pomen komponent tenzorja **malih** deformacij!

2. Zapišite in pojasnite zvezo med normalno napetostjo σ_{xx} in vzdolžno deformacijo pri upogibu z osno silo! Normalno napetost izrazite tudi z notranjimi silami v prečnem prerezu nosilca! Predpostavite, da sta y in z glavni vztrajnostni osi.

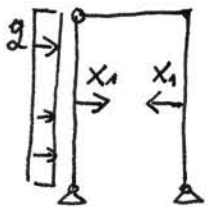
3. Izpeljite formuli za račun pomika u_{Ts} in zasuka ω_{Ts} statično določene linijske konstrukcije! Naredite tudi preprost primer!

TRDNOST - VSŠ 26. 6. 06

1. NALOGA

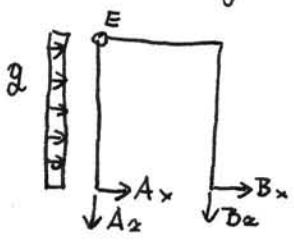
a.) $n = 2 \cdot 2 + 3 \cdot 2 - 3 \cdot 3 = 1$

b.) SPROSTITEV



c.) NOTRANJE SILE ZA SPROŠČENO KONSTRUKCIJO

c1) zunanja obtežba



$A_x + B_x = -g(a+b)$

$A_z + B_z = 0$

$\Sigma M^A: \; -g(a+b)\frac{a+b}{2} - B_z \cdot a = 0$

$\Sigma M^E_{AE} \; A_x \cdot (a+b) + g(a+b)\frac{a+b}{2} = 0$

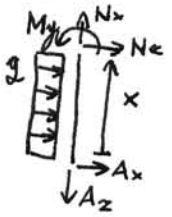
$B_x = -35\ kN$

$A_z = 61{\cdot}25\ kN$

$B_z = -61{\cdot}25\ kN$

$A_x = -35\ kN$

polje AE



$N_x = A_z = 61{\cdot}25\ kN$

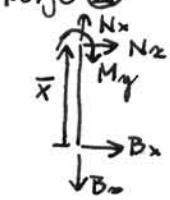
$N_z = -A_x - g\,x$

$M_y = -A_x \cdot x - g\frac{x^2}{2}$

$N_z = 35 - 10x$

$M_y = 35x - 5x^2$ $\quad M_y(3{\cdot}5) = 61{\cdot}25$ (ekstrem)

polje III



$N_x = B_z = -61{\cdot}25\ kN$

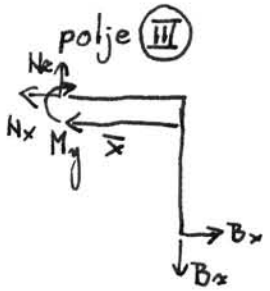
$N_z = -B_x$

$M_y = B_x \cdot \bar{x}$

$N_z = 35\ kN$

$M_y = -35\,\bar{x}$ $\quad M_y(7) = -245\ kNm$

polje III



$N_x = B_x$

$N_z = B_z$

$M_y = B_x \cdot (a+b) - B_z\bar{x}$

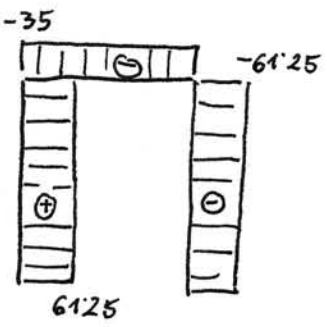
$N_x = -35\ kN$

$N_z = -61{\cdot}25\ kN$

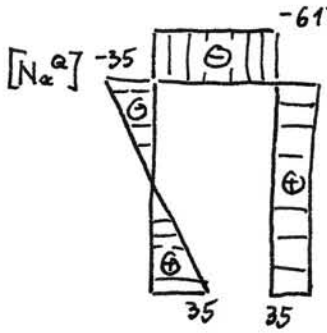
$M_y = -245 + 61{\cdot}25 \cdot \bar{x}$

DIAGRAMI

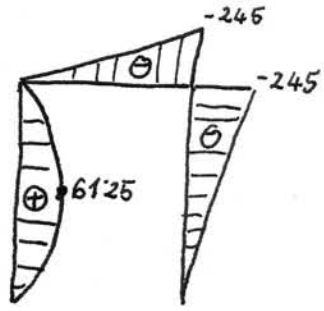
$[N_x^Q]$



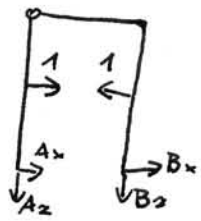
$[N_z^Q]$



$[M_y^Q]$



c.2.) $X_1 = 1$



$A_x + B_x = 0$

$A_z + B_z = 0$

$A_x \cdot 2b + 1 \cdot b = 0$

$B_x \cdot 2b - 1 \cdot b - B_z \cdot a = 0$

$\Rightarrow A_x = -\frac{1}{2}$

$B_x = \frac{1}{2}$

$A_z = B_z = 0$

polje Ⅰ

$N_x = 0$

$N_z = -A_x$

$M_y = -A_x \cdot x$

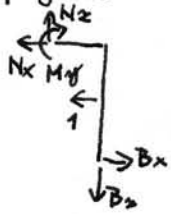
polje Ⅱ

$N_x = 0$

$N_z = -A_x - 1$

$M_y = -1 \cdot \bar{x} - A_x(\bar{x} + a)$

polje Ⅲ

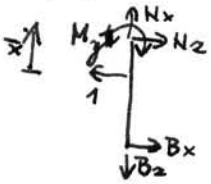


$N_x = B_x - 1$

$N_z = \cancel{B_z}$

$M_y = +B_x(a+b) - 1 \cdot b - \cancel{B_z \bar{x}}$

polje Ⅳ

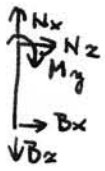


$N_x = \cancel{B_z}$

$N_z = -B_x + 1$

$M_y = B_x \cdot (\bar{x} + a) - 1 \cdot \bar{x}$

polje Ⅴ



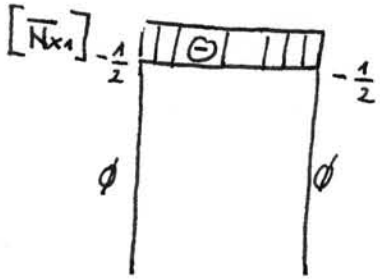
$N_x = \cancel{B_z}$

$N_z = B_x -$

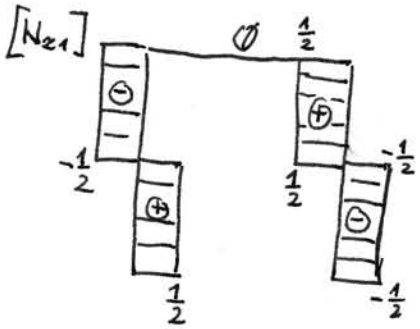
$M_y = B_x \cdot \bar{x}$

DIAGRAMI

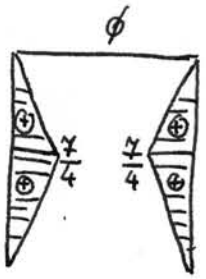
$[\bar{N}_{x1}]$



$[N_{z1}]$



$[\bar{M}_{y1}]$



d.) DOLOČITEV X_1

$$EI_y a_{11} = 4 \cdot \int_0^6 \frac{x}{4} \cdot \frac{x}{4}\, dx = 4 \cdot \frac{1}{3} \cdot \frac{x}{2} \cdot \frac{x}{4} \cdot \frac{x}{4} = \frac{343}{24}$$

