

Dejan Zupan

IZPITNE NALOGE IN REŠITVE NALOG S POSTOPKOM IZ PREDMETA STATIKA NA
UNIVERZITETNEM ŠTUDIJU GRADBENIŠTVA

Igor Planinc

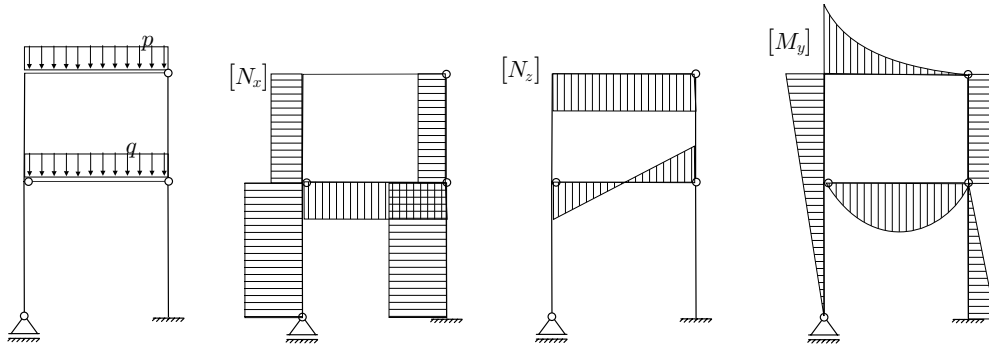
VPRAŠANJA IZ TEORIJE PRI PREDMETU STATIKA NA
UNIVERZITETNEM ŠTUDIJU GRADBENIŠTVA

ŠTUDIJSKO LETO: 2005/06

STATIKA (UNI) - ZIMSKI IZPITNI ROK (30. 1. 2006)

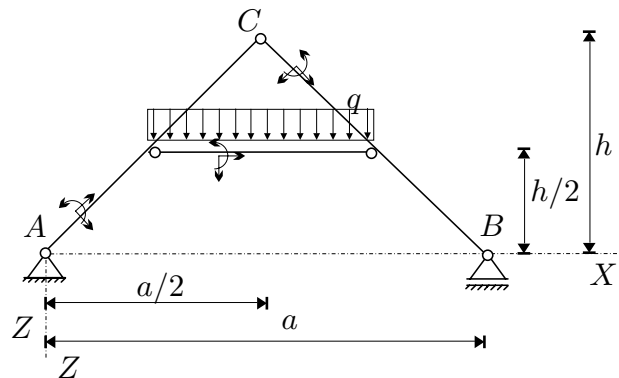
RAČUNSKI DEL IZPITA:

1. Janezek je na izpitu iz statike padel. Njegovi diagrami so polni napak. Pomagaj Janezku in poišči (BREZ RAČUNANJA) vse napake v spodnjih diagramih! Namig: najprej ugotovi katere reakcije so enake nič! (OBVEZNA NALOGA! 25%)



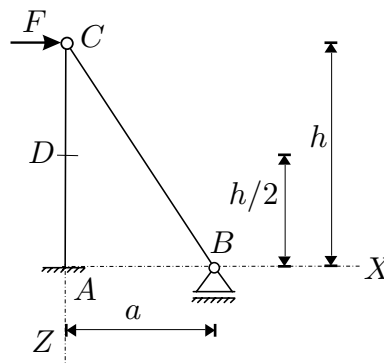
2. Za konstrukcijo na sliki izračunajte stopnjo statične nedoločenosti, reakcije in notranje statične količine (N_x, N_z, M_y)! Rezultate notranjih statičnih količin prikažite z diagrami!

Podatki: $a = 6 \text{ m}$, $h = 2 \text{ m}$,
 $q = 10 \text{ kN/m}$.
 (OBVEZNA NALOGA! 50%)



3. S principom virtualnega dela za konstrukcijo na sliki izračunajte reakcije v podporah in notranji moment v točki $D - M_D$! (25%)

Podatki: $a = 3 \text{ m}$, $h = 4 \text{ m}$,
 $F = 10 \text{ kN}$.

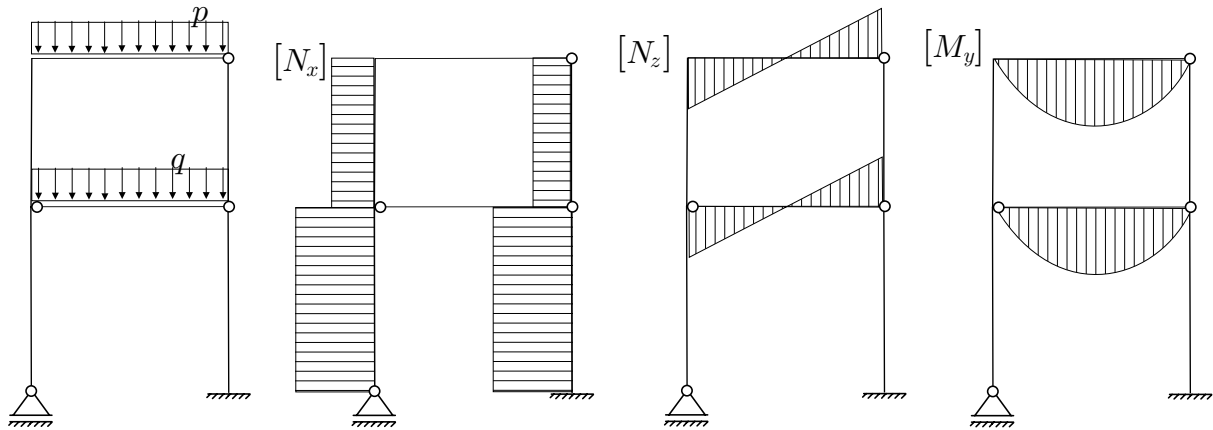


TEORETIČNI DEL IZPITA:

Izmed treh zastavljenih vprašanj si izberete dve, na kateri boste odgovarjali. Izbrani vprašanja jasno označite! Pišite čitljivo.

1. Kdaj govorimo o statično določenih linijskih konstrukcijah! Odgovor komentirajte tudi s preprostimi značilnimi primeri!
2. Izpeljite in opišite nadomestne ravnotežne pogoje (razumevanje podkrepitve s preprostim primerom)!
3. Opišite določanje reakcij in notranjih sil statično določenih linijskih konstrukcij z izrekom o virtualnih pomikih (Razumevanje ilustrirajte na s previsnem nosilcu s točkovno silo naprostem robu)!

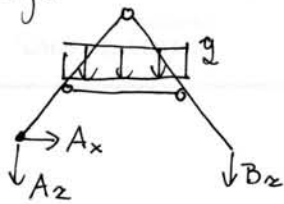
1. Naloga: PRAVILNI DIAGRAMI



2. NALOGA

a.) $\tilde{m}_{PS} = 3 \cdot 3 - 2 - 1 - 3 \cdot 2 = 0$

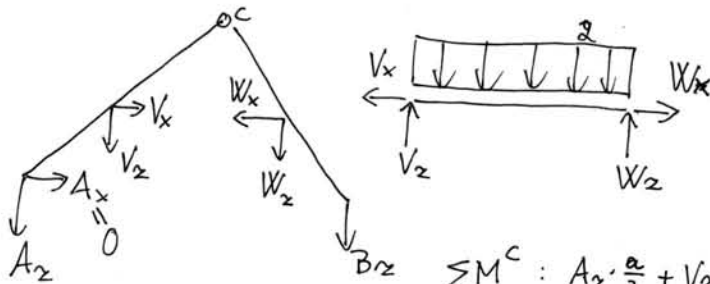
b.) reakcije



$$\begin{aligned} \Sigma x: & A_x = 0 \\ \Sigma z: & A_z + B_z = -g \frac{a}{2} \\ \Sigma M^A: & -B_z \cdot a - g \frac{a}{2} \cdot \frac{a}{2} = 0 \end{aligned}$$

$$\begin{aligned} A_z &= -15 \text{ kN} \\ B_z &= -15 \text{ kN} \end{aligned}$$

c.) sile v vezeh



$$\begin{aligned} \Sigma x: & V_x = W_x \\ \Sigma z: & V_z + W_z = g \frac{a}{2} \\ \Sigma M^V: & W_z \cdot \frac{a}{2} - g \frac{a}{2} \cdot \frac{a}{4} = 0 \Rightarrow \end{aligned}$$

$$\begin{aligned} V_z &= g \frac{a}{4} \\ W_z &= g \frac{a}{4} \end{aligned}$$

$$\Sigma M^C: A_z \cdot \frac{a}{2} + V_z \cdot \frac{a}{4} + V_x \cdot \frac{h}{2} = 0$$

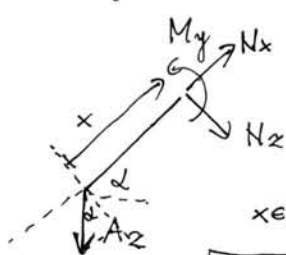
$$V_x = \frac{-2}{h} \left(-g \frac{a^2}{8} + g \frac{a^2}{16} \right) = + \frac{g a^2}{8h}$$

$$W_x = \frac{g a^2}{8h}$$

$V_x = 22.5 \text{ kN}$ $W_x = 22.5 \text{ kN}$ $V_z = 15 \text{ kN}$ $W_z = 15 \text{ kN}$

d.) notranje sile po poljih:

polje I



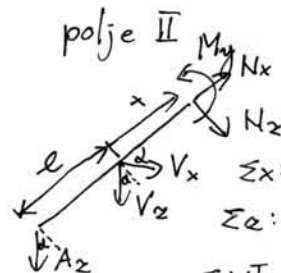
$$\begin{aligned} \Sigma x: & N_x = A_z \cdot \sin \alpha \\ \Sigma z: & N_z = -A_z \cdot \cos \alpha \\ \Sigma M^T: & M_{xy} = -A_z \cdot \cos \alpha \cdot x \end{aligned}$$

$x \in [0, l]$
 $l = \sqrt{\left(\frac{a}{4}\right)^2 + \left(\frac{a}{2}\right)^2} = 1.8 \text{ m}$

$\tan \alpha = \frac{2}{3} \Rightarrow \alpha = 33.69^\circ$

$$\begin{aligned} N_x &= -8.3 \text{ kN} \\ N_z &= 12.5 \text{ kN} \\ M_{xy} &= 12.5x \\ M_{xy}(l) &= 22.5 \text{ kNm} \end{aligned}$$

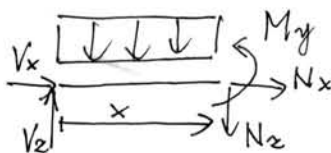
polje II



$$\begin{aligned} \Sigma x: & N_x = A_z \cdot \sin \alpha + V_z \cdot \sin \alpha - V_x \cdot \cos \alpha \\ \Sigma z: & N_z = -A_z \cdot \cos \alpha - V_z \cdot \cos \alpha - V_x \cdot \sin \alpha \\ \Sigma M^T: & M_{xy} = -A_z \cdot \cos \alpha (l+x) \\ & \quad - V_z \cdot \cos \alpha \cdot x \\ & \quad - V_x \cdot \sin \alpha \cdot x \end{aligned}$$

$$\begin{aligned} N_x &= -18.7 \text{ kN} \\ N_z &= -12.5 \text{ kN} \\ M_{xy} &= 22.5 - 12.5x \end{aligned}$$

polje III



$$\begin{aligned} N_x &= V_x \\ N_z &= V_z - g \cdot x \\ M_{xy} &= V_z \cdot x - g \cdot \frac{x^2}{2} \end{aligned}$$

$$\begin{aligned} N_x &= 22.5 \text{ kN} \\ N_z &= 15 - 10x \\ M_{xy} &= 15x - 5x^2 \\ M_{xy}(15) &= 11.25 \text{ kNm} \text{ (ekstrem)} \end{aligned}$$

