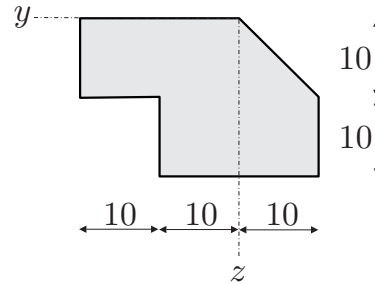


Pazljivo preberite besedilo vsake naloge!
Pišite čitljivo! Uspešno reševanje!

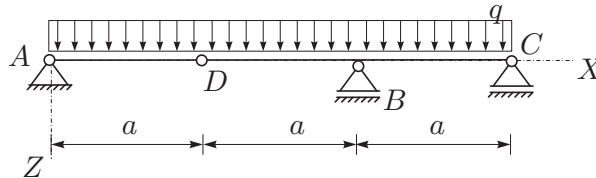
1. Izračunajte geometrijske karakteristike (A , y_T , z_T , I_y , I_z , I_{yz} , I_y^T , I_z^T , I_{yz}^T) lika na sliki! (30%)

Podatki so v centimetrih.



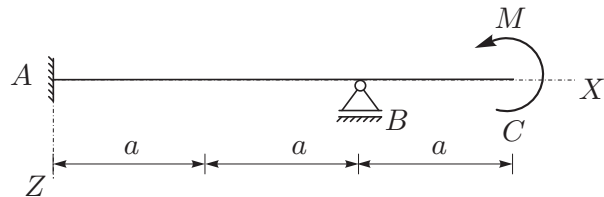
2. Za konstrukcijo na sliki izrazite upogibnico in določite navpični pomik v točki D ! (30%)

Podatki: $a = 3$ m, $q = 2$ kN/m,
 $E = 20000$ kN/cm², $A = 100$ cm²,
 $I_y = 20000$ cm⁴.



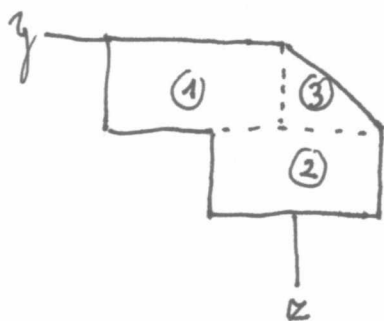
3. Za statično nedoločeno konstrukcijo na sliki izrazite upogibnico, notranje sile in določite navpični pomik v točki C ! Rezultate notranjih statičnih količin prikažite z diagrami! (40%)

Podatki: $a = 2$ m, $M = 10$ kNm,
 $E = 3000$ kN/cm²,
 $A = 1000$ cm², $I_y = 200000$ cm⁴.



TRDNOST 16. 12. 2014

1. NALOGA



	$y_T^{(i)}$	$z_T^{(i)}$	A_i	$I_y^{(i)}$	$I_z^{(i)}$	$I_{yz}^{(i)}$
①	10	5	200	$1666\bar{6}$	$6666\bar{6}$	0
②	0	15	200	$1666\bar{6}$	$6666\bar{6}$	0
③	$-\frac{10}{3}$	$\frac{20}{3}$	50	$277\bar{7}$	$277\bar{7}$	$138\bar{8}$

$$A = 450 \text{ cm}^2$$

$$z_T = \frac{5 \cdot 200 + 15 \cdot 200 + \frac{20}{3} \cdot 50}{450} = 9,629 \text{ cm}$$

$$y_T = \frac{10 \cdot 200 - \frac{10 \cdot 50}{3}}{450} = 4,074 \text{ cm}$$

$$I_y = \sum_i I_y^{(i)} + z_T^{(i)2} A_i = 55833\bar{3} \text{ cm}^4$$

$$I_y^T = I_y - z_T^2 A = 14105 \text{ cm}^4$$

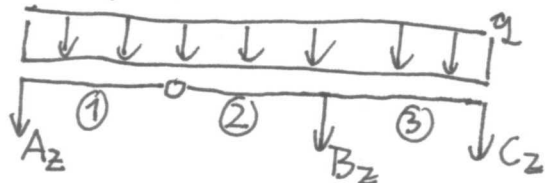
$$I_z = \sum_i I_z^{(i)} + y_T^{(i)2} A_i = 34166\bar{6} \text{ cm}^4$$