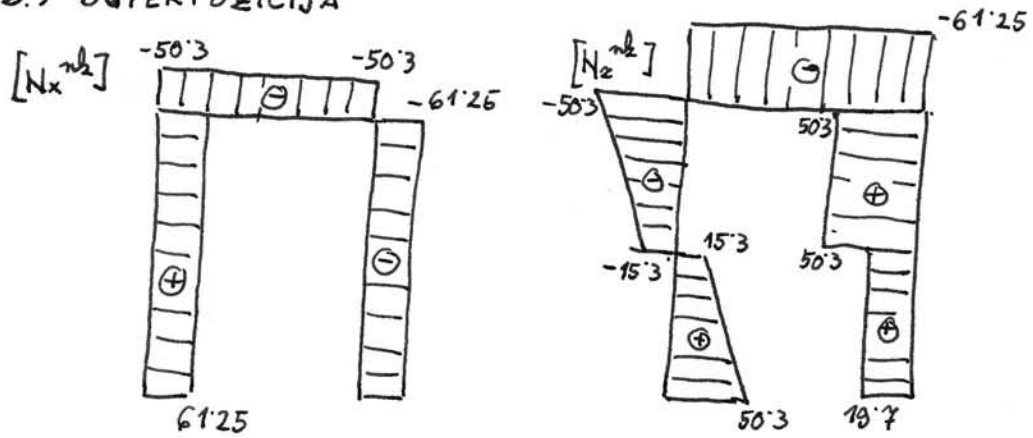
$$EI_y b_1 = 2 \cdot \int_0^6 \frac{61.25}{} \cdot \frac{x}{4}\, dx + \int_0^6 \frac{-122.5}{} \cdot \frac{x}{4}\, dx + \int_0^6 \frac{-122.5}{} \cdot \frac{-245}{} \cdot \frac{x/4}{}\, dx$$

$$= 2 \cdot \frac{5}{12} \cdot \frac{x}{2} \cdot 61.25 \cdot \frac{x}{4} + \frac{1}{3} \cdot \frac{x}{2} \cdot (-122.5) \cdot \frac{x}{4} + \frac{1}{6} \cdot \frac{x}{2} \cdot (-2 \cdot 122.5 - 245) \cdot \frac{x}{4}$$

$$= \frac{49}{8} \left(51.04 - \frac{1}{2} 122.5 - \frac{1}{6} 490 \right) = -437.7$$

$$X_1 = -\frac{b_1}{a_{11}} = +30.625$$

e.) SUPERPOZICIJA



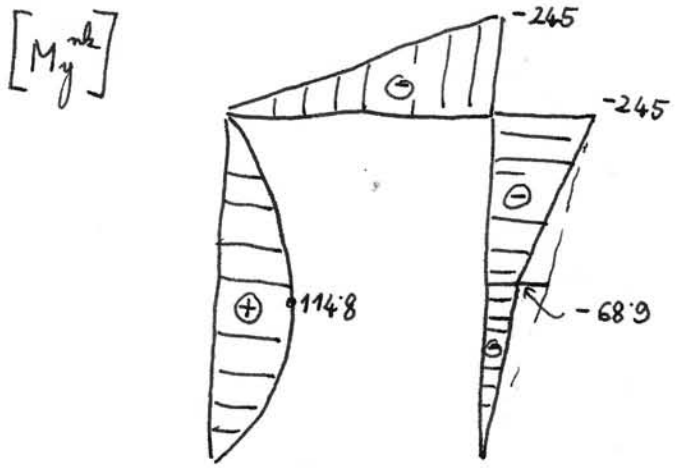


TABELA DOLŽIN, KOSINUSOV IN OSNIH TOGOSTI ZA PODANO PALIČJE

palica	vozel1	vozel2	dolzina	cos(a_ij)	cos(b_ij)	k_ij
1	1	2	2.000	1.000	0.000	1000.000
2	1	3	2.000	0.500	0.866	1000.000
3	2	3	2.000	-0.500	0.866	1000.000
4	2	4	2.000	0.500	0.866	1000.000
5	3	4	2.000	1.000	0.000	1000.000

TOGOSTNA MATRIKA PALIČJA

-1250.000	-433.013	1000.000	0.000	250.000	433.013	0.000	0.000
-433.013	-750.000	0.000	0.000	433.013	750.000	0.000	0.000
1000.000	0.000	-1500.000	0.000	250.000	-433.013	250.000	433.013
0.000	0.000	0.000	-1500.000	-433.013	750.000	433.013	750.000
250.000	433.013	250.000	-433.013	-1500.000	0.000	1000.000	0.000
433.013	750.000	-433.013	750.000	0.000	-1500.000	0.000	0.000
0.000	0.000	250.000	433.013	1000.000	0.000	-1250.000	-433.013
0.000	0.000	433.013	750.000	0.000	0.000	-433.013	-750.000

POMIKI IN REAKCIJE VOZLIŠČ DANEGA PALIČJA

vozel	u_x	u_y	R_x	R_y
1	0.00000	0.00000	-2.887	-5.000
2	0.00000	0.00000	2.887	15.000
3	0.01155	0.00000		
4	0.01732	-0.02333		

TABELA OSNIH SIL ZA PODANO PALIČJE

palica	vozel1	vozel2	N_ij
1	1	2	0.000
2	1	3	5.774
3	2	3	-5.774
4	2	4	-11.547
5	3	4	5.774

TRDNOST - VSŠ 26.6.2006

3. NALOGA

$$\varepsilon_{xx} \approx \frac{\Delta L}{L} = \frac{5}{100} = 0.05$$

$$\varepsilon_{yy} \approx \frac{\Delta a}{a} = \frac{1}{100} = 0.04$$

$$\varepsilon_{zz} \approx \frac{\Delta b}{b} = \frac{2}{50} = 0.04$$

$$\varepsilon_{xy} = \varepsilon_{yz} = 0$$

$$\varepsilon_{\xi\xi} = \frac{\Delta d}{d} = \frac{5.05}{100.6} = 0.050187$$

$$d = \sqrt{100^2 + 10^2 + 5^2} = 100.6$$

$$\vec{\xi} = L\vec{a}_x + a\vec{a}_y + b\vec{e}_z = (100, 10, 5)$$

$$\vec{e}_\xi = \frac{\vec{\xi}}{|\vec{\xi}|} = \frac{(100, 10, 5)}{100.6} = \frac{1}{d}(100, 10, 5)$$

$$\varepsilon_{\xi\xi} = e_\xi^T [\varepsilon] e_\xi = \frac{1}{d^2} \begin{bmatrix} 100 & 10 & 5 \end{bmatrix} \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{xy} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{xz} & \varepsilon_{yz} & \varepsilon_{zz} \end{bmatrix} \begin{bmatrix} 100 \\ 10 \\ 5 \end{bmatrix}$$

$$= \frac{1}{10125} \begin{bmatrix} 100 & 10 & 5 \end{bmatrix} \begin{bmatrix} 0.05 & 0 & \varepsilon_{xz} \\ 0 & 0.04 & 0 \\ \varepsilon_{xz} & 0 & 0.04 \end{bmatrix} \begin{bmatrix} 100 \\ 10 \\ 5 \end{bmatrix}$$

$$= \frac{1}{10125} \begin{bmatrix} 100 & 10 & 5 \end{bmatrix} \begin{bmatrix} 5 + 5\varepsilon_{xz} \\ 0.4 \\ 0.2 + 100\varepsilon_{xz} \end{bmatrix} =$$