polji III in IV

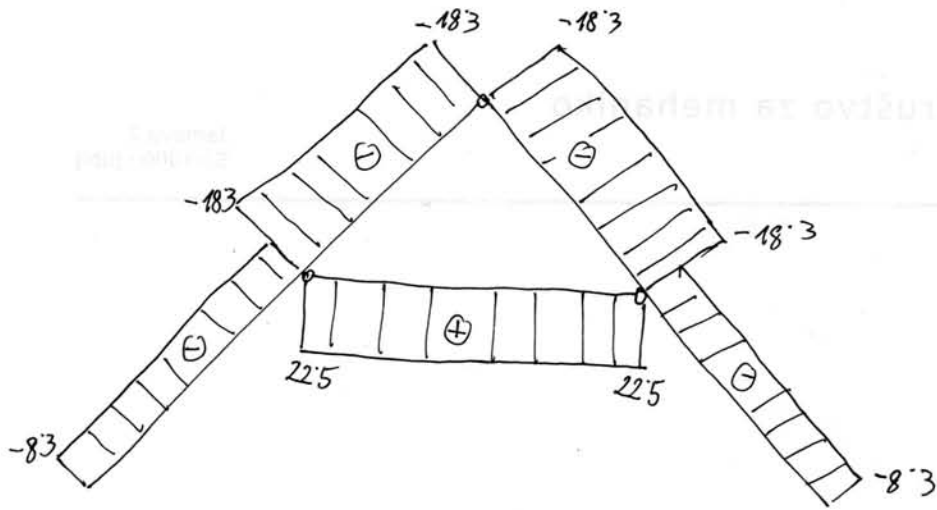
SIMETRIJA



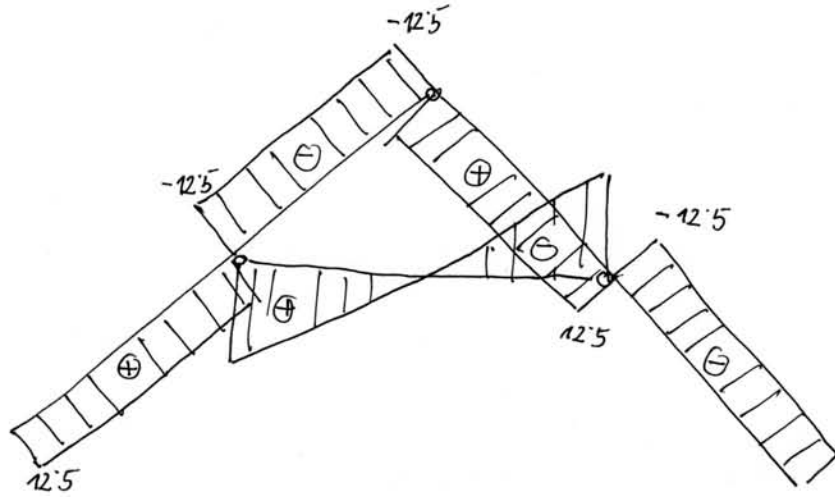
N_x, M_{xy} enaka
 N_z enako velik,
 nasprotno predznaka

e.) DIAGRAMI

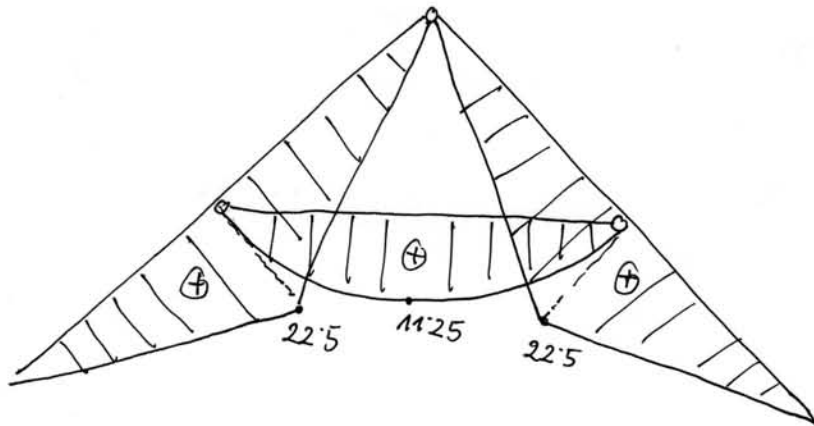
$[N_x]$



$[N_z]$

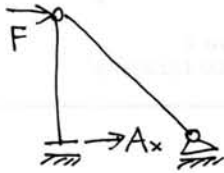


$[M_{xy}]$



3. NALOGA

a.) A_x



$$\delta W = F \delta u_c + A_x \delta u_A = 0$$

$$\delta u_c = \delta u_A - h \delta \varphi_A$$

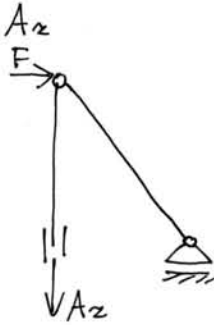
$$\delta \varphi_A = 0 \Rightarrow \delta u_c = \delta u_A$$

$$F \delta u_c + A_x \delta u_c = 0$$

$$\Rightarrow \boxed{A_x = -F}$$

$$\begin{aligned} \delta u &= \delta u_0 + \alpha \delta \varphi_0 \\ \delta w &= \delta w_0 - x \delta \varphi_0 \\ \delta \varphi &= \delta \varphi_0 \end{aligned}$$

b.) A_z

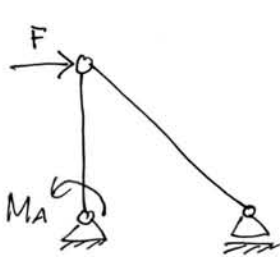


$$\delta W = F \delta u_c + A_z \delta w_z = 0$$

$$\delta u_c = \delta u_A - h \delta \varphi_A \Rightarrow \delta u_c = 0$$

$$A_z \delta w_z = 0 \Rightarrow \boxed{A_z = 0}$$

c.) M_A



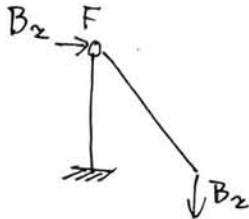
$$\delta W = F \delta u_c + M_A \delta \varphi_A = 0$$

$$\delta u_c = \delta u_A - h \delta \varphi_A$$

$$-F \cdot h \cdot \delta \varphi_A + M_A \delta \varphi_A = 0$$

$$\Rightarrow \boxed{M_A = F \cdot h}$$

d.) B_z



$$\delta W = F \delta u_c + B_z \delta w_B = 0$$

$$\delta u_c = \delta u_B - \alpha \delta \varphi_B$$

$$\delta w_c = \delta w_B - h \delta \varphi_B$$

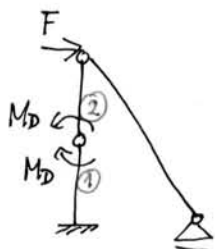
$$\delta w_c = \delta w_A = 0$$

$$\delta u_c = \delta u_A - h \delta \varphi_A = 0$$

$$\Rightarrow B_z \delta w_B = 0$$

$$\boxed{B_z = 0}$$

e.) M_D



$$\delta W = -M_D \delta \varphi_1 + M_D \delta \varphi_2 + F \delta u_c = 0$$

$$\delta \varphi_1 = 0$$

$$\delta u_c = \delta u_D - \frac{h}{2} \delta \varphi_2$$

$$\delta u_D = \delta u_A - \frac{h}{2} \delta \varphi_A = 0$$

$$\delta u_c = -\frac{h}{2} \delta \varphi_2$$

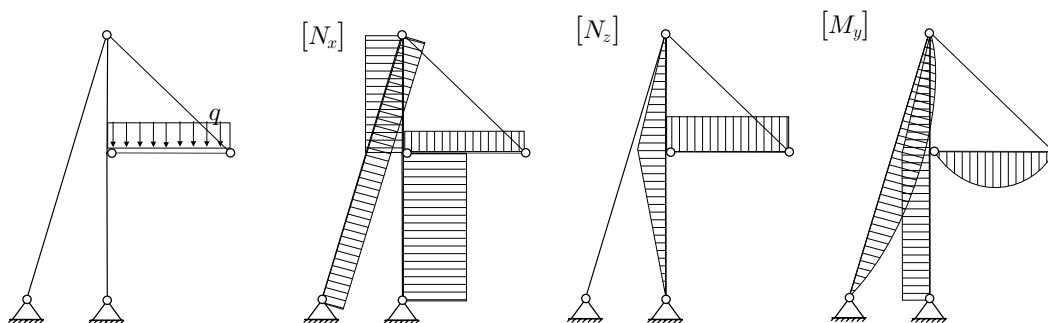
$$M_D \delta \varphi_2 - \frac{h}{2} F \delta \varphi_2 = 0$$

$$\Rightarrow \boxed{M_D = \frac{h}{2} F}$$

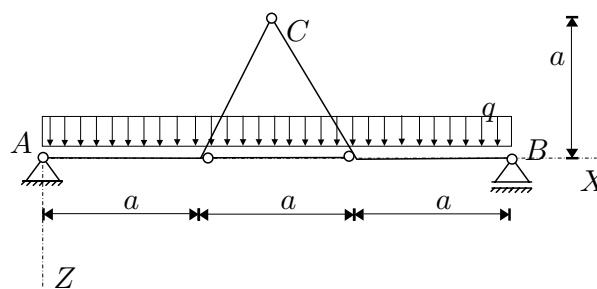
STATIKA (UNI) - 1. IZREDNI IZPITNI ROK (14. 3. 2006)

RAČUNSKI DEL IZPITA:

1. Janezek je na izpitu iz statike padel. Njegovi diagrami so polni napak. Pomagaj Janezku in poišči (BREZ RAČUNANJA) vse napake v spodnjih diagramih ! (25%)

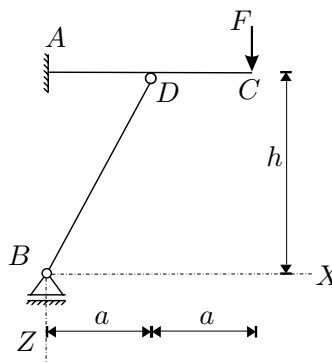


2. Za konstrukcijo na sliki izračunajte stopnjo statične nedoločenosti, reakcije in notranje statične količine (N_x, N_z, M_y)! Rezultate notranjih statičnih količin prikažite z diagrami!



Podatki: $a = 2 \text{ m}$, $q = 5 \text{ kN/m}$.
(OBVEZNA NALOGA! 50%)

3. S principom virtualnega dela za konstrukcijo na sliki izračunajte reakcije v podporah! (25%)

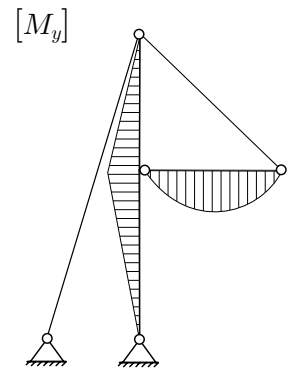
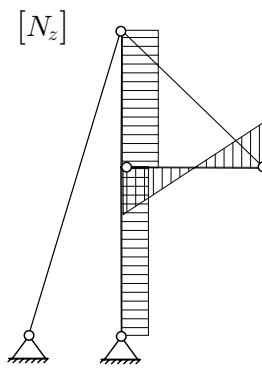
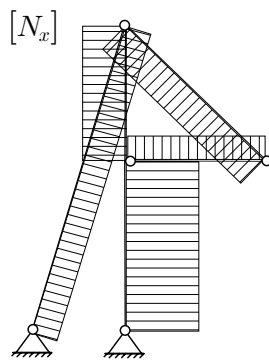
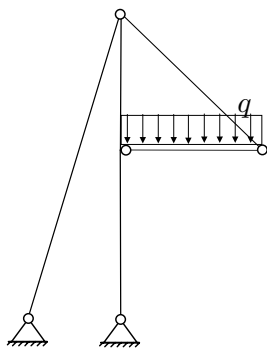


TEORETIČNI DEL IZPITA:

Izmed treh zastavljenih vprašanj si izberete dve, na kateri boste odgovarjali. Izbrani vprašanji jasno označite! Pišite čitljivo.

1. Izpeljite ravnotežne pogoje za sile, ki delujejo na sistemu delcev s togimi vezmi in togem telesu!
2. Pomiki in zasuki togega telesa (izpeljava enačb za ravninsko gibanje togega telesa)!
3. Opišite določanje reakcij in notranjih sil statično določenih linijskih konstrukcij z izrekom o virtualnih pomikih! Razumevanje ilustrirajte na obojestransko previsnem prostoležečem nosilcu s prečnima točkovnima silama na prostih robovih! Izračunajte vse reakcije ter notranje sile na sredini razpona!

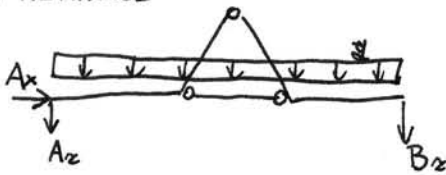
1. Naloga: PRAVILNI DIAGRAMI



2. NALOGA

a.) $\tilde{m}_{ps} = 3 \cdot 3 - 3 \cdot 2 - 2 - 1 = 0$

b.) REAKCIJE



$A_x = 0$

$A_z + B_z = -g \cdot 3a$

$-B_z \cdot 3a - g \cdot 3a \cdot \frac{3a}{2} = 0$

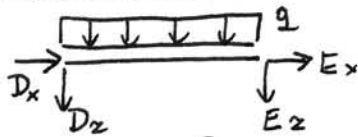
$A_z = -g \cdot \frac{3a}{2}$

$B_z = -g \cdot \frac{3a}{2}$

$A_z = -15 \text{ kN}$

$B_z = -15 \text{ kN}$

c.) RAZSTAVLJANJE



$D_x + E_x = 0$

$D_x = -E_x$

$D_z + E_z = -g \cdot a$

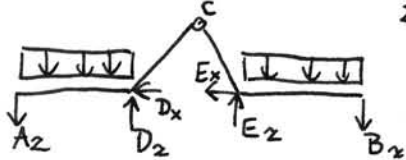
$D_z = -g \cdot \frac{a}{2}$

$\sum M^D: -E_z \cdot a - g \cdot a \cdot \frac{a}{2} = 0$

$E_z = -g \cdot \frac{a}{2}$

$D_z = -5 \text{ kN}$

$E_z = -5 \text{ kN}$



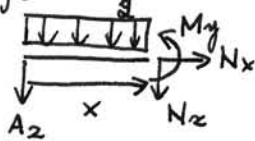
$\sum M^C: A_z \cdot \frac{3}{2}a - D_z \cdot \frac{a}{2} - D_x \cdot a + g \cdot a \cdot (\frac{a}{2} + \frac{a}{2}) = 0$