$$I_z^T = I_z - y_T^2 A = 26697,5 \text{ cm}^4$$

$$I_{yz} = \sum_i I_{yz}^{(i)} - y_T^{(i)} z_T^{(i)} A_i = -8750 \text{ cm}^4$$

$$I_{yz}^T = I_{yz} + y_T z_T A = 8904 \text{ cm}^4$$

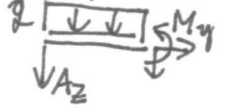
2. NALOGA



$$A_z = -\frac{qa}{2} = -3 \text{ kN}$$

$$C_z = \frac{qa}{2} = 3 \text{ kN}$$

$$B_z = 3qa = -18 \text{ kN}$$

polje ① $x \in [0, 3]$ 

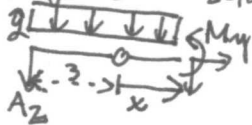
$$M_y = -A_z \cdot x - q \frac{x^2}{2}$$

$$M_y = 3x - x^2$$

$$\frac{d^2 w}{dx^2} = \frac{1}{EI_y} (x^2 - 3x)$$

$$\frac{dw}{dx} = \frac{1}{EI_y} \left(\frac{x^3}{3} - 3 \frac{x^2}{2} + C_1 \right)$$

$$w^{(1)}(x) = \frac{1}{EI_y} \left(\frac{x^4}{12} - \frac{3x^3}{6} + C_1 x + C_2 \right)$$

polje ② $x \in [0, 3]$ 

$$M_y = -A_z(3+x) + q \frac{(x+3)^2}{2}$$

$$M_y = -x^2 - 3x$$

$$\frac{d^2 w}{dx^2} = \frac{1}{EI_y} (x^2 + 3x)$$

$$\frac{dw}{dx} = \frac{1}{EI_y} \left(\frac{x^3}{3} + 3 \frac{x^2}{2} + D_1 \right)$$

$$w^{(2)}(x) = \frac{1}{EI_y} \left(\frac{x^4}{12} + \frac{3x^3}{6} + D_1 x + D_2 \right)$$

polje ③ $x \in [0, 3]$ 

$$M_y = -C_z x - q \frac{x^2}{2}$$

$$M_y = -x^2 + 9x - 18$$

$$\frac{d^2 w}{dx^2} = \frac{1}{EI_y} (x^2 - 9x + 18)$$

$$\frac{dw}{dx} = \frac{1}{EI_y} \left(\frac{x^3}{3} - 9 \frac{x^2}{2} + 18x + E_1 \right)$$

$$w^{(3)}(x) = \frac{1}{EI_y} \left(\frac{x^4}{12} - \frac{3x^3}{2} + 9x^2 + E_1 x + E_2 \right)$$

ROBNI POGOVI

$$w^A = 0 \Rightarrow w^{(1)}(0) = 0 \rightarrow C_2 = 0$$

$$w^B = 0 \Rightarrow w^{(2)}(3) = 0$$

$$\rightarrow w^{(3)}(0) = 0 \rightarrow E_2 = 0$$

$$w^C = 0 \Rightarrow w^{(3)}(3) = 0$$

$$\frac{3^4}{12} - \frac{3^4}{2} + 9 \cdot 9 + 3E_1 = 0 \Rightarrow E_1 = -15 \cdot 75$$

$$\varphi_C^{(2)} = \varphi_C^{(3)} \Rightarrow \frac{dw^{(2)}}{dx} \Big|_{x=3} = \frac{dw^{(3)}}{dx} \Big|_{x=0}$$

$$\frac{3^3}{3} + \frac{3^3}{2} + D_1 = E_1 \Rightarrow D_1 = -38 \cdot 25$$

$$w_D^{(1)} = w_D^{(2)} \Rightarrow w^{(1)}(3) = w^{(2)}(0)$$

$$\frac{3^4}{12} + \frac{3^4}{6} + 3D_1 + D_2 = 0 \Rightarrow D_2 = 94 \cdot 5$$

$$\frac{3^4}{12} - \frac{3^4}{6} + C_1 \cdot 3 = D_2 \Rightarrow C_1 = 33 \cdot 75$$