$$= \frac{1}{10125}(500 + 500\varepsilon_{xz} + 4 + 1 + 500\varepsilon_{xz})$$

$$= \frac{1}{10125}(505 + 1000\varepsilon_{xz})$$

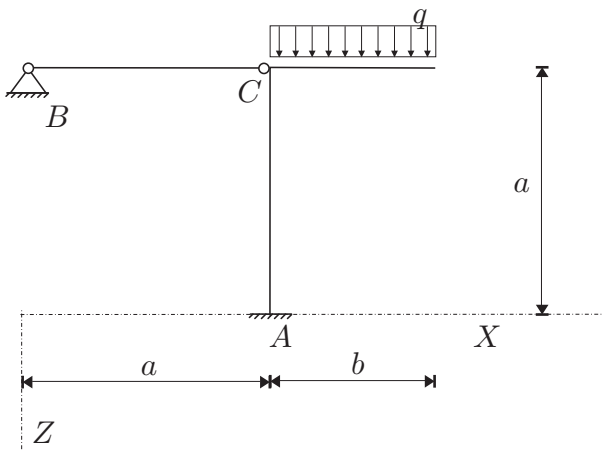
$$\varepsilon_{xz} = \frac{1}{1000}(10125\,\varepsilon_{\xi\xi} - 505) = 0.00314$$

$$[\varepsilon] = \begin{bmatrix} 0.05 & 0 & 0.031 \\ 0 & 0.04 & 0 \\ 0.031 & 0 & 0.04 \end{bmatrix}$$

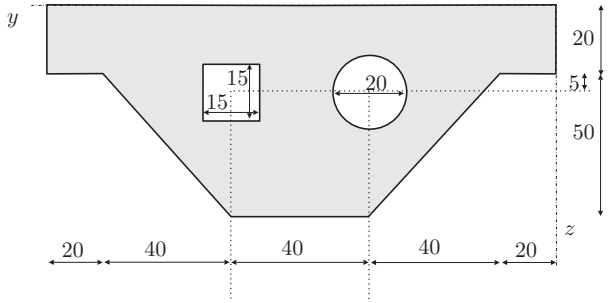
TRDNOST (VSŠ) - 4. IZPITNI ROK (06. 09. 2006)

RAČUNSKI DEL IZPITA:

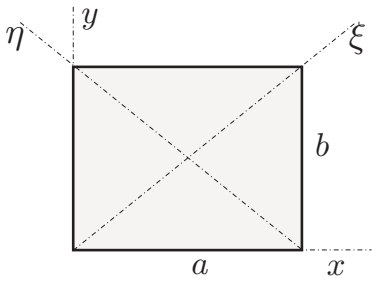
1. Za konstrukcijo na sliki izračunajte notranje statične količine! Vpliva osnih in prečnih sil ni potrebno upoštevati
(OBVEZNA NALOGA! 40%)
Podatki: $a = 3\,\mathrm{m}$, $b = 2\,\mathrm{m}$,
$q = 3\,\mathrm{kN/m}$,
$E = 3000\,\mathrm{kN/cm^2}$, $J_y = 1667\,\mathrm{cm^4}$,
$A_x = 200\,\mathrm{cm^2}$.



2. Za prerez na sliki določite geometrijske karakteristike: ploščino A, težišče y_T, z_T, vztrajnostne momente glede na podane koordinate I_y, I_z, I_{yz} in vztrajnostne momente glede na težišče I_y^T, I_z^T, I_{yz}^T! (30%)
Podatki so v centimetrih.



3. V tanki steni dimenzije $a = 5\,\mathrm{cm}$ in $b = 4\,\mathrm{cm}$ (glej sliko) izmerimo normalni napetosti v smereh ξ in η. Specifična sprememba dolžine v smeri osi x je enaka 0. Določite deformacijski in napetostni tenzor ob predpostavki linearno elastičnega izotropnega materiala. (30%)
Podatki:
$\sigma_{\xi\xi} = 10\,\mathrm{kN/cm^2}$, $\sigma_{\eta\eta} = 15\,\mathrm{kN/cm^2}$,
$\nu = 0.3$, $E = 2 \cdot 10^4\,\mathrm{kN/cm^2}$.



TEORETIČNI DEL IZPITA:
Izmed treh zastavljenih vprašanj si izberete dve, na kateri boste odgovarjali. Izbrani vprašanji jasno označite! Pišite čitljivo.

1. Opišite tenzor malih deformacij! Pojasnite geometrijski pomen njegovih komponent!

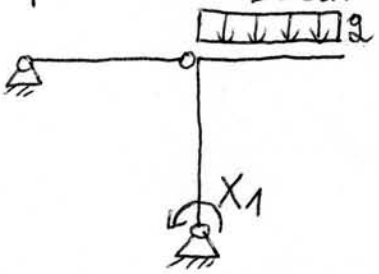
2. Opišite metodo pomikov za primer ravninskega paličja! Opišite postopek sestavljanja togostne matrike konstrukcije! Kako upoštevamo robne pogoje? (Pomagajte si s preprostim paličjem!)

3. Opišite metodo sil za reševanje statično nedoločenih linijskih konstrukcij! Naredite preprosti primer!

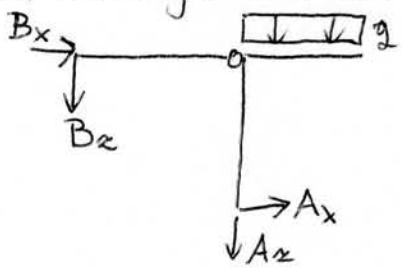
TRDNOST - VSŠ 6.9.2006

1. NALOGA

a.) Sprostitev - zasuk v A



b.) Zunanja obtežba



reakcije

$\Sigma X: \; A_x + B_x = 0$

$\Sigma Z: \; A_z + B_z + qb = 0$

$\Sigma M^C_{BC}: \; B_z \cdot a = 0 \quad \boxed{B_z = 0}$

$\Sigma M^A: \; -qb\frac{b}{2} - B_x \cdot a = 0$

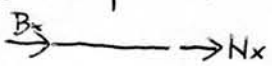
$\boxed{A_z = -6\,kN}$

$\boxed{A_x = 2\,kN}$

$\boxed{B_x = -2\,kN}$

notranje sile

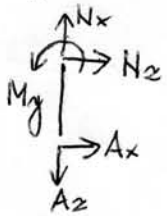
polje I (palica)



$N_x = -B_x$

$\boxed{N_x = 2\,kN} \quad \boxed{N_z = M_y = 0}$

polje II



$N_x = A_z$

$N_z = -A_x$

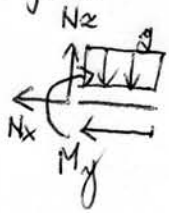
$M_y = -A_x x$

$\boxed{N_x = -6\,kN}$

$\boxed{N_z = -2\,kN}$

$\boxed{M_y = -2x}$ $\quad M_y(3) = -6\,kNm$

polje III



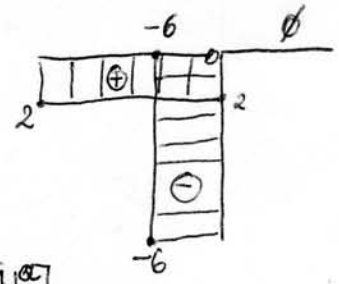
$\boxed{N_x = 0}$

$\boxed{N_z = q\bar{x}}$ $\quad N_z(0) = 0 \quad N_z(2) = 6$

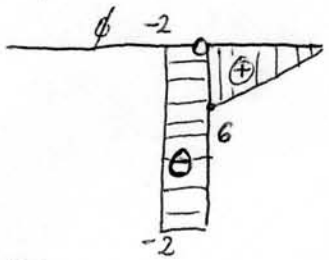
$\boxed{M_y = -q\bar{x}\frac{\bar{x}}{2}}$ $\quad M_y(2) = -6\,kNm$