$D_x = A_z \cdot \frac{3}{2} - D_z \cdot \frac{1}{2} + g \cdot a$

$D_x = -10 \text{ kN}$

d. NOTRANJE SILE

polje AD



$N_x = 0$

$N_z = -A_z - g \cdot x$

$N_z = 15 - 5x$

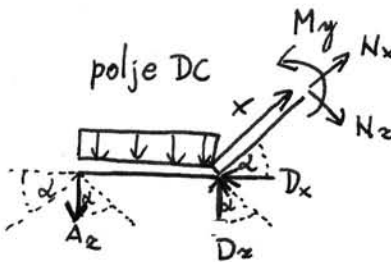
$N_z(2) = 5 \text{ kN}$

$M_y = -A_z \cdot x - g \cdot \frac{x^2}{2}$

$M_y = 15x - \frac{5}{2}x^2$

$M_y(2) = 20 \text{ kNm}$

polje DC



$x \in [0, l] \quad l = 2.2 \text{ m}$

$N_x = A_z \cdot \sin \alpha + D_x \cdot \cos \alpha - D_z \cdot \sin \alpha + g \cdot \sin \alpha \cdot a$

$N_z = -A_z \cdot \cos \alpha + D_x \cdot \sin \alpha + D_z \cdot \cos \alpha - g \cdot \cos \alpha \cdot a$

$M_y = -A_z \cdot (a + x \cdot \cos \alpha) - g \cdot a \cdot (\frac{a}{2} + x \cdot \cos \alpha) + D_z \cdot x \cdot \cos \alpha + D_x \cdot x \cdot \sin \alpha$

$\text{tg} \alpha = \frac{a}{a/2}$

$\alpha = 63.4^\circ$

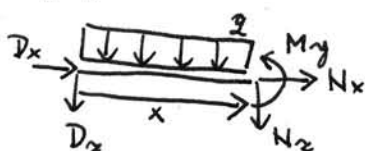
$N_x = -4.47 \text{ kN}$

$N_z = -8.94 \text{ kN}$

$M_y = 20 - 8.94x$

$M_y(l) = 0 \checkmark$

polje DE



$N_x = D_x$

$N_z = -D_z - g \cdot x$

$M_y = -D_z \cdot x - g \cdot \frac{x^2}{2}$

$N_x = 10 \text{ kN}$

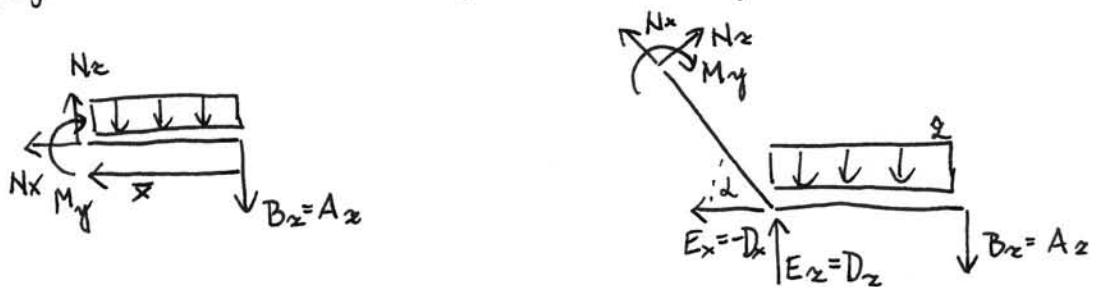
$N_z = 5 - 5x$

$M_y = 5x - \frac{5}{2}x^2$

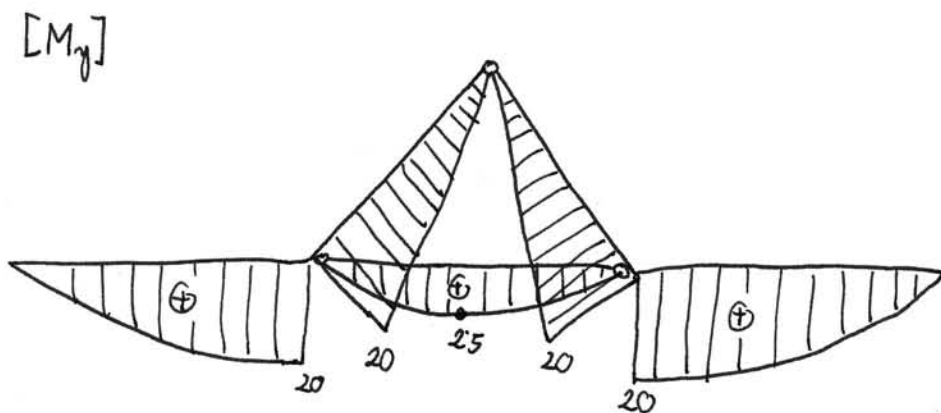
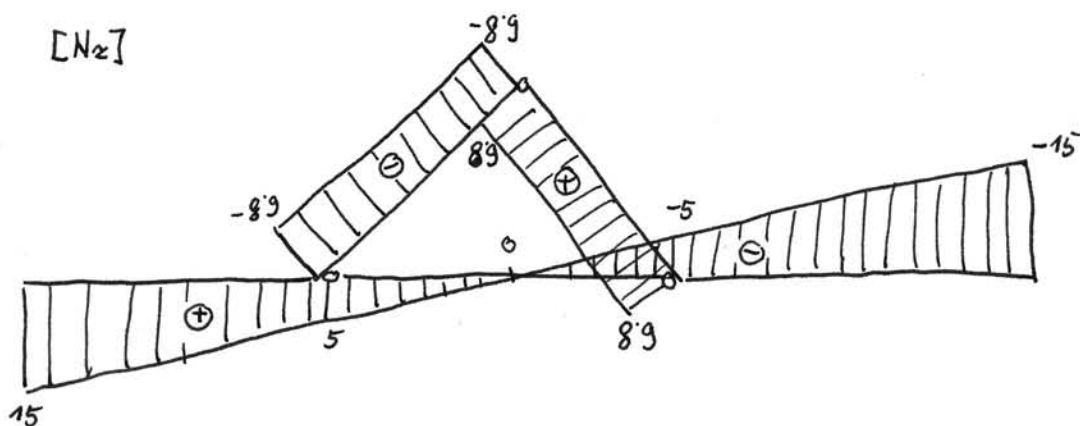
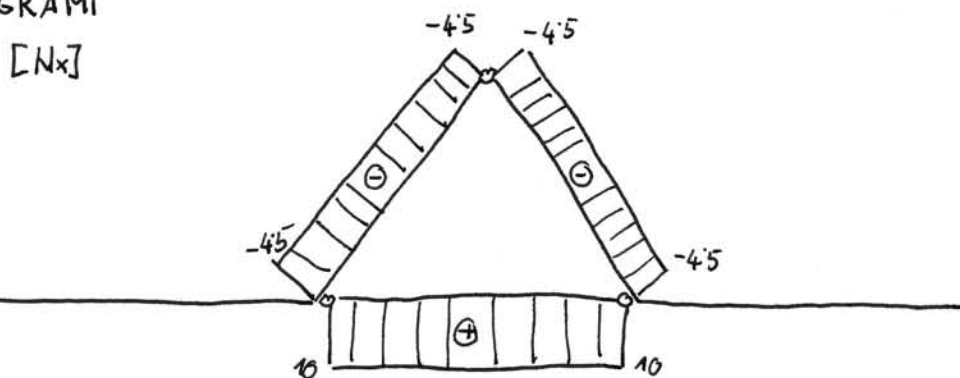
$N_z(2) = -5 \text{ kN}$

$M_y(1) = 2.5 \text{ kNm}$
(ekstrem)

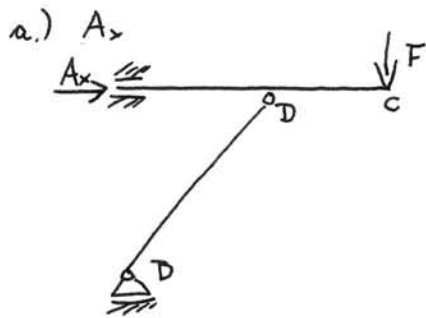
pri poljih BE in EC lahko upoštevamo simetrijo :



e.) DIAGRAMI



3. NALOGA



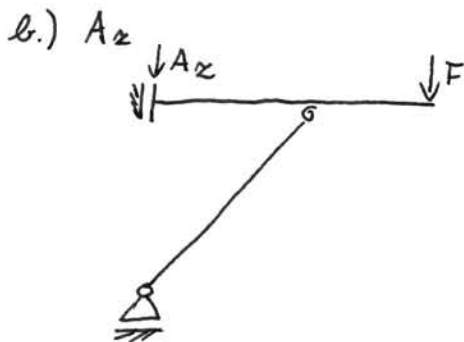
$$A_x \cdot \delta u_A + F \cdot \delta w_C = 0$$

$$\delta w_C = \delta w_A - 2a \cdot \delta \varphi_A = 0$$

$$\delta w_A = 0 \quad \delta \varphi_A = 0$$

$$A_x \cdot \delta u_A = 0$$

$$\Rightarrow \boxed{A_x = 0}$$



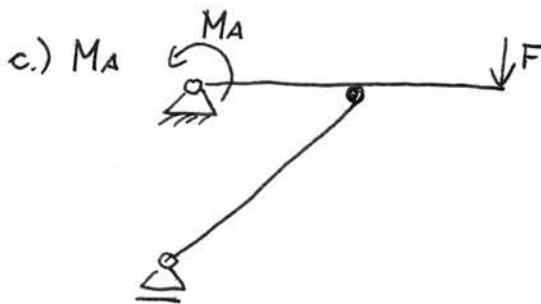
$$A_z \cdot \delta w_A + F \cdot \delta w_C = 0$$

$$\delta w_C = \delta w_A - 2a \cdot \delta \varphi_A = \delta w_A$$

$$\delta \varphi_A = 0$$

$$A_z \delta w_A + F \cdot \delta w_A = 0$$

$$\boxed{A_z = -F}$$



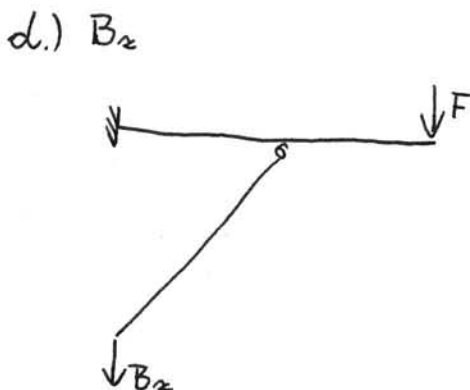
$$M_A \cdot \delta \varphi_A + F \cdot \delta w_C = 0$$

$$\delta w_C = \delta w_A - 2a \cdot \delta \varphi_A$$

$$\delta w_A = 0$$

$$M_A \cdot \delta \varphi_A - F \cdot 2a \delta \varphi_A = 0$$

$$\boxed{M_A = F \cdot 2a}$$



$$B_z \cdot \delta w_B + F \delta w_C = 0$$

$$\delta w_C = \delta w_A - 2a \cdot \delta \varphi_A = 0$$

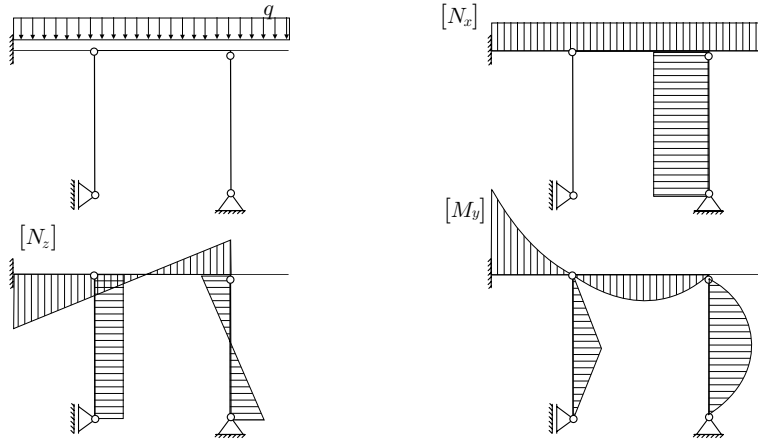
$$\delta w_A = 0 \quad \delta \varphi_A = 0$$

$$\Rightarrow B_z \cdot \delta w_B = 0$$

$$\boxed{B_z = 0}$$

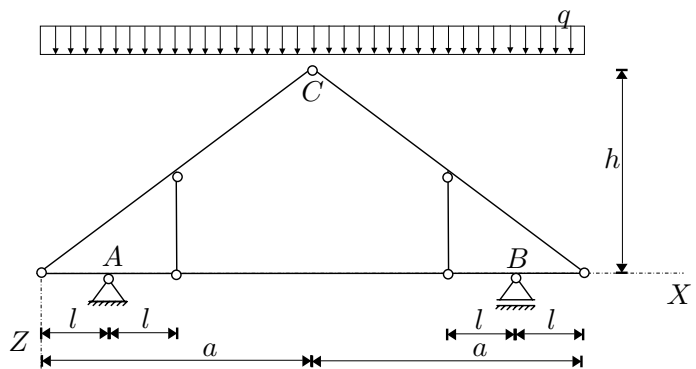
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1. Janezek je na izpitu iz statike padel. Njegovi diagrami so polni napak. Pomagaj Janezku in poišči (BREZ RAČUNANJA) vse napake v spodnjih diagramih ! (OBVEZNA NALOGA! 25%)

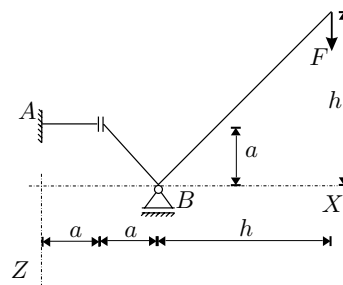


2. Za konstrukcijo na sliki izračunajte stopnjo statične nedoločenosti, reakcije in notranje statične količine (N_x, N_z, M_y)! Rezultate notranjih statičnih količin prikažite z diagrami!

