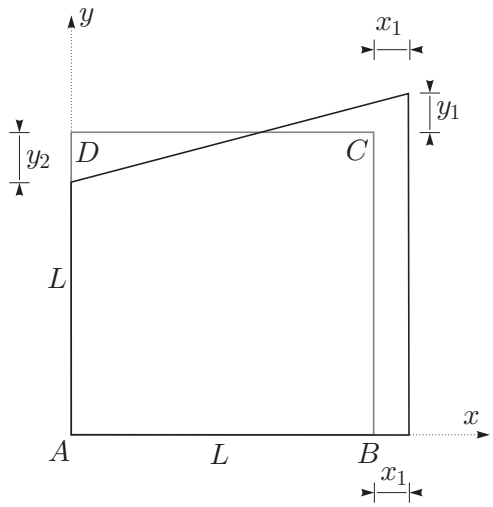
TRDNOST (VSŠ) - 1. KOLOKVIJ (17. 12. 2009)

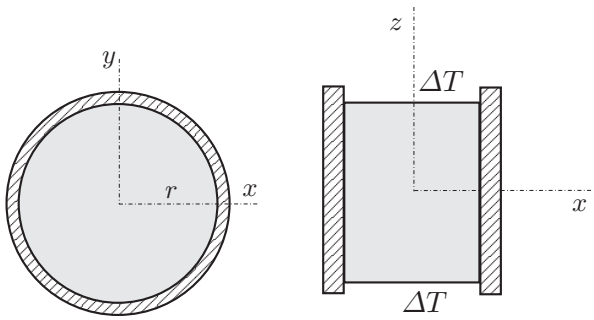
Pazljivo preberite besedilo vsake naloge! Naloge so točkovane enakovredno (vsaka 25%)! Pišite čitljivo! Uspešno reševanje!

1. Kvadratna plošča s stranico dolžine $L = 1$ m se deformira v svoji ravnini, kot kaže slika. Deformacije iz ravnine lahko zanemarimo. Tako deformiranje opišemo s pomiki oblike $\vec{u} = (ax, by + cxy)$. Lega točke A se ne spremeni. Nove koordinate ostalih točk pa so podane s pomiki: $x_1 = 2\,\mathrm{mm}$, $y_1 = 3\,\mathrm{mm}$ in $y_2 = 1\,\mathrm{mm}$.

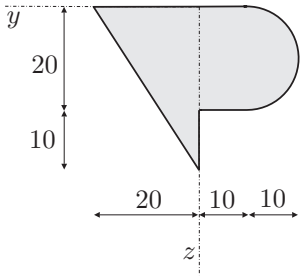


Izračunajte:
a) konstante a, b in c;
b) tenzor majhnih deformacij in njegovo vrednost v točki C;
c) specifično spremembo dolžine vlakna v točki C v smeri vektorja $\vec{AC}$;
d) spremembo pravega kota v točki C med vektorjema v smereh $\vec{AC}$ in $\vec{BD}$.

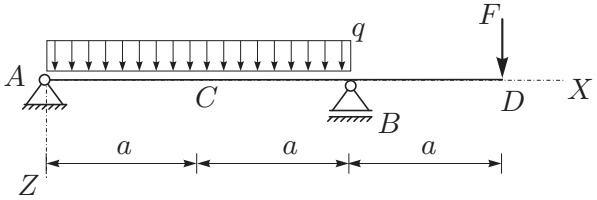
2. V togo, nerazteglivo cev je postavljen valj iz izotropnega, linearno elastičnega materiala. Polmer valja je 5 cm, višina pa 20 cm. Trenje med valjem in cevjo zanemarimo. Valj segrejemo za za $\Delta T = 30$ K.
a) Določite normalne napetosti med valjem in cevjo!
b) Določite tudi spremembo višine valja.
Podatki: $\nu = 0.2$, $E = 2 \cdot 10^4\,\mathrm{kN/cm^2}$,
$\alpha = 1.1 \cdot 10^{-5}\ \mathrm{K}^{-1}$.