diagrami

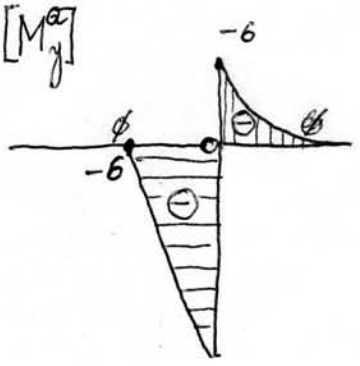
$[N_x^a]$



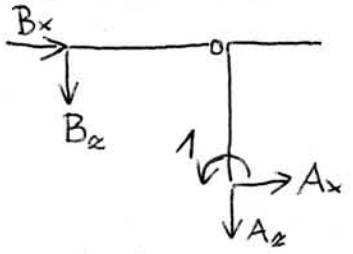
$[N_z^a]$



$[M_y^a]$



c.) Virtualna sila $X_1 = 1$



reakcije

$\Sigma X: \; A_x + B_x = 0$

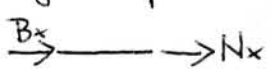
$\Sigma Z: \; A_z + B_z = 0$

ΣM^C_{BC} $\boxed{B_z = 0}$ $\boxed{A_z = 0}$ $\boxed{A_x = -\frac{1}{3}\,kN}$

$\Sigma M^A: \; 1 - B_x a = 0$ $\boxed{B_x = \frac{1}{3}\,kN}$

notranje sile

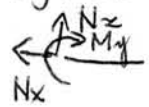
polje I (palica)



$N_x = -B_x$

$\boxed{N_x = -\frac{1}{3}\,kN}$ $\boxed{N_z = M_y = 0}$

polje III



$\boxed{N_x = N_z = M_y = 0}$

polje II

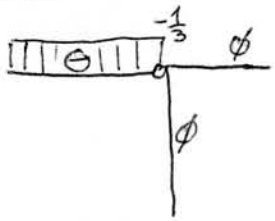
$\boxed{N_x = 0}$

$N_z = -A_x$ $\boxed{N_z = \frac{1}{3}\,kN}$

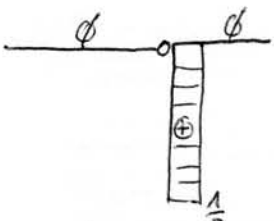
$M_y = -1 - A_x \bar{x}$ $\boxed{M_y = -1 + \frac{1}{3}\bar{x}}$

diagrami

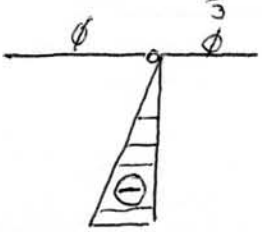
$[N_x]$



$[N_z]$



$[M_y]$



d.) Določitev sile X_1

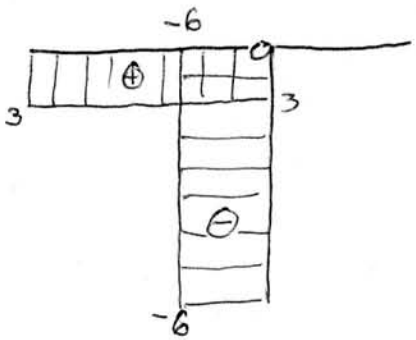
$$EI_y\, a_{11} = \int_0^a \overset{-1}{\triangle}\,\overset{-1}{\triangle}\, dx = \frac{1}{3}\cdot 3\cdot(-1)\cdot(-1) = 1$$

$$EI_y\, b_1 = \int_0^a \overset{-6}{\triangle}\,\overset{-1}{\triangle}\, dx = \frac{1}{6}\cdot 3\,(-6)\cdot(-1) = 3$$

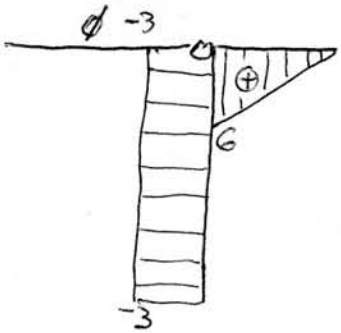
$$X_1 = -\frac{b_1}{a_{11}} = -3$$

e.) Superpozicija $\left(N^{nk} = N^{Q} + X_1 N^{X_1}\right)$

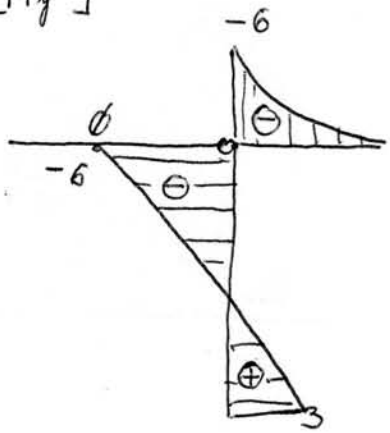
$[N_x^{nk}]$



$[N_z^{nk}]$

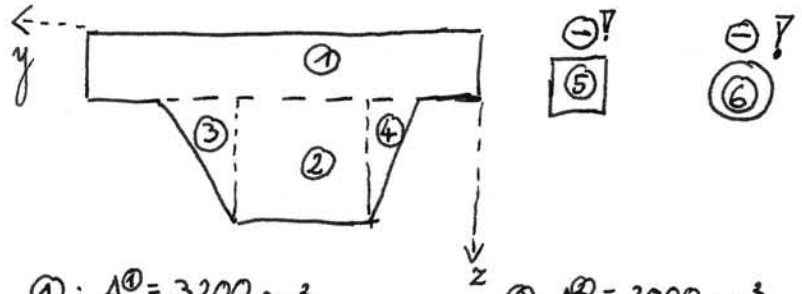


$[M_y^{nk}]$



TRDNOST - VSŠ 6.9.2006

2. NALOGA

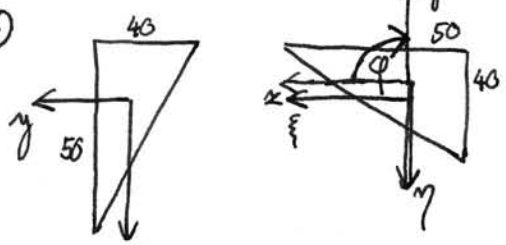


① : $A^{①} = 3200\ cm^2$
$y_T^{①} = 80\ cm$
$x_T^{①} = 10\ cm$
$I_y^{①} = 106\,666{\cdot}7\ cm^4$
$I_x^{①} = 6\,826\,666{\cdot}7\ cm^4$

② $A^{②} = 2000\ cm^2$
$y_T^{②} = 80\ cm$
$x_T^{②} = 45\ cm$
$I_y^{②} = 416\,666{\cdot}7\ cm^4$
$I_x^{②} = 266\,666{\cdot}7\ cm^4$

③ $A^{③} = 1000\ cm^2$
$y_T^{③} = 113{\cdot}3\ cm$
$x_T^{③} = 36{\cdot}7\ cm$
$I_y^{③} = 138\,888{\cdot}9\ cm^4$
$I_z^{③} = 88\,888{\cdot}9\ cm^4$
$I_{yz}^{③} = 55\,555{\cdot}5\ cm^4$

④



ξ in η po formulah
I_y in I_z zasuk za $-90°$ (NEGATIVEN KOT)
$\cos 90° = 0$ $\sin -90 = -1$