Podatki: $a = 4 \text{ m}, h = 3 \text{ m},$
 $l = 1 \text{ m}, q = 5 \text{ kN/m}.$
 (OBVEZNA NALOGA! 50%)



3. S principom virtualnega dela za konstrukcijo na sliki izračunajte reakcije v podporah in notranji moment v točki B – M_B !! (25%)

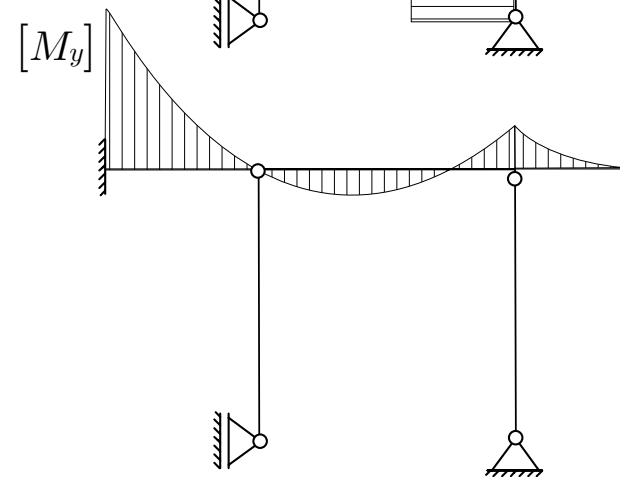
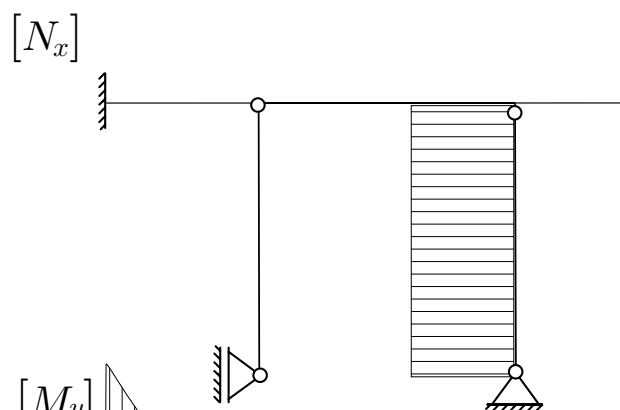
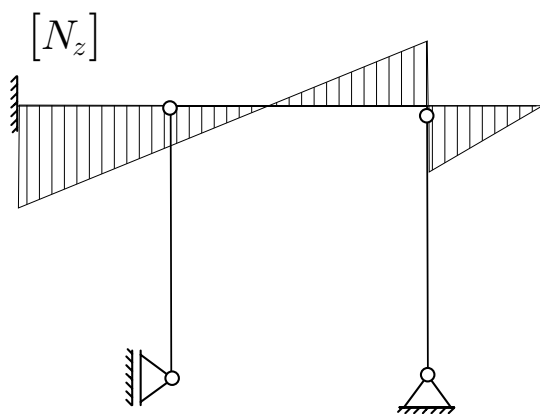
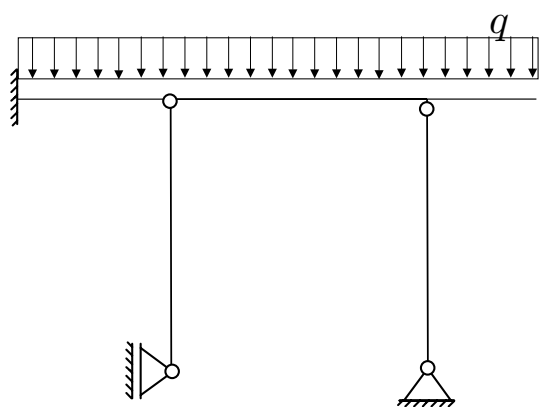


TEORETIČNI DEL IZPITA:

Izmed treh zastavljenih vprašanj si izberete dve, na kateri boste odgovarjali. Izbrani vprašanji jasno označite! Pišite čitljivo.

1. Računski modeli za opis medsebojnega vpliva med telesi!
2. Izpeljite in opišite nadomestne ravnotežne pogoje (razumevanje podkrepitve s preprostim primerom)!
3. Opišite določanje reakcij in notranjih sil statično določenih linijskih konstrukcij z izrekom o virtualnih pomikih (Razumevanje ilustrirajte na enostransko previsnem nosilcu s prečno in horizontalno točkovno silo na previsnem robu. Izračunajte vse notranje sile v izbranih značilnih točkah nosilca in vse reakcije)!

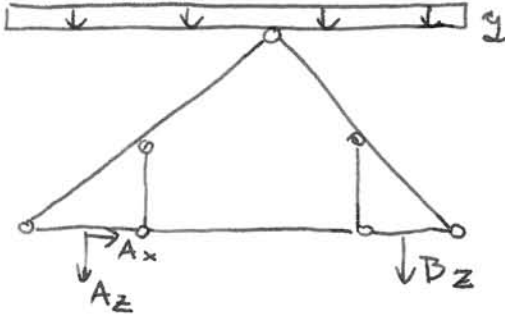
1. Naloga: PRAVILNI DIAGRAMI



2. NALOGA

a.) $\tilde{m}_{ps} = 7 \cdot 3 - 3 - 5 \cdot 2 - 2 \cdot 4 = 0$

b.) REAKCIJE



$$\Sigma X: A_x = 0$$

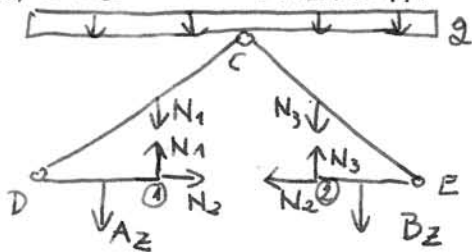
$$\Sigma Z: A_z + B_z = -2ga$$

$$\Sigma M^C: A_z \cdot a - B_z \cdot a = 0$$

$$A_z = B_z = -ga$$

$$A_z = -20 \text{ kN} \quad B_z = -20 \text{ kN}$$

c.) SILE V PALICAH



$$\Sigma M^D \text{ (D)}: -N_1 \cdot 2 + A_z \cdot 1 = 0$$

$$N_1 = + \frac{A_z}{2} \quad N_1 = -10 \text{ kN}$$

$$\Sigma M^E \text{ (E)}: -N_3 \cdot 2 + B_z \cdot 1 = 0$$

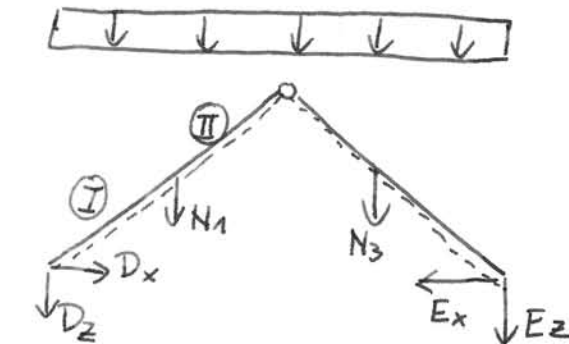
$$N_3 = \frac{B_z}{2} \quad N_3 = -10 \text{ kN}$$

$$\Sigma M^C \text{ (DC)}: A_z \cdot 3 + g \cdot 4 \cdot 2 + N_2 \cdot 3 = 0$$

$$N_2 = - \frac{-20 \cdot 3 + 5 \cdot 4 \cdot 2}{3} = \frac{20}{3}$$

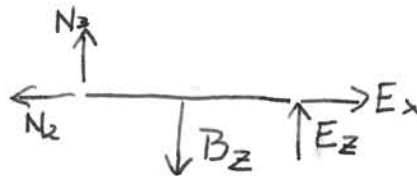
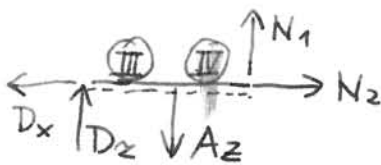
$$N_2 = +6.67 \text{ kN}$$

d.) RAZREZ



$$D_x = N_2 \quad D_x = \frac{20}{3} \text{ kN}$$

$$E_x = N_2 \quad E_x = \frac{20}{3} \text{ kN}$$



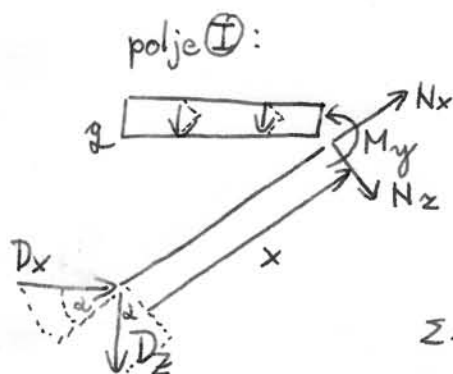
$$D_z = A_z - N_1$$

$$E_z = B_z - N_3$$

$$D_z = -10 \text{ kN}$$

$$E_z = -10 \text{ kN}$$

e.) NOTRANJE SILE PO POLJIH
(le polovica, ostalo je simetrično!)



$$\cos \alpha = \frac{4}{5} \quad \sin \alpha = \frac{3}{5}$$

$$x \in [0, 2.5]$$

$$\Sigma x: N_x + D_x \cos \alpha - D_z \sin \alpha - q \sin \alpha \cdot x \cos \alpha = 0$$

$$\Sigma z: N_z + D_x \sin \alpha + D_z \cos \alpha + q \cos \alpha \cdot x \cos \alpha = 0$$

$$\Sigma M^T: M_y + D_x \cdot x \sin \alpha + D_z \cdot x \cos \alpha + q \cdot x \cos \alpha \cdot \frac{x \cos \alpha}{2} = 0$$

$$N_x = -\frac{20}{3} \cdot \frac{4}{5} - 10 \cdot \frac{3}{5} + 5 \cdot \frac{3}{5} \cdot \frac{4}{5} x$$

$$N_z = -\frac{20}{3} \cdot \frac{3}{5} + 10 \cdot \frac{4}{5} - 5 \cdot \frac{4}{5} \cdot \frac{4}{5} x$$

$$M_y = -\frac{20}{3} \cdot \frac{3}{5} x + 10 \cdot \frac{4}{5} x - 5 \cdot \frac{4}{5} \cdot \frac{4}{5} \cdot \frac{x^2}{2}$$

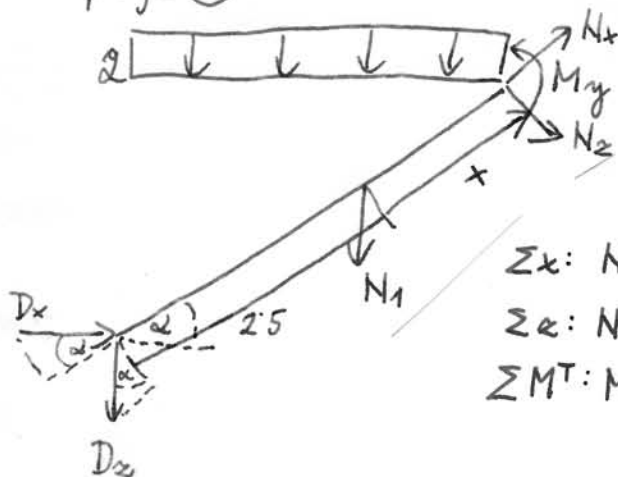
$N_x = -11.33 + 2.4x$
$N_z = 4 - 3.2x$
$M_y = 4x - 1.6x^2$

$$N_x(2.5) = -5.3 \text{ kN}$$

$$N_z(2.5) = -4 \text{ kN}$$

$$M_y(1.25) = 2.5 \text{ kNm (extrem)}$$

polje II:



$$x \in [0, 2.5]$$

$$\Sigma x: N_x + D_x \cdot \cos \alpha - D_z \sin \alpha - N_1 \sin \alpha - q \sin \alpha (2.5+x) \cos \alpha = 0$$

$$\Sigma z: N_z + D_x \sin \alpha + D_z \cos \alpha + N_1 \cos \alpha + q \cos \alpha (2.5+x) \cos \alpha = 0$$

$$\Sigma M^T: M_y + D_x (x+2.5) \cdot \sin \alpha + D_z \cdot (x+2.5) \cdot \cos \alpha + N_1 \cdot x \cos \alpha + q \cdot (x+2.5) \cos \alpha \cdot (x+2.5) \frac{\cos \alpha}{2} = 0$$

$$N_x = -\frac{20}{3} \cdot \frac{4}{5} - 10 \cdot \frac{3}{5} - 10 \cdot \frac{3}{5} + 5 \cdot \frac{3}{5} \cdot \frac{4}{5} (2.5+x)$$

$$N_z = -\frac{20}{3} \cdot \frac{3}{5} + 10 \cdot \frac{4}{5} + 10 \cdot \frac{4}{5} - 5 \cdot \frac{4}{5} \cdot \frac{4}{5} (2.5+x)$$

$$M_y = -\frac{20}{3} (x+2.5) \cdot \frac{3}{5} + 10 \cdot (x+2.5) \frac{4}{5} + 10x \cdot \frac{4}{5} - 5 \cdot \frac{4}{5} \cdot \frac{4}{5} \cdot \frac{(x+2.5)^2}{2}$$