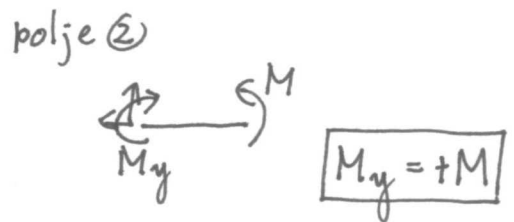
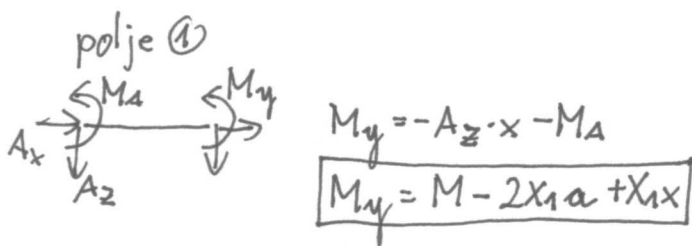
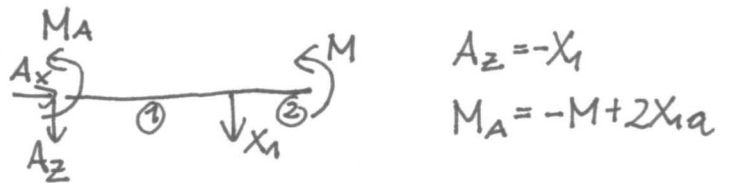
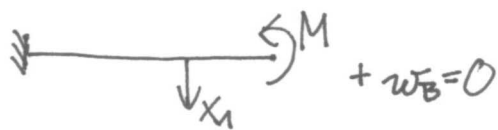
$$w^{(1)}(x) = \frac{1}{EI_y} \left(\frac{x^4}{12} - \frac{x^3}{2} + 33 \cdot 75 x \right)$$

$$w^{(2)}(x) = \frac{1}{EI_y} \left(\frac{x^4}{12} + \frac{x^3}{2} - 38 \cdot 25 x + 94 \cdot 5 \right)$$

$$w^{(3)}(x) = \frac{1}{EI_y} \left(\frac{x^4}{12} - \frac{3x^3}{2} + 9x^2 - 15 \cdot 75 x \right)$$

$$w^{(1)}(3) = \frac{1}{20000 \cdot 2} \left(\frac{3^4}{12} - \frac{3^3}{2} + 3 \cdot 33 \cdot 75 \right) = 0.00236 \text{ m} = 2.36 \text{ mm}$$

3. NALOGA



$$\frac{d^2 w}{dx^2} = +\frac{1}{EI_y} (-X_1 x + 2X_1 a - M)$$

$$\frac{d^2 w}{dx^2} = \frac{1}{EI_y} (-M)$$

$$\frac{dw}{dx} = \frac{1}{EI_y} (-X_1 \frac{x^2}{2} + 2X_1 a x - Mx + C_1)$$

$$\frac{dw}{dx} = \frac{1}{EI_y} (-Mx + D_1)$$

$$w^{(1)}(x) = \frac{1}{EI_y} (-X_1 \frac{x^3}{6} + X_1 a x^2 - M \frac{x^2}{2} + C_1 x + C_2)$$

$$w^{(2)}(x) = \frac{1}{EI_y} (-M \frac{x^2}{2} + D_1 x + D_2)$$

ROBNI POGOJI

$$w_A = 0 \Rightarrow w^{(1)}(0) = 0 \Rightarrow C_2 = 0$$

$$\varphi_A = 0 \Rightarrow \left. \frac{dw^{(1)}}{dx} \right|_{x=0} = 0 \Rightarrow C_1 = 0$$

$$w_B = 0 \Rightarrow w^{(1)}(2a) = 0 \longrightarrow \frac{1}{EI_y} (-X_1 \frac{4a^3}{3} + X_1 4a^3 - M 2a^2) = 0$$