$A^{④} = 1000\ cm^2$
$y_T^{④} = 46{\cdot}7\ cm$
$x_T^{④} = 36{\cdot}7\ cm$

$I_\xi^{④} = 88\,888{\cdot}9\ cm^4$
$I_\eta^{④} = 138\,888{\cdot}9\ cm^4$
$I_{\xi\eta}^{④} = 55\,555{\cdot}5\ cm^4$

$I_y^{④} = I_\eta^{④} = 138\,888{\cdot}9\ cm^4$
$I_z^{④} = I_\xi^{④} = 88\,888{\cdot}9\ cm^4$
$I_{yz}^{④} = -I_{\xi\eta}^{④} = -55\,555{\cdot}5\ cm^4$

⑤ $A^{⑤} = 225\ cm^2$
$y_T^{⑤} = 100\ cm$
$x_T^{⑤} = 25\ cm$
$I_y^{⑤} = I_z^{⑤} = 4218{\cdot}75\ cm^4$

⑥ $A^{⑥} = 314\ cm^2$
$y_T^{⑥} = 60\ cm$
$x_T^{⑥} = 25\ cm$
$I_y^{⑥} = I_z^{⑥} = 7850\ cm^4$

$$A = A^{①} + A^{②} + A^{③} + A^{④} - A^{⑤} - A^{⑥} = 6661\ cm^2$$

$$y_T = \frac{\sum_{i=1}^{4} y_i A^{(i)} - y_⑤ A^{⑤} - y_⑥ A^{⑥}}{A} = 80{\cdot}27\ cm$$

$$x_T = \frac{\sum_{i=1}^{4} x_i A^{(i)} - x_⑤ A^{⑤} - x_⑥ A^{⑥}}{A} = 27{\cdot}3\ cm$$

$$I_y = I_y^{①} + I_y^{②} + I_y^{③} + I_y^{④} - I_y^{⑤} - I_y^{⑥} + z_{①}^2 A^{①} + z_{②}^2 A^{②} + z_{③}^2 A^{③} + z_{④}^2 A^{④} - z_{⑤}^2 A^{⑤} - z_{⑥}^2 A^{⑥} =$$
$$= 789\,042 + 6\,722\,062 = 7\,511\,105 \text{ cm}^4$$

$$I_z = 7\,259\,042 + 44\,921\,778 = 52\,180\,820 \text{ cm}^4$$

$$I_{yz} = 0 - y_T^{①} z_T^{①} A^{①} - \ldots - y_T^{④} z_T^{④} A^{④} + y_T^{⑤} z_T^{⑤} A^{⑤} + y_T^{⑥} z_T^{⑥} A^{⑥}$$
$$= -14\,593\,220 \text{ cm}^4$$

$$I_y^T = I_y - z_T^2 A = 2\,546\,728 \text{ cm}^4$$
$$I_z^T = I_z - y_T^2 A = 9\,262\,179 \text{ cm}^4$$
$$I_{yz}^T = I_{yz} + y_T z_T A = 3\,502 \text{ cm}^4$$

KONTROLA Z MATHEMATICO O.K.

TRDNOST - VSŠ 9. 6. 2006

3. NALOGA

TANKA STENA → RAVNINSKO NAPETOSTNO STANJE

$$[\sigma] = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & 0 \\ \sigma_{xy} & \sigma_{yy} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\vec{\xi} = (5,4)$ $\quad |\vec{\xi}| = \sqrt{25+16} = \sqrt{41}$ $\quad \vec{e}_\xi = \frac{1}{\sqrt{41}}(5,4)$

$\vec{\eta} = (-5,4)$ $\quad |\vec{\eta}| = \sqrt{25+16} = \sqrt{41}$ $\quad \vec{e}_\eta = \frac{1}{\sqrt{41}}(-5,4)$

$$\sigma_{\xi\xi} = e_\xi^T [\sigma] e_\xi = \frac{1}{41} \begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{xy} & \sigma_{yy} \end{bmatrix} \begin{bmatrix} 5 \\ 4 \end{bmatrix} =$$

$$[5 \;\; 4]$$

$$= \frac{1}{41} [5 \;\; 4] \begin{bmatrix} 5\sigma_{xx} + 4\sigma_{xy} \\ 5\sigma_{xy} + 4\sigma_{yy} \end{bmatrix} = \frac{1}{41}(25\sigma_{xx} + 40\sigma_{xy} + 16\sigma_{yy})$$

$$\Rightarrow \quad \boxed{41\sigma_{\xi\xi} = 25\sigma_{xx} + 40\sigma_{xy} + 16\sigma_{yy}}$$

ANALOGNO: $\boxed{41\sigma_{\eta\eta} = 25\sigma_{xx} + 16\sigma_{yy} - 40\sigma_{xy}}$ $\ominus$:

$$41(\sigma_{\xi\xi} - \sigma_{\eta\eta}) = 80\sigma_{xy}$$

velja tudi: $41(\sigma_{\xi\xi} + \sigma_{\eta\eta}) = 50\sigma_{xx} + 32\sigma_{yy}$

$\sigma_{xy} = -2{\cdot}56 \frac{kN}{cm^2}$

HOOKOV ZAKON:

$$\varepsilon_{\xi\xi} = \frac{1+\nu}{E}\sigma_{\xi\xi} - \frac{\nu}{E}(\sigma_{\xi\xi} + \sigma_{\eta\eta} + \overbrace{\sigma_{\varphi\varphi}}^{=0})$$

$$\varepsilon_{\eta\eta} = \frac{1+\nu}{E}\sigma_{\eta\eta} - \frac{\nu}{E}(\sigma_{\xi\xi} + \sigma_{\eta\eta} + \overbrace{\sigma_{\varphi\varphi}}^{=0})$$

$$\varepsilon_{xx} = \varepsilon_{\varphi\varphi} = -\frac{\nu}{E}(\sigma_{\xi\xi} + \sigma_{\eta\eta}) = -\frac{0{\cdot}3}{2\cdot 10^4} \cdot 25 = -3{\cdot}75 \cdot 10^{-4}$$

$$\varepsilon_{\xi\xi} = \frac{1{\cdot}3}{2\cdot 10^4} \cdot 10 - \frac{0{\cdot}3}{2\cdot 10^4} \cdot 25 = 2{\cdot}75 \cdot 10^{-4}$$

$$\varepsilon_{\eta\eta} = \frac{1{\cdot}3}{2\cdot 10^4} \cdot 15 - \frac{0{\cdot}3}{2\cdot 10^4} \cdot 25 = 6 \cdot 10^{-4}$$

$$\varepsilon_{xz} = 0, \quad \varepsilon_{yz} = 0$$

PODATEK:

$$\varepsilon_{xx} = 0$$

KOORDINATNA TRANSFORMACIJA

$$\varepsilon_{\xi\xi} = e_\xi^T \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & 0 \\ \varepsilon_{xy} & \varepsilon_{yy} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{bmatrix} e_\xi$$

$$= \frac{1}{41}\begin{bmatrix} 5 & 4 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & \varepsilon_{xy} & 0 \\ \varepsilon_{xy} & \varepsilon_{yy} & 0 \\ 0 & 0 & -3{\cdot}75\cdot 10^{-4} \end{bmatrix} \begin{bmatrix} 5 \\ 4 \\ 0 \end{bmatrix}$$

$$= \frac{1}{41}\begin{bmatrix} 5 & 4 & 0 \end{bmatrix} \begin{bmatrix} 4\varepsilon_{xy} \\ 5\varepsilon_{xy} + 4\varepsilon_{yy} \\ 0 \end{bmatrix} = \frac{1}{41}\left(20\varepsilon_{xy} + 20\varepsilon_{xy} + 16\varepsilon_{yy}\right)$$