3. Izračunajte geometrijske karakteristike (A, y_T, z_T, I_y, I_z, I_{yz}, I_y^T, I_z^T, I_{yz}^T) lika na sliki!
Podatki so v centimetrih.



4. Za konstrukcijo na sliki izrazite upogibnico in izvrednotite vertikalna pomika v točkah C in D! Podatki: $a = 3\,\mathrm{m}$, $q = 2\,\mathrm{kN/m}$, $F = 10\,\mathrm{kN}$, $E = 2500\,\mathrm{kN/cm^2}$, $A = 400\,\mathrm{cm^2}$, $I_y = 200000\,\mathrm{cm^4}$.



1. NALOGA

a.) $A = A' \quad (0,0) = (0,0)$ ✓

$B + u_B = B' \quad (L,0) + (aL, 0) = (L + x_1, 0)$

$$\Rightarrow \boxed{a = \frac{x_1}{L}} = \frac{2}{10^3} = 2 \cdot 10^{-3}$$

$C + u_C = C' \quad (L,L) + (aL, bL + cL^2) = (L + x_1, L + y_1)$

$$a = \frac{x_1}{L} \checkmark \qquad bL + cL^2 = y_1$$

$D + u_D = D' \quad (0,L) + (0, bL) = (0, L - y_2)$

$$bL = -y_2$$

$$\boxed{b = -\frac{y_2}{L}} \qquad \boxed{c = \frac{y_1 + y_2}{L^2}}$$

$$b = -1\,10^{-3} \qquad c = \frac{4}{10^6} = 4 \cdot 10^{-6}\ mm^{-1}$$

b.) $F = \begin{bmatrix} a & 0 \\ cy & b + cx \end{bmatrix}$

$$\varepsilon = \begin{bmatrix} a & \frac{cy}{2} \\ \frac{cy}{2} & b + cx \end{bmatrix}$$

$$\varepsilon|_{1000,1000} = \begin{bmatrix} 2 \cdot 10^{-3} & 2 \cdot 10^{-6} \cdot 10^{3} \\ 2 \cdot 10^{-6} \cdot 10^{-3} & -1 \cdot 10^{-3} + 4 \cdot 10^{-6} \cdot 10^{3} \end{bmatrix}$$

$$\varepsilon|_C = 10^{-3} \begin{bmatrix} 2 & 2 \\ 2 & 3 \end{bmatrix}$$

c.) $\vec{AC} = (1,1) \cdot L$

$e_{AC} = \frac{1}{\sqrt{2}}(1,1)$

$$\varepsilon_{AC} = \frac{1}{2} \cdot 10^{-3} \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{10^{-3}}{2} \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \end{bmatrix} = \frac{9}{2} \cdot 10^{-3} = 4{,}5 \cdot 10^{-3}$$

d.) $\vec{BD} = (-1,1)L$

$e_{BD} = \frac{1}{\sqrt{2}}(-1,1)$

$$\varepsilon_{AC\,BD} = \frac{1}{2} \cdot 10^{-3} \begin{bmatrix} -1 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \end{bmatrix} = \frac{1}{2} \cdot 10^{-3}$$

$$D_{AC,BD} = 10^{-3}\ rad$$

2. NALOGA

a.) $[\sigma][\begin{smallmatrix}\phi\\ \phi\\ 1\end{smallmatrix}] = \phi$

$\sigma_{xz} = 0$

$\sigma_{yz} = 0$

$\sigma_{zz} = 0$

$\varepsilon_{xx} = \varepsilon_{yy} = 0$

$$\left.\begin{aligned} 0 = \varepsilon_{xx} &= \tfrac{1+\nu}{E}\sigma_{xx} - \tfrac{\nu}{E}(\sigma_{xx}+\sigma_{yy}) + \alpha_T \Delta T \\ 0 = \varepsilon_{yy} &= \tfrac{1+\nu}{E}\sigma_{yy} - \tfrac{\nu}{E}(\sigma_{xx}+\sigma_{yy}) + \alpha_T \Delta T \end{aligned}\right) - \; +$$