$$\hookrightarrow w^{(2)}(0) = 0 \Rightarrow D_2 = 0$$

$$\boxed{X_1 = \frac{3M}{4a}}$$

$$\varphi_B^{(1)} = \varphi_B^{(2)} \Rightarrow \left. \frac{dw^{(1)}}{dx} \right|_{x=2a} = \left. \frac{dw^{(2)}}{dx} \right|_{x=0}$$

$$\hookrightarrow -X_1 2a^2 + 4X_1 a^2 - 2Ma = D_1$$

$$\boxed{D_1 = -\frac{Ma}{2}}$$

$$w^{(1)}(x) = \frac{M}{24EI_y} \left(-3\frac{x^3}{a} + 6x^2 \right)$$

$$w^{(2)}(x) = \frac{M}{2EI_y} (-x^2 - ax)$$

$$w^{(2)}(a) = \frac{10(-4-4)}{2 \cdot 3000 \cdot 20} = -\frac{2}{3000} \text{ m} = -0.00067 \text{ m} = -0.67 \text{ mm}$$

GeometrijskeKarakteristike[

{ {0, 0}, {20, 0}, {20, 10}, {10, 10}, {10, 20}, {-10, 20}, {-10, 10}, {0, 0} }]

Ax = 450.

Sy = 4333.33

Sz = 1833.33

Y_T = 4.07407

z_T = 9.62963

I_y = 55833.3

I_z = 34166.7

I_{yz} = -8750.