$$\boxed{41\varepsilon_{\xi\xi} = 40\varepsilon_{xy} + 16\varepsilon_{yy}}$$

ANALOGNO: $\boxed{41\varepsilon_{\eta\eta} = -40\varepsilon_{xy} + 16\varepsilon_{yy}}$

$$41(\varepsilon_{\xi\xi} + \varepsilon_{\eta\eta}) = 32\,\varepsilon_{yy} \Rightarrow \varepsilon_{yy} = \frac{41}{32}(2{\cdot}75 + 6)\cdot 10^{-4}$$

$$\varepsilon_{yy} = 11{\cdot}2\cdot 10^{-4}$$

$$\varepsilon_{xy} = \frac{1}{40}(41\varepsilon_{\xi\xi} - 16\varepsilon_{yy}) = -1{\cdot}666\cdot 10^{-4}$$

$$[\varepsilon] = 10^{-4}\cdot \begin{bmatrix} 0 & -1{\cdot}7 & 0 \\ -1{\cdot}7 & 11{\cdot}2 & 0 \\ 0 & 0 & -3{\cdot}75 \end{bmatrix}$$

HOOKOV ZAKON II.

$$\varepsilon_{xy} = \frac{1+\nu}{E}\sigma_{xy} \Rightarrow \sigma_{xy} = \frac{2\cdot 10^4}{1{\cdot}3}\cdot(-1{\cdot}666)\cdot 10^{-4} = 2{\cdot}56\ \text{kN/cm}^2$$ ✓

$$\varepsilon_{xx} = \frac{1+\nu}{E}\sigma_{xx} - \frac{\nu}{E}(\sigma_{xx} + \sigma_{yy} + \sigma_{zz})$$

$$\sigma_{xx} + \sigma_{yy} + \sigma_{zz} = \sigma_{\eta\eta} + \sigma_{\xi\xi} + \sigma_{qq} = 25\ \text{kN/cm}^2$$

$$0 = \frac{1{\cdot}3}{2\cdot 10^4}\sigma_{xx} - \frac{0{\cdot}3}{2\cdot 10^4}\cdot 25$$

$$\Rightarrow \sigma_{xx} = 5{\cdot}77\ \text{kN/cm}^2$$

ANALOGNO $11{\cdot}2\cdot 10^{-4} = \frac{1{\cdot}3}{2\cdot 10^4}\cdot\sigma_{yy} - \frac{0{\cdot}3}{2\cdot 10^4}\cdot 25$

$$\Rightarrow \sigma_{yy} = 23\ \text{kN/cm}^2$$

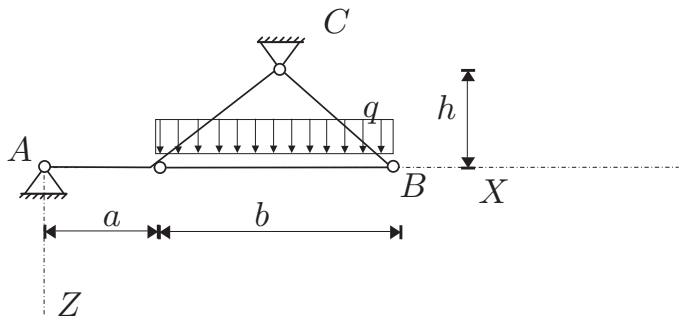
KONTROLA $50\cdot 5{\cdot}77 + 32{\cdot}23 = 41{\cdot}25$ ✓

$$[\sigma] = \begin{bmatrix} 5{\cdot}77 & -2{\cdot}56 & 0 \\ -2{\cdot}56 & 23 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

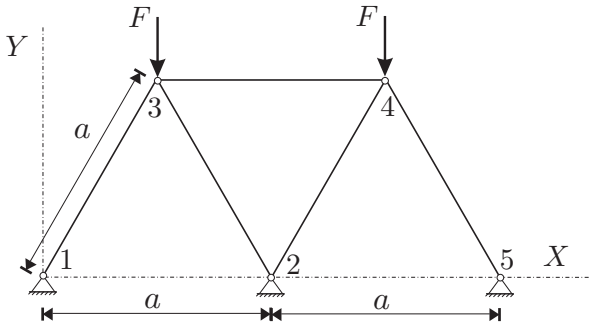
TRDNOST (VSŠ) - 2. IZREDNI IZPITNI ROK (30. 11. 2006)

RAČUNSKI DEL IZPITA:

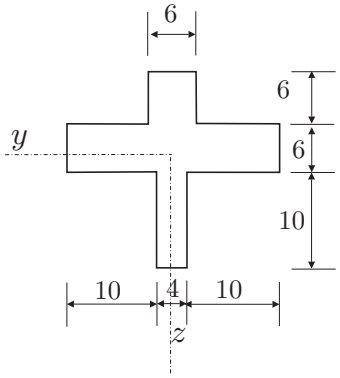
1. Za konstrukcijo na sliki izračunajte notranje statične količine! Upoštevajte tudi vpliv osnih sil. **(OBVEZNA NALOGA! 40%)**
Podatki: $a = 1\,\mathrm{m}$, $b = 2\,\mathrm{m}$
$h = 0.5\,\mathrm{m}$, $q = 5\,\mathrm{kN/m}$,
$E = 21000\,\mathrm{kN/cm^2}$, $J_y = 4000\,\mathrm{cm^4}$,
$A_x = 45\,\mathrm{cm^2}$.



2. Za paličje na sliki določite pomike vozlišč in osne sile v palicah po metodi pomikov! (30%)
Podatki: $a = 2\,\mathrm{m}$, $F = 15\,\mathrm{MN}$,
$E = 2 \cdot 10^5\,\mathrm{MPa}$, $A = 0.01\,\mathrm{m^2}$.



3. Prerez na sliki je obremenjen s prečno silo $N_z = 10\,\mathrm{kN}$. Določite nekaj značilnih vrednosti in skicirajte diagram strižne napetosti σ_{xy} v tem prerezu! (30%)
Podatki za prerez so v centimetrih.



TEORETIČNI DEL IZPITA:
1.12.2006 ob 9:00

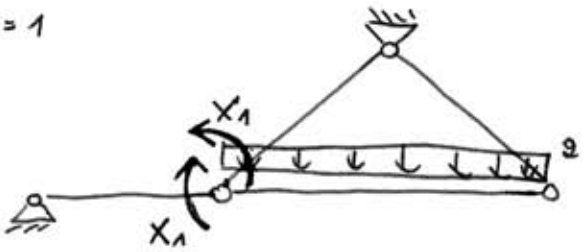
TRDNOST - VSŠ 30. 11. 2006

1. NALOGA

a.) $n = 2 \cdot 2 + 3 \cdot 2 - 3 \cdot 3 = 1$

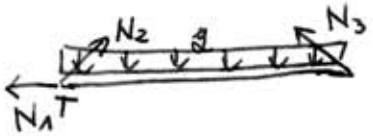
b.) SPROSTITEV

vrinemo členek :