$$0 = \tfrac{1+\nu}{E}(\sigma_{xx} - \sigma_{yy}) \Rightarrow \sigma_{xx} = \sigma_{yy}$$

$$0 = \tfrac{1-\nu}{E}(\sigma_{xx} + \sigma_{yy}) + 2\alpha_T \Delta T$$

$$\Rightarrow \sigma_{xx} = \sigma_{yy} = -\frac{E}{1-\nu}\alpha_T \Delta T$$

$$= -\frac{2\cdot 10^4 \, kN}{cm^2 \cdot 0{,}8} \cdot 1{,}1 \cdot 10^{-5} K^{-1} \cdot 30 K$$

$$\boxed{\sigma_{xx} = \sigma_{yy} = -8{,}25 \; kN/cm^2}$$

b.) $$\varepsilon_{zz} = -\frac{\nu}{E}(\sigma_{xx} + \sigma_{yy}) + \alpha_T \Delta T$$

$$= -\frac{2\nu}{E}\sigma_{xx} + \alpha_T \Delta T$$

$$= \frac{-0{,}4}{2\cdot 10^4}(-8{,}25) + 1{,}1 \cdot 10^{-5} \cdot 30$$

$$\boxed{\varepsilon_{zz} = 4{,}95 \cdot 10^{-4}}$$

$$\Delta h = \varepsilon_{zz} \cdot h = 0{,}099 \; mm$$

3. NALOGA

① $A^{①} = 300\,cm^2$

$y^{①} = \frac{20}{3} = 6.7\,cm$

$z^{①} = \frac{30}{3} = 10\,cm$

$I_y^{①} = \frac{A \cdot 30^2}{18} = 15000\,cm^4$

$I_z^{①} = \frac{A \cdot 20^2}{18} = 6666.7\,cm^4$

$I_{yz}^{①} = \frac{A^2}{18} = 5000\,cm^4$

② $A^{②} = 200\,cm^2$

$y^{②} = -5\,cm$

$z^{②} = 10\,cm$

$I_y^{②} = \frac{20^3 \cdot 10}{12} = 6666.7\,cm^4$

$I_z^{②} = \frac{10^3 \cdot 20}{12} = 1666.7\,cm^4$

$I_{yz}^{②} = 0$

③ $A^{③} = \frac{\pi \cdot 10^2}{2} = 157\,cm^2$

$y^{③} = \frac{-4 \cdot 10}{3\pi} - 10 = -14.25\,cm$

$z^{③} = 10\,cm$

$I_y^{③} = \frac{\pi 10^4}{8} = 3925\,cm^4$

$I_z^{③} = 10^4\left(\frac{\pi}{8} - \frac{8}{9\pi}\right) = 1094\,cm^4$

$I_{yz}^{③} = 0$

a.) $A = A^{①} + A^{②} + A^{③} = 657\,cm^2$

b.) $y_T = \frac{\sum y^{(i)} A^{(i)}}{A} = -1.88\,cm$

$z_T = \frac{\sum z^{(i)} A^{(i)}}{A} = 10\,cm$

c.) $I_y = I_y^{①} + I_y^{②} + I_y^{③} + z^{①2} A^{①} + z^{②2} A^{②} + z^{③2} A^{③}$

$= 91292\,cm^4$

$I_z = I_z^{①} + I_z^{②} + I_z^{③} + y^{①2} A^{①} + y^{②2} A^{②} + y^{③2} A^{③}$

$= 59642\,cm^4$

$I_{yz} = 5000 - 6.7 \cdot 10 \cdot 300 + 5 \cdot 10 \cdot 200 + 14.25 \cdot 10 \cdot 157 = 17242\,cm^4$