I_y^T = 14104.9

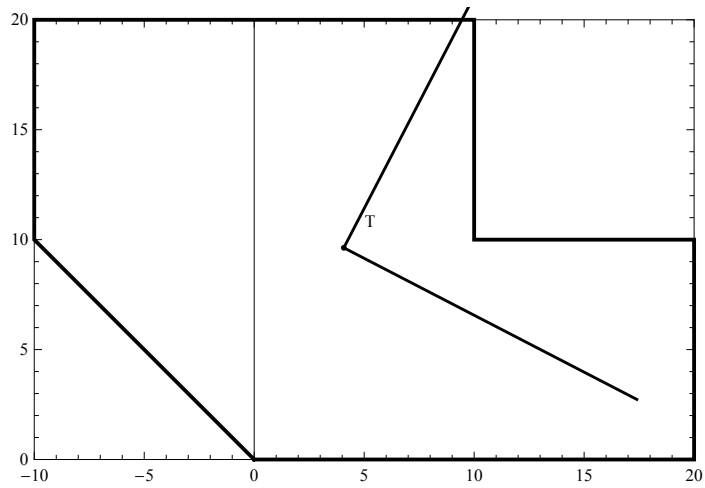
I_z^T = 26697.5

I_{yz}^T = 8904.32

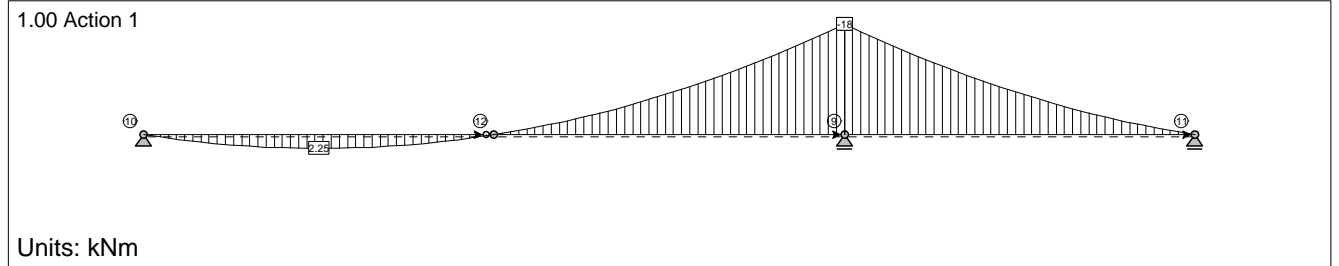
α_G = -27.3678

I₁ = 9495.72

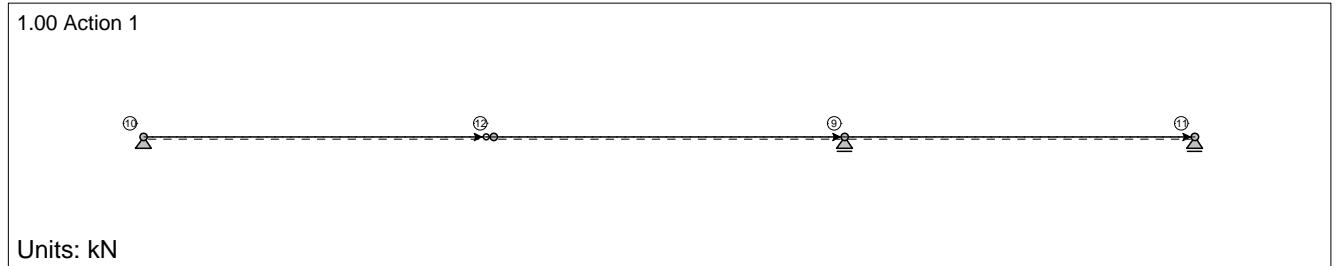
I₂ = 31306.8



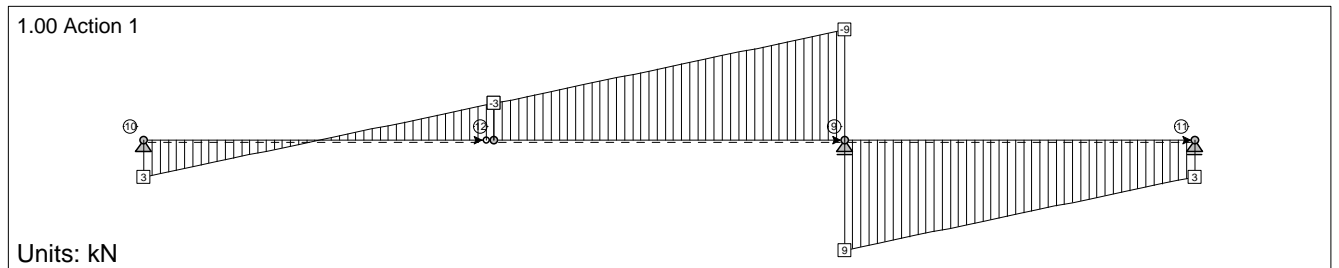
LC1: Load case 2: Bending Moments My



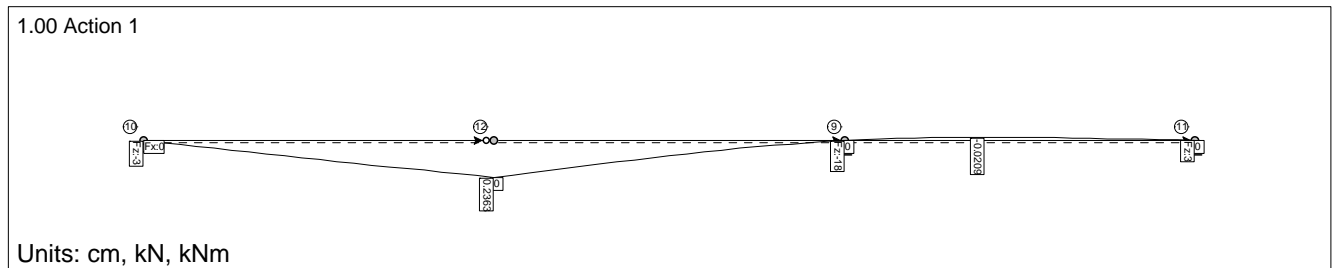
LC1: Load case 2: Axial Forces Fx