c.) NOTRANJE SILE

c1.) Zunanja obtežba



$\operatorname{tg}\alpha = \frac{0.5}{1} \Rightarrow \alpha = 26.56°$

$\Sigma X: -N_1 + N_2 \cdot \cos\alpha - N_3 \cos\alpha = 0$

$\Sigma Z: q \cdot b - N_2 \sin\alpha - N_3 \sin\alpha = 0$

$\Sigma M^T: -q \cdot b \cdot \frac{b}{2} + N_3 \cdot \sin\alpha \cdot b = 0$

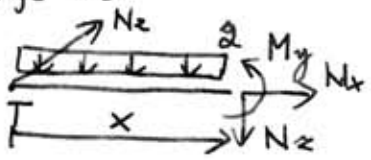
$N_2 = N_3$

$N_1 = 0$

$N_2 = 11.18\ kN$

$N_3 = 11.18\ kN$

polje TB



$N_x = -N_2 \cos\alpha$

$N_z = -q x + N_2 \sin\alpha$

$M_y = N_2 \sin\alpha \, x - q \cdot x \cdot \frac{x}{2}$

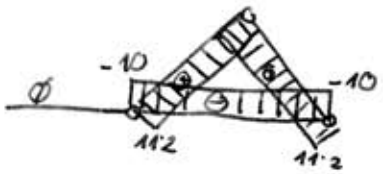
$N_x = -10\ kN$

$N_z = 5 - 5x$

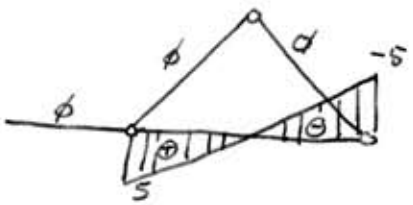
$M_y = 5x - \frac{5}{2}x^2$

$M_y(1) = 2.5\ kNm$

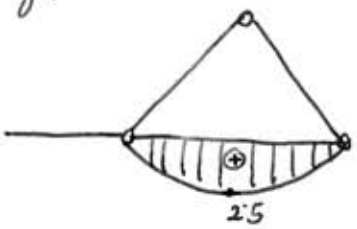
$[N_x^q]$



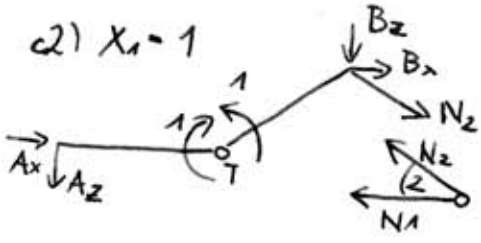
$[N_z^q]$



$[M_y^q]$



c2) $X_1 = 1$



$N_1 = N_2 = 0$

$\Sigma X: A_x + B_x = 0$

$\Sigma Z: A_z + B_z = 0$

$\Sigma M^A: -B_z \cdot 2 - B_x \cdot 0.5 = 0$

$\Sigma M^T_{AT}: A_z \cdot 1 - 1 = 0$

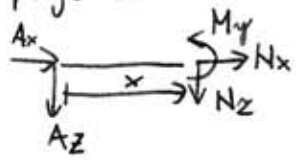
$A_x = -4\ kN$

$B_z = -1\ kN$

$B_x = +4\ kN$

$A_z = 1\ kN$

polje AT



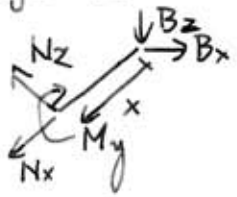
$N_x = -A_x$

$N_z = -A_z$

$M_y = -A_z \cdot x$

$N_x = +4$ kN
$N_z = -1$ kN
$M_y = -x$

polje TB



$\Sigma x: \; N_x - B_x \cdot \cos\alpha + B_z \cdot \sin\alpha = 0$

$\Sigma z: \; N_z - B_x \cdot \sin\alpha - B_z \cdot \cos\alpha = 0$

$\Sigma M: \; -M_y - B_x \cdot x \cdot \sin\alpha - B_z \cdot x \cdot \cos\alpha = 0$

$N_x = 4{\cdot}025$ kN

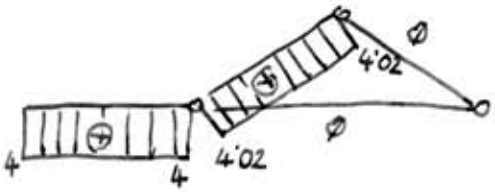
$N_z = 0{\cdot}89$ kN

$M_y = -0{\cdot}89x \qquad M_y(\ell) = -1$

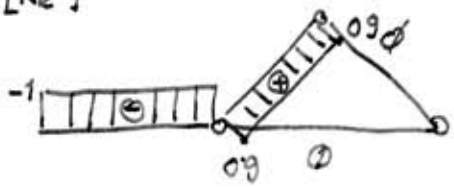
$x \in [0, \ell]$

$\ell = \sqrt{1^2 + 0{\cdot}5^2} = 1{\cdot}118$

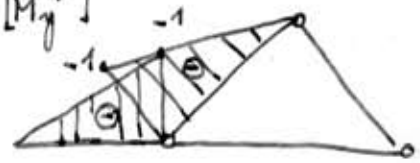
$[\overline{N_x^1}]$



$[\overline{N_z^1}]$



$[\overline{M_y^1}]$



d.) DOLOČITEV X_1

$$EI_y\, a_{11} = \int_0^a \overline{M_y^1}\,\overline{M_y^1}\,dx + \int_0^\ell \overline{N_z^1}\,\overline{N_z^1}\,dx$$

$$EI_y\, b_1 = 0$$

⇒ Upoštevamo tudi osne sile

$$b_1 = \frac{1}{EA_x} \int_0^\ell \overline{N_x^1}\,N_x\,dx$$

$$E\,b_{1} = \frac{1}{45\cdot 10^{-4}} \cdot 1{\cdot}118 \cdot 11{\cdot}18 \cdot 4{\cdot}025 = 1{\cdot}118 \cdot 10^4$$

$$a_{11} = \frac{1}{EI_y}\left(\int_0^a \overline{M_y^1}\,\overline{M_y^1}\,dx + \int_0^\ell \overline{N_z^1}\,\overline{N_z^1}\,dx\right) + \frac{1}{EA_x}\left(\int_0^\ell \overline{N_x^1}\,N_x\,dx + \int_0^a \overline{N_x^1}\,\overline{N_x^1}\,dx\right)$$

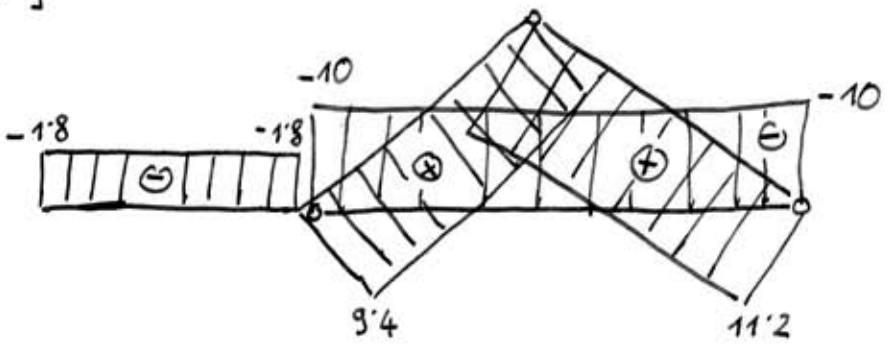
$$E\,a_{11} = \frac{1}{4000\cdot 10^{-2}E} \cdot \frac{1}{3}(1 + 1{\cdot}118) + \frac{1}{45\cdot 10^{-4}} \cdot (1{\cdot}118 \cdot 4{\cdot}025^2 + 1 \cdot 4^2)$$