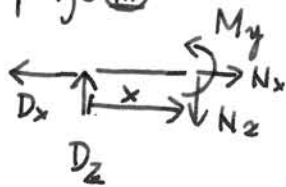
$N_x = -11.3 + 2.4x$
$N_z = 4 - 3.2x$
$M_y = 4x - 1.6x^2$

$$N_x(2.5) = -5.3 \text{ kN}$$

$$N_z(2.5) = -4 \text{ kN}$$

$$M_y(1.25) = 2.5 \text{ kNm (extrem)}$$

polje III



$$x \in [0, 1]$$

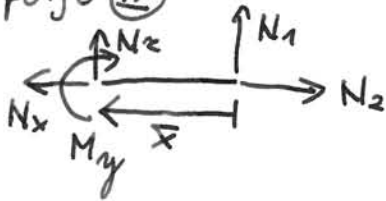
$$\sum x: N_x = D_x = \frac{20}{3} \text{ kN}$$

$$\sum z: N_z = D_z = -10 \text{ kN}$$

$$\sum M^T: M_y = D_z x = -10x$$

$$M_y(1) = -10 \text{ kNm}$$

polje IV



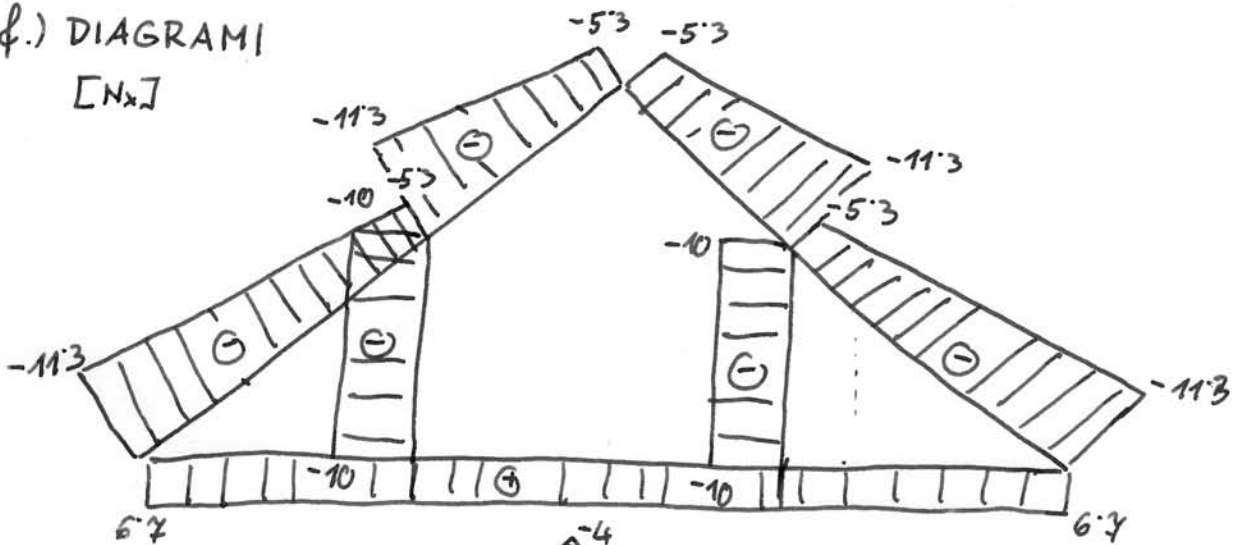
$$\sum x: N_x = N_2 = \frac{20}{3} \text{ kN}$$

$$\sum z: N_z = -N_1 = 10 \text{ kN}$$

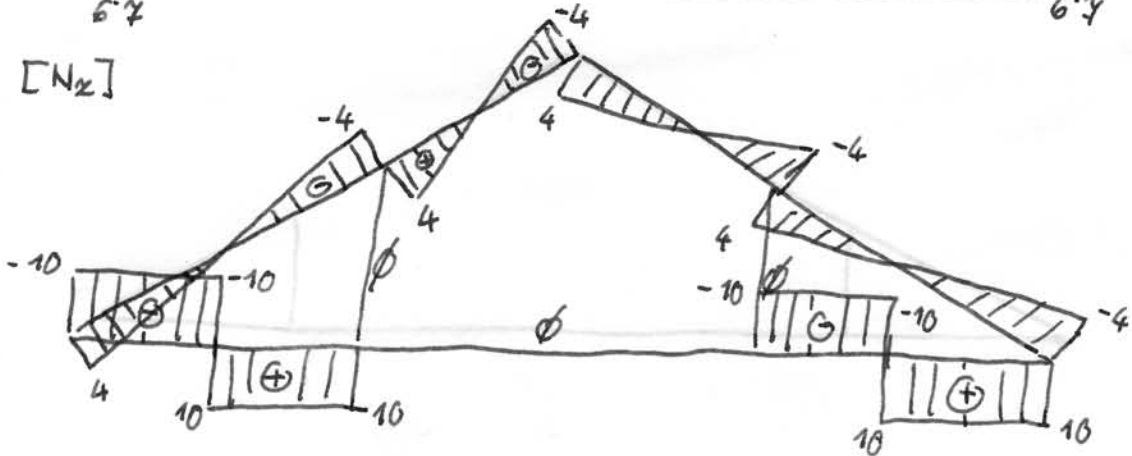
$$\sum M^T: M_y = N_1 x = -10x$$

$$M_y(1) = -10 \text{ kNm}$$

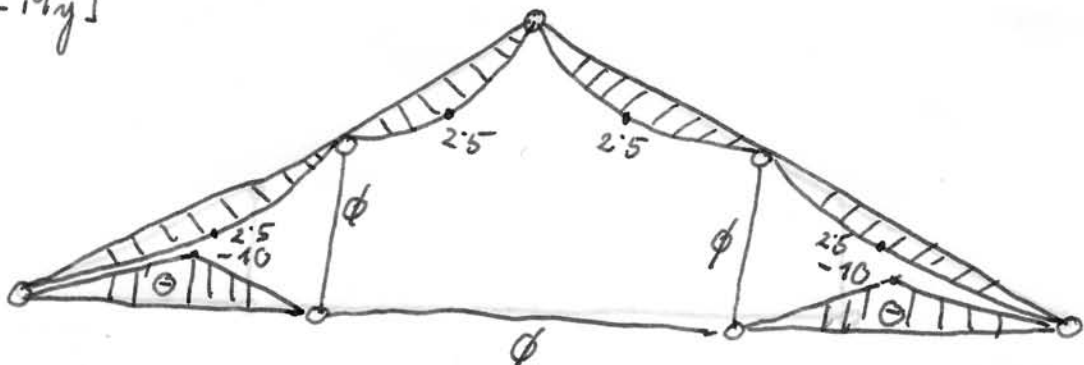
f.) DIAGRAMI
[Nx]



[Nz]

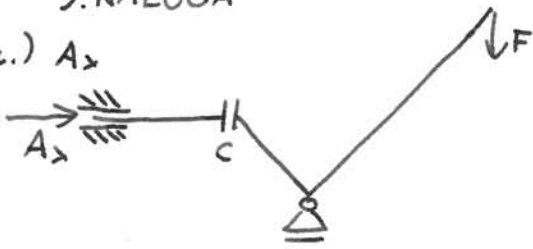


[My]



3. NALOGA

a.) A_x



$$\delta W = A_x \delta u_A + F \delta w_F = 0$$

$$\delta w_C = \delta w_A - a \delta \varphi_A = 0$$

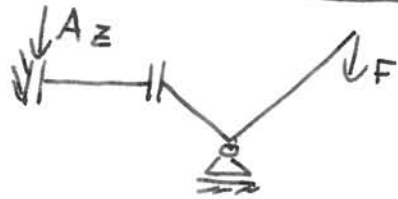
$$\delta \varphi_C = \delta \varphi_A = 0$$

$$\delta w_F = \delta w_C - (a+h) \delta \varphi_C = 0$$

$$A_x \delta u_A = 0$$

$$\boxed{A_x = 0}$$

b.)



$$\delta W = A_z \delta w_A + F \delta w_F = 0$$

$$\delta \varphi_C = \delta \varphi_A = 0$$

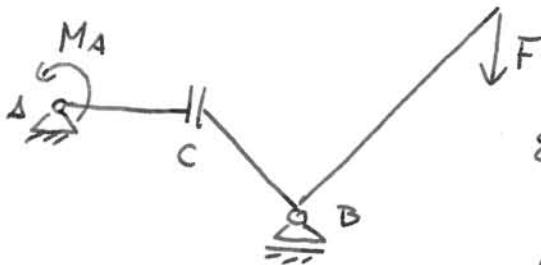
$$\delta \varphi_F = \delta \varphi_C = 0$$

$$\delta w_B = \delta w_F + h \delta \varphi_F$$

$$\Rightarrow \delta w_F = 0$$

$$\Rightarrow \boxed{A_z = 0}$$

c.)



$$\delta W = M_A \delta \varphi_A + F \delta w_F = 0$$

$$\delta \varphi_C = \delta \varphi_A$$

$$\delta \varphi_F = \delta \varphi_C = \delta \varphi_A$$

$$\delta w_B = \delta w_F + h \delta \varphi_F$$

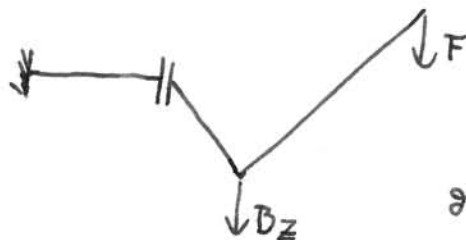
$$\delta w_F = -h \delta \varphi_A$$

$$M_A \delta \varphi_A - Fh \delta \varphi_A = 0$$

$$(M_A - Fh) \delta \varphi_A = 0$$

$$\boxed{M_A = Fh}$$

d.)



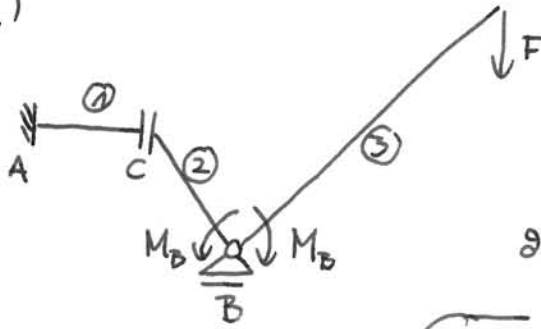
$$\delta W = B_z \delta w_B + F \delta w_F = 0$$

$$\delta \varphi_A = 0 \Rightarrow \delta \varphi_F = 0$$

$$\delta w_B = \delta w_F + h \delta \varphi_F$$

$$\Rightarrow \boxed{B_z = -F}$$

e.)



$$\delta W = M_B \delta \varphi_B^{(2)} - M_B \delta \varphi_B^{(3)} + F \delta w_F = 0$$

$$\delta \varphi_B^{(3)} = \delta \varphi_C = \delta \varphi_A = 0$$

$$\delta w_B = \delta w_F + h \delta \varphi_F$$

$$0 = \delta w_F + h \delta \varphi_F \Rightarrow \delta w_F = -h \delta \varphi_B^{(3)}$$

$$-M_B \delta \varphi_B^{(3)} - F h \delta \varphi_B^{(3)} = 0$$

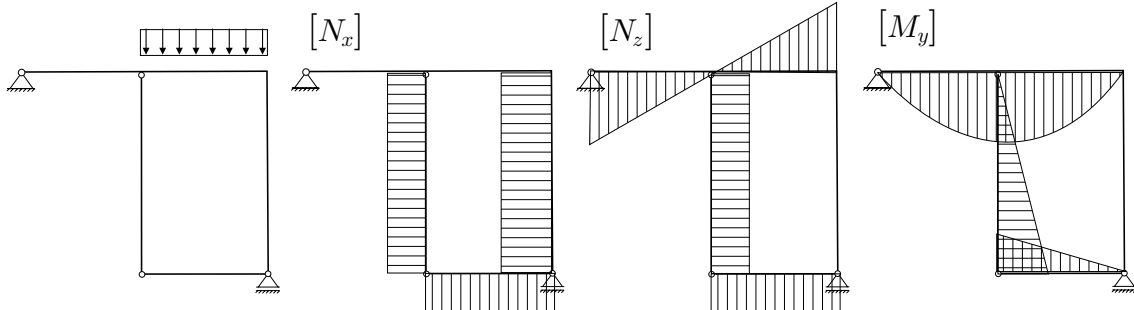
$$(M_B + Fh) \delta \varphi_B^{(3)} = 0$$

$$M_B + Fh = 0$$

$$\boxed{M_B = -Fh}$$

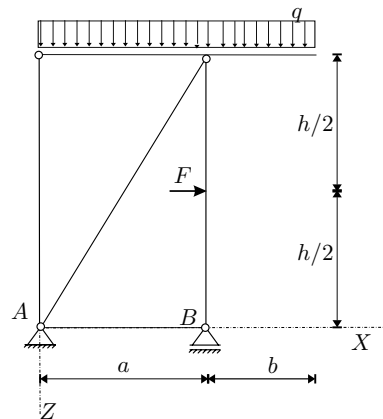
RAČUNSKI DEL IZPITA:

1. Janezek je na izpitu iz statike padel. Njegovi diagrami so polni napak. Pomagaj Janezku in poišči (BREZ RAČUNANJA) vse napake v spodnjih diagramih ! (OBVEZNA NALOGA! 25%)



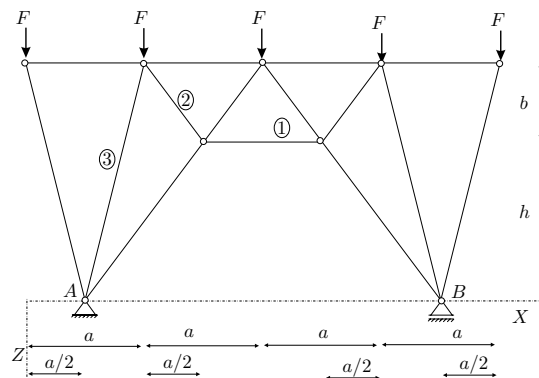
2. Za konstrukcijo na sliki izračunajte stopnjo statične nedoločnosti, reakcije in notranje statične količine (N_x, N_z, M_y)! Rezultate notranjih statičnih količin prikažite z diagrami!