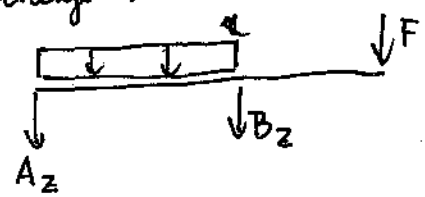
d.) $I_y^T = I_y - z^2 A = 25592\,cm^4$

$I_z^T = I_z - y_T^2 A = 57319\,cm^4$

$I_{yz}^T = 4921\,cm^4$

4. NALOGA

a.) notranje sile



$A_x = 0$

$A_z + B_z = -F - q \cdot 2a = -10 - 2 \cdot 6 = -22\,kN$

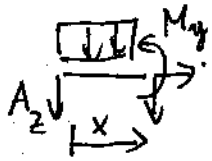
$-B_z \cdot 6 - F \cdot 9 - q \cdot 6 \cdot 3 = 0$

$B_z = -F \cdot \frac{3}{2} - q \cdot 3$

$B_z = -21\,kN$

$A_z = -1\,kN$

polje ①



$M_y = -A_z x - q\frac{x^2}{2}$

$\boxed{M_y = x - x^2}$

$M_y(6) = 6 - 36 = -30\,kNm$

polje ②

$M_y = -F\bar{x}$

$\bar{x} = 3 - x$

~~$M_y = +10x - 20$~~

$\boxed{M_y = -30 + 10x}$

$M_y(0) = -30\,kNm$

kontrola zveznost momentov O.K.

b.) upogibnica

$$\frac{d^2w}{dx^2} = -\frac{x}{EI_y} + \frac{x^2}{EI_y}$$

$$\frac{dw}{dx} = C_1 - \frac{x^2}{2EI_y} + \frac{x^3}{3EI_y}$$

$$w^{①}(x) = C_2 + C_1 x - \frac{x^3}{6EI_y} + \frac{x^4}{12EI_y}$$

$$\frac{d^2w}{dx^2} = \frac{30}{EI_y} - \frac{10x}{EI_y}$$

$$\frac{dw}{dx} = D_1 + \frac{30x}{EI_y} - \frac{10x^2}{2EI_y}$$

$$w^{②}(x) = D_2 + D_1 x + \frac{15x^2}{EI_y} - \frac{5x^3}{3EI_y}$$

c.) robni pogoji

$w^{①}(0) = 0 \Rightarrow C_2 = 0$

$w^{①}(6) = 0 \Rightarrow C_1 \cdot 6 - \frac{6^3}{6EI_y} + \frac{6^4}{12EI_y} = 0$

$C_1 = -\frac{12}{EI_y}$

$w^{②}(0) = 0 \Rightarrow D_2 = 0$

$\left.\frac{dw^{①}}{dx}\right|_{x=6} = \left.\frac{dw^{②}}{dx}\right|_{x=0}$

$-\frac{12}{EI_y} - \frac{6^2}{2EI_y} + \frac{6^3}{3EI_y} = D_1$

$D_1 = \frac{42}{EI_y}$

$$w^{①}(x) = \frac{1}{EI_y}\left(-12x - \frac{x^3}{6} + \frac{x^4}{12}\right)$$

$$w^{②}(x) = \frac{1}{EI_y}\left(42x + 15x^2 - \frac{5}{3}x^3\right)$$

$$w_C = w^{①}(3) = \frac{1}{20\cdot10^4\,cm^4 \cdot 2500\,kN/cm^2}\left(-12\cdot 3 - \frac{3^3}{6} + \frac{3^4}{12}\right) kN\,m^3$$

$$= -\frac{33{,}75\cdot 10^6\,cm}{500\cdot 10^6} = \underline{\underline{-0{,}0675\,cm}}$$

$$w_D = w^{②}(3) = \frac{1}{20\cdot10^4\,cm^4 \cdot 2500\,kN/cm^2}\left(42\cdot 3 + 15\cdot 9 - 5\cdot 9\right) kN\cdot m^3$$

$$= \frac{216\cdot 10^6\,cm}{500\cdot 10^6} = \underline{\underline{0{,}432\,cm}}$$