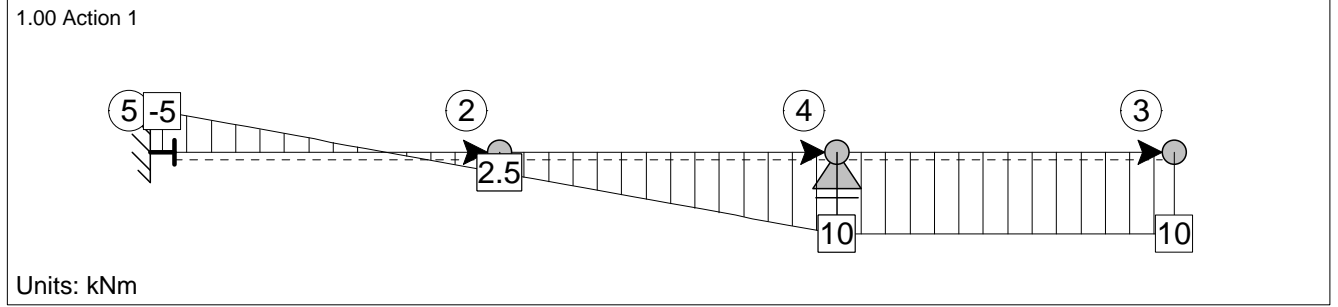
LC1: Load case 2: Shear Forces Fz



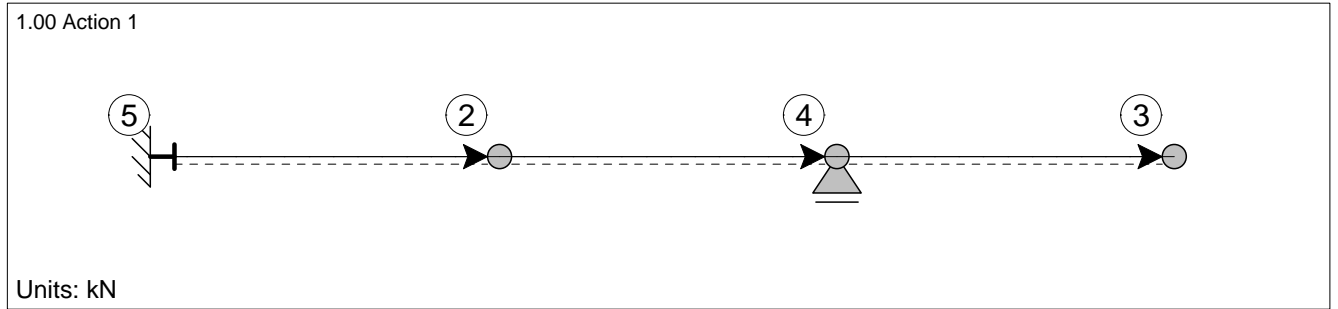
LC1: Load case 2: Displacements and Reactions



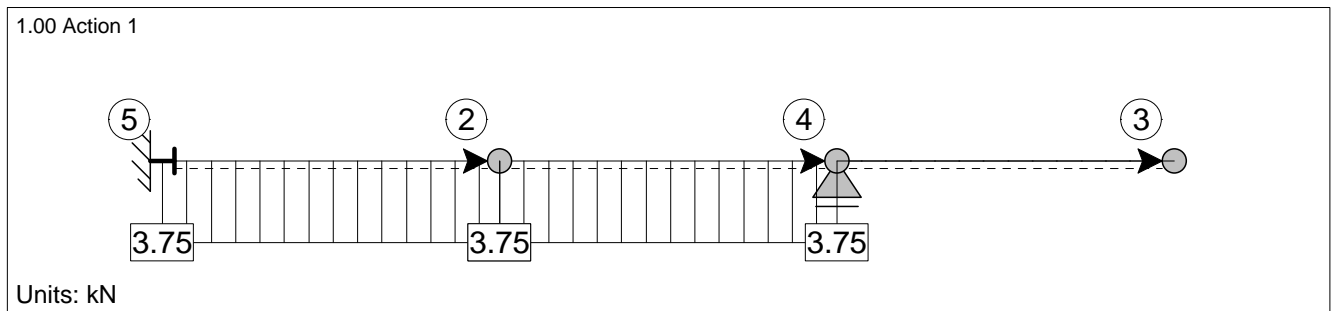
LC1: Load case 1: Bending Moments My



LC1: Load case 1: Axial Forces Fx



LC1: Load case 1: Shear Forces Fz



LC1: Load case 1: Displacements and Reactions