Podatki: $a = 3\text{ m}$, $h = 5\text{ m}$,
 $b = 2\text{ m}$, $F = 10\text{ kN}$, $q = 5\text{ kN/m}$.
 (OBVEZNA NALOGA! 50%)



3. Za palično konstrukcijo na sliki izračunajte stopnjo statične nedoločnosti in osne sile v palicah 1, 2 in 3! (25%)

Podatki: $a = 3\text{ m}$, $b = 2\text{ m}$,
 $h = 4\text{ m}$, $F = 5\text{ kN}$.

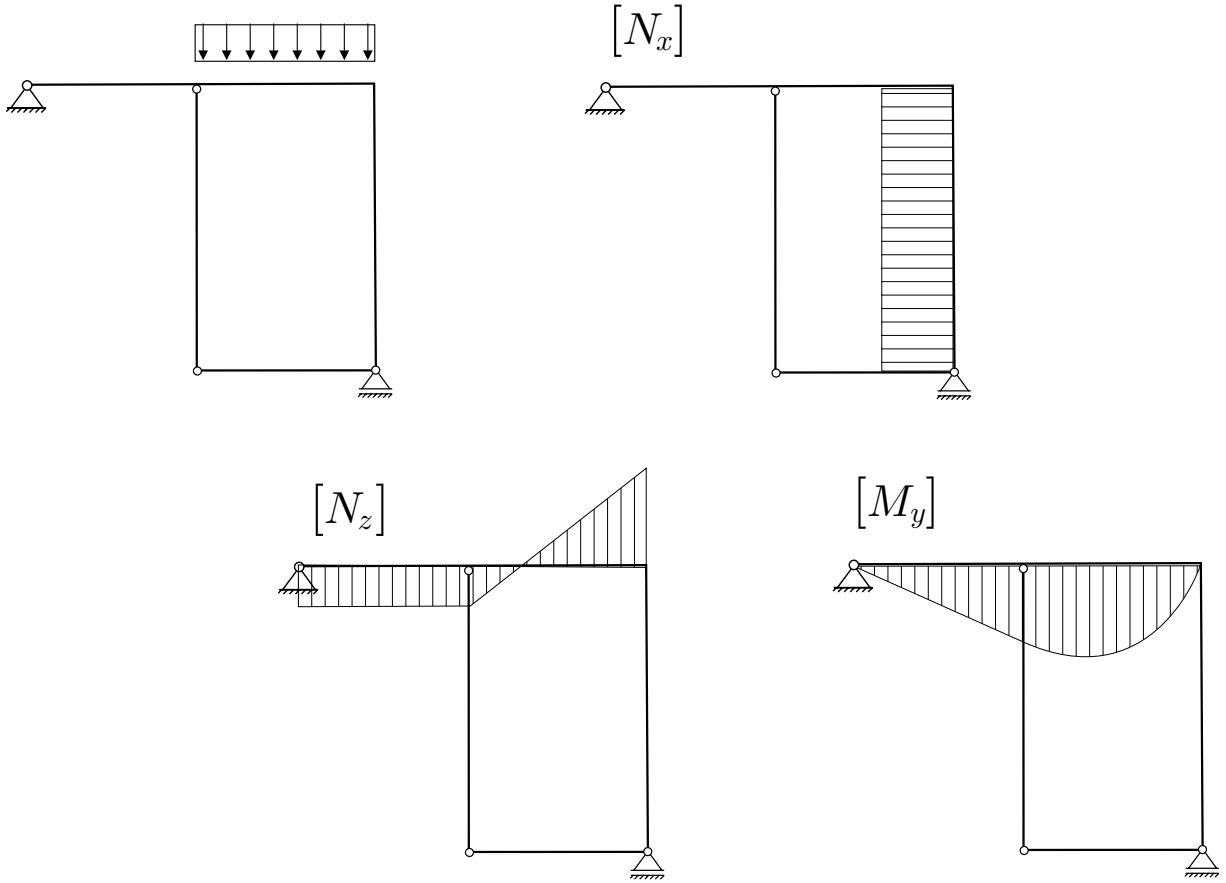


TEORETIČNI DEL IZPITA:

Izmed treh zastavljenih vprašanj si izberete dve, na kateri boste odgovarjali. Izbrani vprašanja jasno označite! Pišite čitljivo.

1. Izpeljite ravnotežne pogoje za sile, ki delujejo na sistemu delcev s togimi vezmi in togem telesu!
2. Opišite kinematične enačbe sistema togih teles ter postopek računanja dejanskega števila prostostnih stopenj sistema togih teles (ilustracija z značilnim primerom)!
3. Izpeljite in opišite izraz za število odvzetih prostostnih stopenj, ki jih vez odvzame k nepovezanim telesom! Obravnavajte tudi primer, ko imajo vsa telesa na mestu vezi enake nekatere kinematične količine, preostale kinematične količine pa so možne za vsa telesa! (ilustracija z značilnimi primeri).

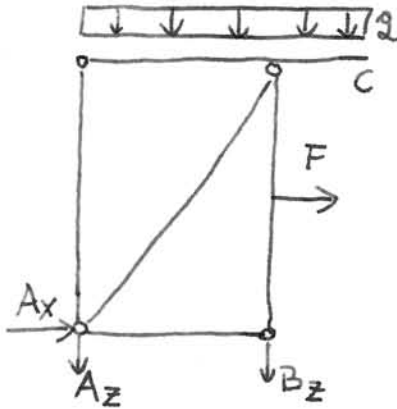
1. Naloga: PRAVILNI DIAGRAMI



2. NALOGA

a.) $\tilde{m}_{P_0} = 5 \cdot 3 - 3 - 2 \cdot 2 - 2 \cdot 4 = 0$

b.) REAKCIJE



$\Sigma X: A_x = -F$ $A_x = -10 \text{ kN}$

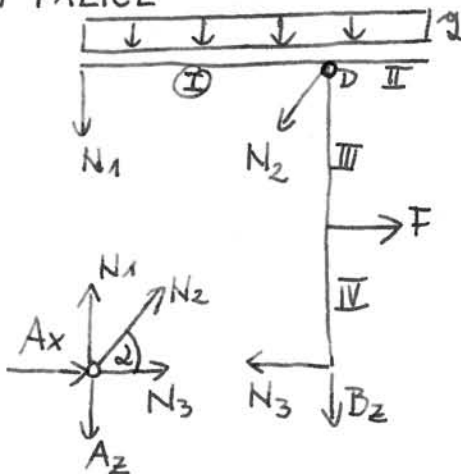
$\Sigma Z: A_z + B_z + g(a+b) = 0$

$\Sigma M^A: -B_z \cdot a - F \cdot \frac{a}{2} - g \cdot \frac{(a+b)^2}{2} = 0$

$B_z = -29.16 \text{ kN}$
 $A_z = 4.16 \text{ kN}$

KONTROLA: $\Sigma M^C: A_z \cdot (a+b) + B_z \cdot b + A_x \cdot h + F \cdot \frac{h}{2} + g \cdot \frac{(a+b)^2}{2} = 0$

c.) PALICE



$\tan \alpha = \frac{5}{3} \Rightarrow \alpha = 59.036^\circ$

nod. A:

$N_2 \cdot \cos \alpha + N_3 = -A_x$

$N_2 = 9.718 \text{ kN}$

$N_1 + N_2 \cdot \sin \alpha = A_z$

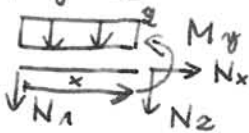
$N_1 = -4.166 \text{ kN}$

$\Sigma M^D: F \cdot \frac{h}{2} - N_3 \cdot h = 0$

$N_3 = \frac{F}{2}$ $N_3 = 5 \text{ kN}$

d.) NOTRANJE SILE

polje I:



$x \in [0, 3]$

$N_x = 0$

$N_z = -N_1 - g \cdot x$

$N_z = 4.17 - 5x$

$N_z(3) = -10.83 \text{ kN}$

$M_y = -N_1 \cdot x - g \cdot \frac{x^2}{2}$

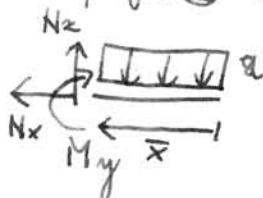
$M_y = +4.17x - \frac{5}{2}x^2$

$M_y(3) = -10 \text{ kNm}$

$N_z = 0 \Rightarrow x_E = \frac{4.17}{5} \approx 0.83 \text{ m}$

$M_y(x_E) = 1.74 \text{ kNm (obstojna)}$

polje II (z desne):



$\bar{x} \in [0, 2]$

$N_x = 0$

$N_z = g \cdot \bar{x}$

$M_y = -g \cdot \frac{\bar{x}^2}{2}$

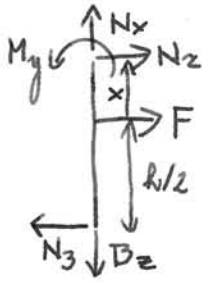
$N_z = 5\bar{x}$

$N_z(2) = 10 \text{ kN}$

$M_y = -2.5 \cdot \bar{x}^2$

$M_y(2) = -10 \text{ kNm}$

polje III



$$x \in [0, 2.5]$$

$$N_x = B_z$$

$$N_z = N_3 - F$$

$$M_y = N_3 \left(\frac{l}{2} + x \right) - Fx$$

$$N_x = -29.17 \text{ kN}$$

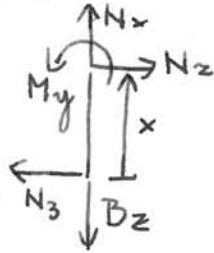
$$N_z = -5 \text{ kN}$$

$$M_y = \frac{25}{2} - 5x$$

$$M_y(0) = 12.5 \text{ kNm}$$

$$M_y(2.5) = 0$$

polje IV



$$x \in [0, 2.5]$$

$$N_x = B_z$$

$$N_z = N_3$$

$$M_y = N_3 \cdot x$$

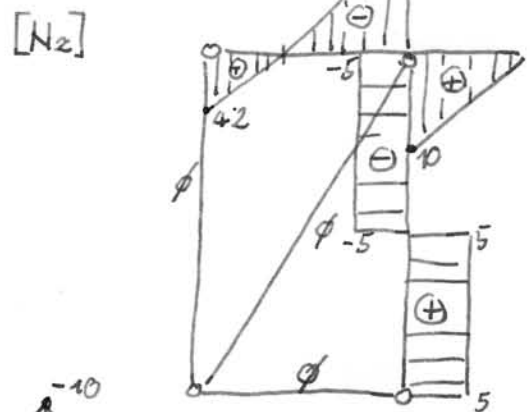
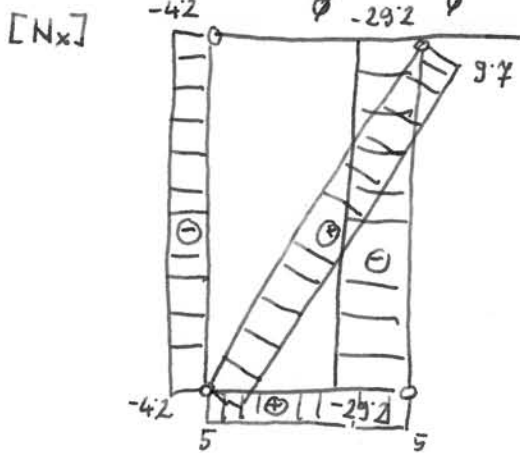
$$N_x = -29.17 \text{ kN}$$

$$N_z = 5 \text{ kN}$$

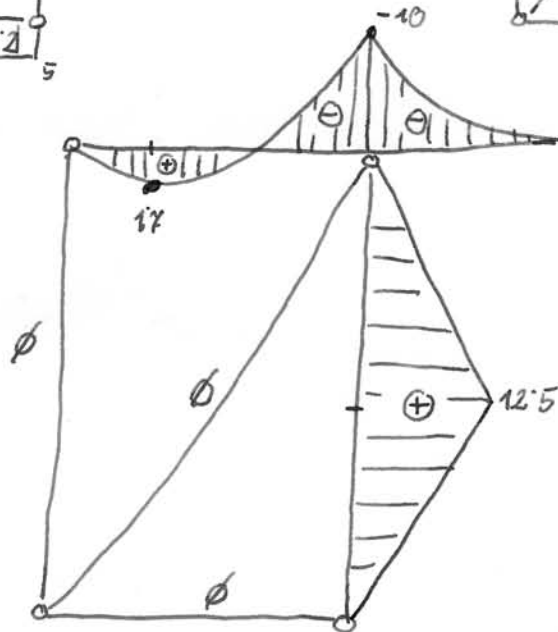
$$M_y = 5x$$

$$M_y(2.5) = 12.5 \text{ kNm}$$

e.) DIAGRAMI



[My]