$$= 2{\cdot}5231$$

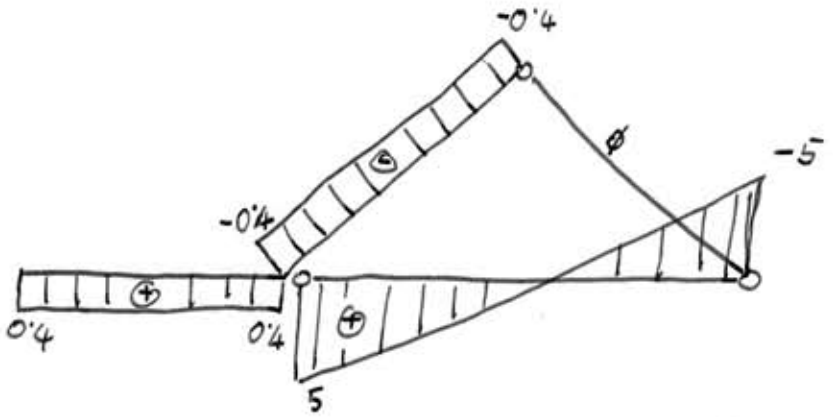
$$X_1 = -\frac{b_1}{a_{11}} = -0{\cdot}4431$$

e.) SUPERPOZICIJA

$[N_x^{mk}]$



$[N_z^{mk}]$



$[M_y^{mk}]$

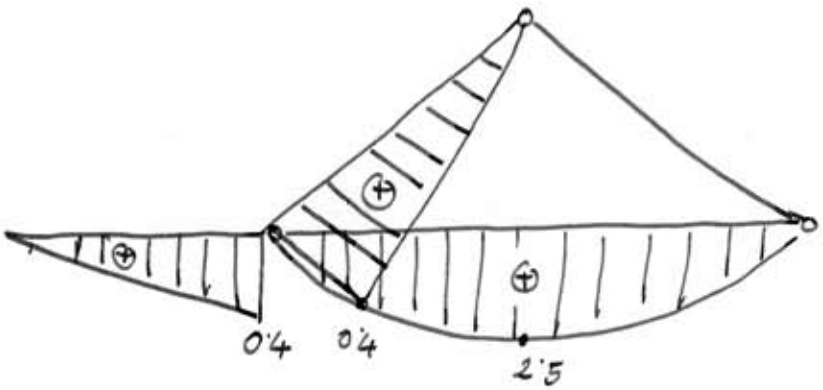


TABELA DOLŽIN, KOSINUSOV IN OSNIH TOGOSTI ZA PODANO PALIČJE

palica	vozel1	vozel2	dolzina	cos(a_ij)	cos(b_ij)	k_ij
1	1	3	2.000	0.500	0.866	1000.000
2	2	3	2.000	-0.500	0.866	1000.000
3	2	4	2.000	0.500	0.866	1000.000
4	3	4	2.000	1.000	0.000	1000.000
5	4	5	2.000	0.500	-0.866	1000.000

TOGOSTNA MATRIKA PALIČJA

-250.000	-433.013	0.000	0.000	250.000	433.013	0.000	0.000
0.000	0.000						
-433.013	-750.000	0.000	0.000	433.013	750.000	0.000	0.000
0.000	0.000						
0.000	0.000	-500.000	0.000	250.000	-433.013	250.000	433.013
0.000	0.000						
0.000	0.000	0.000	-1500.000	-433.013	750.000	433.013	750.000
0.000	0.000						
250.000	433.013	250.000	-433.013	-1500.000	0.000	1000.000	0.000
0.000	0.000						
433.013	750.000	-433.013	750.000	0.000	-1500.000	0.000	0.000
0.000	0.000						
0.000	0.000	250.000	433.013	1000.000	0.000	-1500.000	0.000
250.000	-433.013						
0.000	0.000	433.013	750.000	0.000	0.000	0.000	-1500.000
-433.013	750.000						
0.000	0.000	0.000	0.000	0.000	0.000	250.000	-433.013
-250.000	433.013						
0.000	0.000	0.000	0.000	0.000	0.000	-433.013	750.000
433.013	-750.000						

POMIKI IN REAKCIJE VOZLIŠČ DANEGA PALIČJA

vozel	u_x	u_y	R_x	R_y
1	0.00000	0.00000	4.330	7.500
2	0.00000	0.00000	0.000	15.000
3	0.00000	-0.01000		
4	0.00000	-0.01000		
5	0.00000	0.00000	-4.330	7.500

TABELA OSNIH SIL ZA PODANO PALIČJE

palica	vozel1	vozel2	N_ij
1	1	3	-8.660

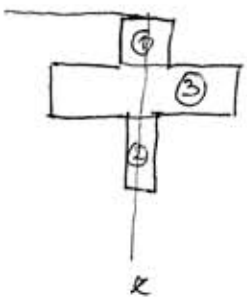
```
     2         2         3        -8.660
 ----------------------------------------------
     3         2         4        -8.660
 ----------------------------------------------
     4         3         4         0.000
 ----------------------------------------------
     5         4         5        -8.660
 ----------------------------------------------
```

TRDNOST - VSŠ 5. 7. 2005

3. NALOGA

$$\sigma_{xy} = -\frac{1}{\rho^*}\cdot\frac{S_y^*}{I_y}N_x$$

a.) Karakteristike prereza (I_y^T)



① $A^{①} = 36\,cm^2$ $z^{①} = 3\,cm$ $I_y^{①} = \frac{6\cdot 6^3}{12} = 108\,cm^4$

② $A^{②} = 40\,cm^2$ $z^{②} = 17\,cm$ $I_y^{②} = \frac{4\cdot 10^3}{12} = 333.\bar{3}\,cm^4$

③ $A^{③} = 144\,cm^2$ $z^{③} = 9\,cm$ $I_y^{③} = \frac{24\cdot 6^3}{12} = 432\,cm^4$

$A = 220\,cm^2$

$$z^T = \frac{3\cdot 36 + 17\cdot 40 + 9\cdot 144}{220} = 9.47\,cm$$

$$I_y = I_y^{①} + I_y^{②} + I_y^{③} + z^{①2}A^{①} + z^{②2}A^{②} + z^{③2}A^{③} = 24421\,cm^4$$

$$I_y^T = I_y - z^{T2}A = 4680\,cm^4$$

b.) Statični momenti $(S_y(y^*))$

Ⓘ $y^* \in [-12, -3]$

$$S_y^* = \int_{-12}^{y^*}\int_{-3.47}^{2.53} z\,dz\,dy = (y^*+12)\cdot\frac{1}{2}(2.53^2 - 3.47^2) = -2.8y^* - 33.8$$

$S_y^*(-12) = 0$

$S_y^*(-3) = -25.4$

Ⓘ $y^* \in [-3, -2]$

$$S_y^* = -25.4 + \int_{-3}^{y^*}\int_{-9.47}^{2.53} z\,dz\,dy = -25.4 + (y^*+3)\tfrac{1}{2}(2.53^2 - 9.47^2) = -41.6y^* - 150$$

$S_y^*(-2) = -67.1$

Ⓘ $y^* \in [-2, 2]$

$$S_y^* = -67.1 + \int_{-2}^{y^*}\int_{-9.47}^{12.53} z\,dz\,dy = -67.1 + (y^*+2)\cdot\frac{1}{2}(12.53^2 - 9.47^2) = 33.7y^*$$

$S_y^*(2) = 67.1$

OSTALO SIMETRIČNO

$S_y^*(3) = 25.4$

$S_y^*(12) = 0$

NALOGA 3

y*	h*	Sy*	Sigma_xy [N/cm2]
-12	6	0	0.00
-3	6	-25	8.90
-3	12	-25	4.45
-2	12	-67	11.93
-2	22	-67	6.51
2	22	67	-6.51
2	12	67	-11.93
3	12	25	-4.45
3	6	25	-8.90
12	6	0	0.00