3. NALOGA

a.) $\tilde{m}_{PS} = 7 \cdot 2 + 1 - 15 = 0$

b.) REAKCIJE

$\Sigma X: A_x = 0$

$\Sigma Z: A_z + B_z + 5F = 0$

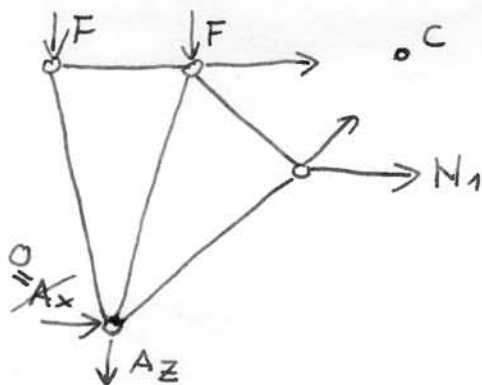
$\Sigma M^A: F \cdot \frac{a}{2} - F \cdot \frac{a}{2} - 3F \cdot \frac{a}{2} - 5F \cdot \frac{a}{2} - 4F \cdot \frac{a}{2} - B_z \cdot 3a = 0$

$B_z = -\frac{15F}{6}$

$B_z = -\frac{5}{2}F$

$A_z = -\frac{5}{2}F$

c.) REZ PREKO ①

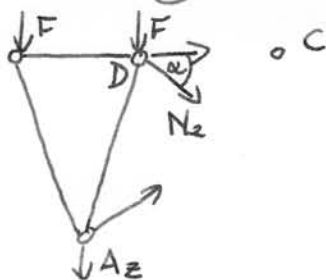


$\Sigma M^C: N_1 \cdot b + F \cdot a + F \cdot 2a + A_z \cdot \frac{3}{2}a = 0$

$N_1 = +\frac{3Fa}{4b}$

$N_1 = 5.625 \text{ kN}$

d.) REZ PREKO ②



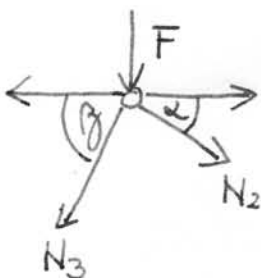
$\Sigma M^C: N_2 \cdot \sin \alpha \cdot a + F \cdot a + F \cdot 2a + A_z \cdot \frac{3}{2}a = 0$

$N_2 \cdot \sin \alpha = +\frac{3F}{4}$

$N_2 = 4.69 \text{ kN}$

$\text{tg} \alpha = \frac{2b}{a} = \frac{4}{3}$

e.) IZREZ VOZLIŠČA D



$\Sigma Z: N_2 \cdot \sin \alpha + N_3 \cdot \sin \beta + F = 0$

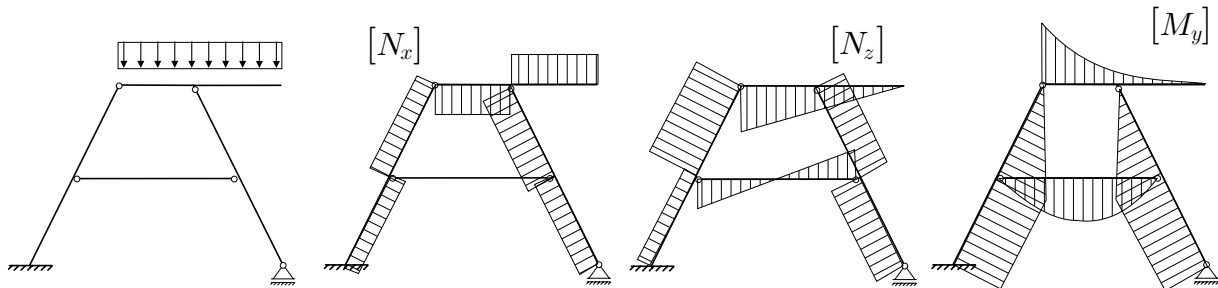
$\text{tg} \beta = \frac{6}{15} = \frac{12}{3}$

$N_3 = \frac{-F - N_2 \sin \alpha}{\sin \beta}$

$N_3 = -9.02 \text{ kN}$

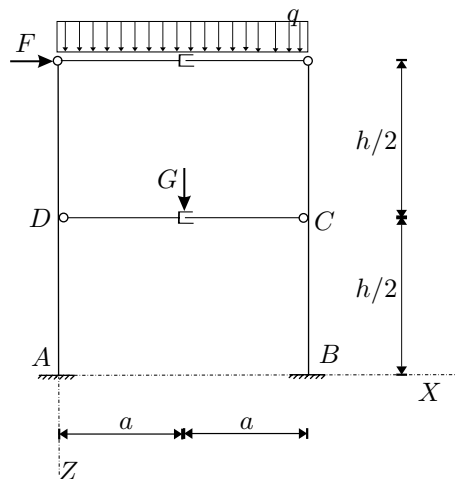
RAČUNSKI DEL IZPITA:

1. Janezek je na izpitu iz statike padel. Njegovi diagrami so polni napak. Pomagaj Janezku in poišči (BREZ RAČUNANJA) vse napake v spodnjih diagramih ! (OBVEZNA NALOGA! 25%)

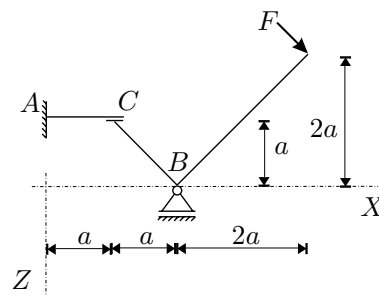


2. Za konstrukcijo na sliki izračunajte stopnjo statične nedoločenosti, reakcije in notranje statične količine (N_x , N_z , M_y)! Rezultate notranjih statičnih količin prikažite z diagrami!

Podatki: $a = 2.5 \text{ m}$, $h = 6 \text{ m}$,
 $F = 2 \text{ kN}$, $G = 5 \text{ kN}$, $q = 10 \text{ kN/m}$.
 (OBVEZNA NALOGA! 50%)



3. Za konstrukcijo na sliki izračunajte računsko in dejansko število prostostnih stopenj! (25%)

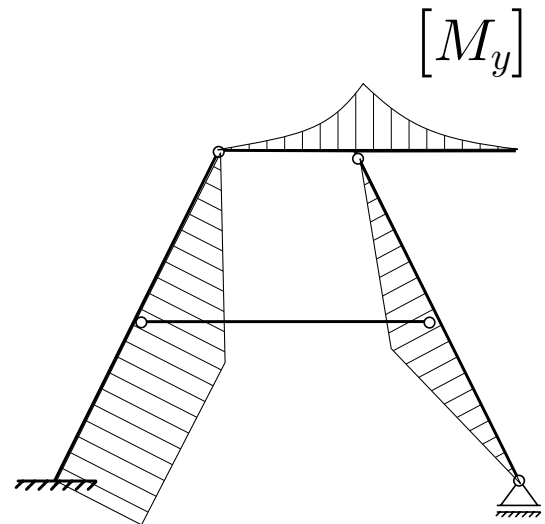
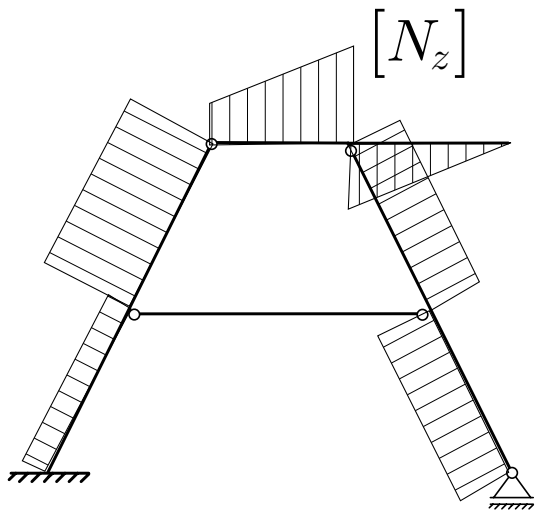
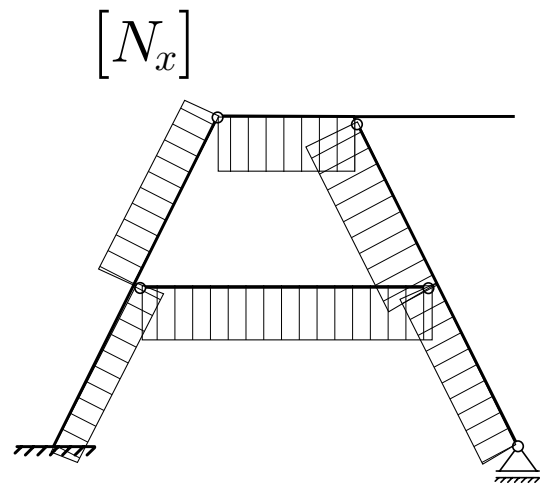
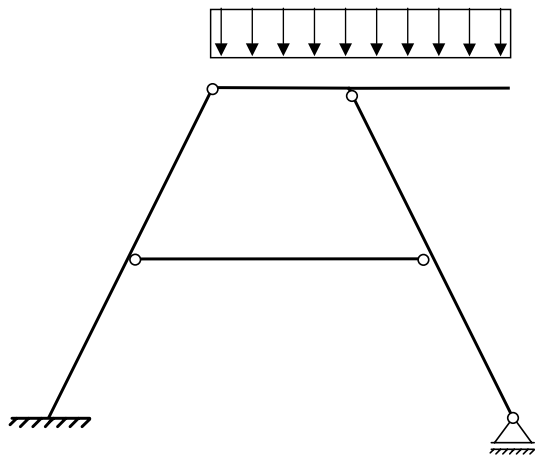


TEORETIČNI DEL IZPITA:

Izmed treh zastavljenih vprašanj si izberete dve, na kateri boste odgovarjali. Izbrani vprašanji jasno označite! Pišite čitljivo.

1. Vzporedna prestavitev sile! Kako lahko nadomestimo silo in moment, ki sta med seboj pravokotna? Dokaz!
2. Zapišite izraz za računsko število prostostnih stopenj sistema togih teles (razumevanje podkrepitve s preprostim primerom)! Kako se računsko število prostostnih stopenj razlikuje od dejanskega? Ali je lahko dejansko in računsko število prostostnih stopenj različno! Če je, to ilustrirajte s preprostim primerom.
3. Pomiki in zasuki togega telesa (izpeljavo ilustrirajte z ravninskim gibanjem togega telesa)!

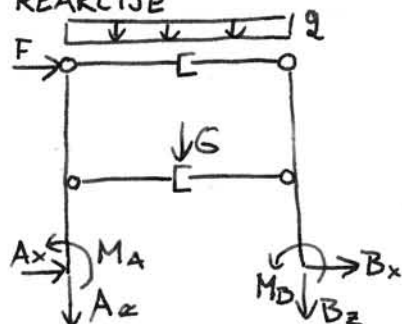
1. Naloga: PRAVILNI DIAGRAMI



2. NALOGA

a.) $\tilde{m}_{ps} = 6 \cdot 3 - 4 \cdot 2 - 2 \cdot 2 - 2 \cdot 3 = 0$

b.) REAKCIJE

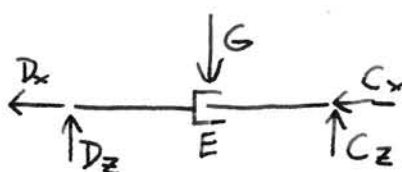
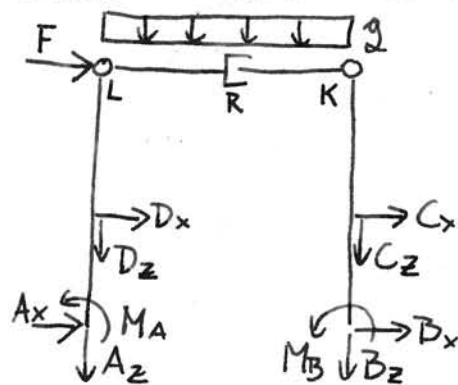


$\Sigma X: A_x + B_x = -F$

$\Sigma Z: A_z + B_z = -G - q \cdot 2a$

$\Sigma M^A: M_A + M_B - B_z \cdot 2a - G \cdot a - F \cdot h - q \cdot 2a \cdot a = 0$

c.) IZREZ V VEZEH C IN D



$\Sigma X: C_x + D_x = 0$

$\Sigma Z: C_z + D_z = G$

$\Sigma M^P: -G \cdot a + C_z \cdot 2a = 0$

dodatna

$\Sigma X_{CE}: C_x = 0$

$D_x = 0$

$C_z = \frac{G}{2} \quad D_z = \frac{G}{2}$

$C_z = 2.5 \text{ kN} \quad D_z = 2.5 \text{ kN}$

DODATNE ENACBE

$\Sigma M_{AL}^L: D_x \cdot \frac{a}{2} + A_x \cdot h + M_A = 0$

$\Sigma M_{BK}^K: C_x \cdot \frac{a}{2} + B_x \cdot h + M_B = 0$

$-B_z = qa + \frac{G}{2}$

$\Sigma X_{RKB}: C_x + B_x = 0$

$B_x = 0 \quad A_x = -F$

$A_x = -2 \text{ kN} \quad M_A = 12 \text{ kNm} \quad M_B = 0$

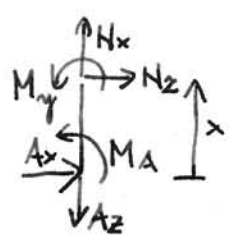
$A_z = B_z = -27.5 \text{ kN}$

d.) KONTROLA

$\Sigma M^R: A_z \cdot a + A_x \cdot h + M_A + B_x \cdot h - B_z \cdot a + M_B =$
 $= -27.5 \cdot 2.5 - 2 \cdot 6 + 12 + 0 + 27.5 \cdot 2.5 + 0 = 0$

e.) NOTRANJE SILE PO POLJIH

POLJE AD:



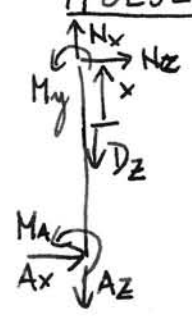
$$N_x = A_z = -27.5 \text{ kN}$$

$$N_z = A_x = +2 \text{ kN}$$

$$M_y = -M_A - A_x \cdot x$$

$$M_y = -12 + 2x$$

POLJE DL:

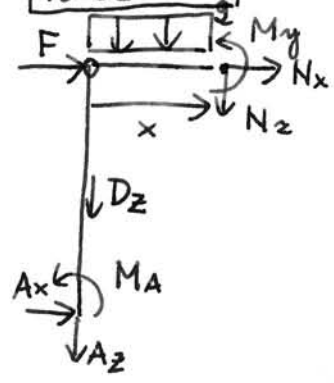


N_z in M_y se gleda na polje AD ne spreminata!

$$N_x = A_z + D_z$$

$$N_x = -25 \text{ kN}$$

POLJE LK:



$$\sum x: N_x + F + A_x = 0$$

$$N_x = 0$$

$$\sum z: N_z + A_z + D_z + g \cdot x = 0$$

$$\sum M^I: M_y + A_z \cdot x + D_z \cdot x + A_x \cdot h + M_A + g \cdot x \cdot \frac{x}{2} = 0$$

$$N_z = 25 - 10x \quad N_z(5) = -25 \text{ kN}$$

$$M_y = 25x - 5x^2$$

$$M_y(0) = 0$$

$$M_y(5) = 0$$

$$M_y(2.5) = 31.25 \text{ kNm (ekstremum)}$$

POLJE BC (z derne):



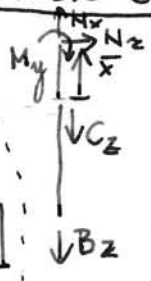
$$N_x = B_z$$

$$N_z = 0$$

$$M_y = 0$$

$$N_x = -27.5 \text{ kN}$$

POLJE CK (z derne):



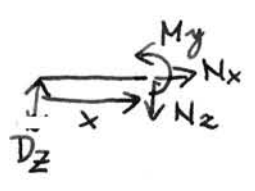
$$N_x = B_z + C_z$$

$$N_z = 0$$

$$M_y = 0$$

$$N_x = -25 \text{ kN}$$

POLJE DG:



$$N_x = 0$$

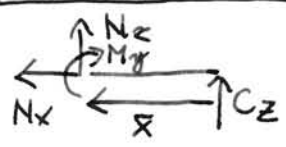
$$N_z = +D_z = +2.5 \text{ kN}$$

$$M_y = +D_z \cdot x$$

$$M_y = +2.5x$$

$$M_y(2.5) = 6.25 \text{ kNm}$$

POLJE GC (z derne):



$$N_x = 0$$

$$N_z = -C_z$$

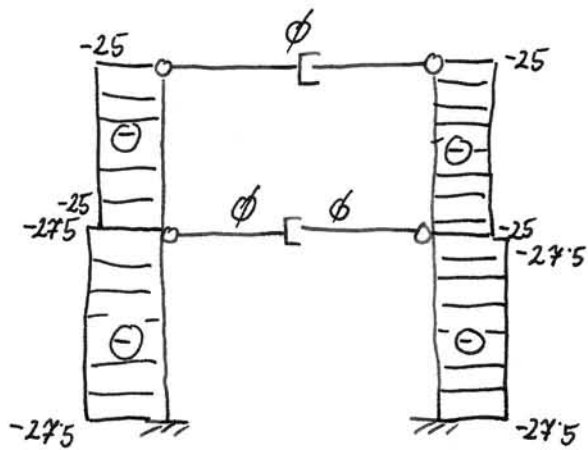
$$N_z = -2.5 \text{ kN}$$

$$M_y = C_z \cdot x$$

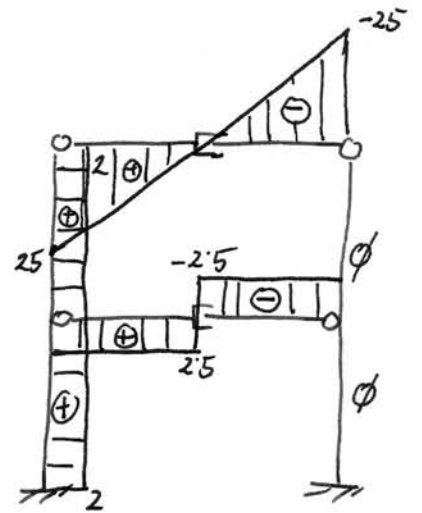
$$M_y = 2.5x$$

f.) DIAGRAMMI

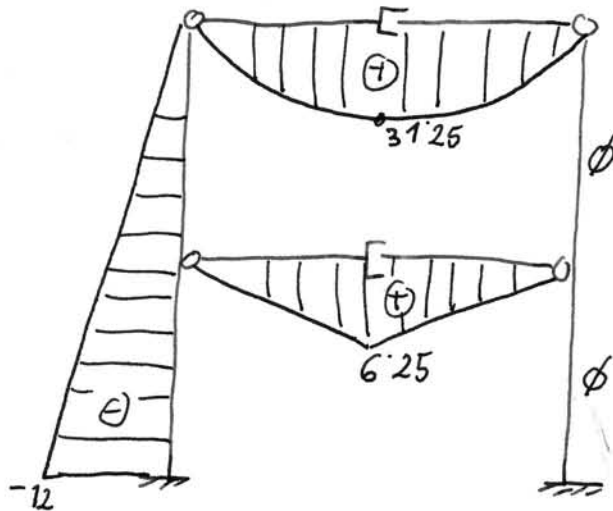
[N_x]



[N_z]



[M_y]



3. NALOGA

a.) $\tilde{n}_{ps} = 2 \cdot 3 - 2 - 3 - 1 = 0$

b.) n_{ps}

PODPORE:

$$\begin{aligned} w_A^{(1)} &= 0 & w_B^{(2)} &= 0 \\ w_A^{(2)} &= 0 \\ \varphi_A^{(1)} &= 0 \end{aligned}$$

VEZI:

$$\begin{aligned} w_C^{(1)} &= w_C^{(2)} \\ \varphi_C^{(1)} &= \varphi_C^{(2)} \end{aligned}$$

TELESA

$$w_C^{(1)} = w_A^{(1)} + 0$$

$$w_C^{(2)} = w_A^{(2)} - a \varphi_A^{(1)}$$

$$\varphi_C^{(1)} = \varphi_A^{(1)}$$

$$w_B^{(2)} = w_C^{(2)} + a \cdot \varphi_C^{(2)} \leftarrow \text{to enačbo vrnemo na četrto mesto}$$

$$w_B^{(2)} = w_C^{(2)} - a \varphi_C^{(2)}$$

$$\varphi_B^{(2)} = \varphi_C^{(2)} \leftarrow \text{to ra na šestu}$$

12 ENAČB IN 12 NEZNANK

1											
0	1										
0	0	1									
-a	0	0	1	0	0	0	0	0	-1	0	-a
0	0	0	0	0	1	0	0	0	0	0	0
0	0	0	0	0	0	1	0	0	0	0	-1
-1	0	0	0	0	0	0	1	0	0	0	0
0	-1	a	0	0	0	0	1	0	0	0	0
0	0	-1	0	0	0	0	0	1	0	0	0
0	0	0	0	-1	0	0	0	0	1	-a	0
0	0	0	0	0	0	0	-1	0	0	1	0
0	0	0	0	0	0	0	0	-1	0	0	1

~

takoj opazimo, da sta ta stolpca linearno odvisna; vseno naredimo še eliminacijo po vrsticah

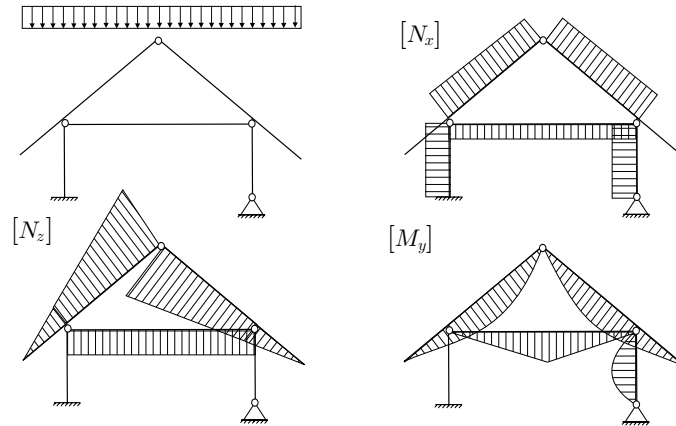
1											
0	1										
0	0	1									
0	0	0	1	0	0	0	0	-1	0	-a	
0	0	0	0	1	0	0	0	0	0	0	
0	0	0	0	0	1	0	0	0	0	-1	
0	0	0	0	0	0	1	0	0	0	0	
0	0	0	0	0	0	0	1	0	0	0	
0	0	0	0	0	0	0	0	1	0	0	
0	0	0	0	0	0	0	0	0	1	-a	
0	0	0	0	0	0	0	0	0	1	0	
0	0	0	0	0	0	0	0	0	0	1	

rang = 11
 \Rightarrow $n_{ps} = 1$

← linearno odvisna vrstica

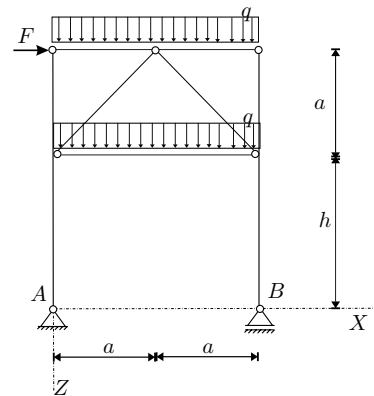
RAČUNSKI DEL IZPITA:

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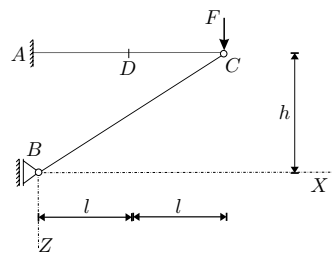


2. Za konstrukcijo na sliki izračunajte stopnjo statične nedoločenosti, reakcije in notranje statične količine (N_x , N_z , M_y)! Rezultate notranjih statičnih količin prikažite z diagrami!

Podatki: $a = 2\text{ m}$, $h = 3\text{ m}$,
 $F = 2\text{ kN}$, $q = 5\text{ kN/m}$.
 (OBVEZNA NALOGA! 50%)



3. S principom virtualnega dela za konstrukcijo na sliki izračunajte reakcije v podporah in notranji moment v točki D – M_D ! (25%)

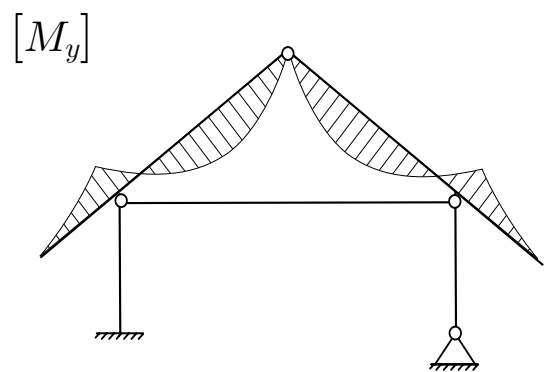
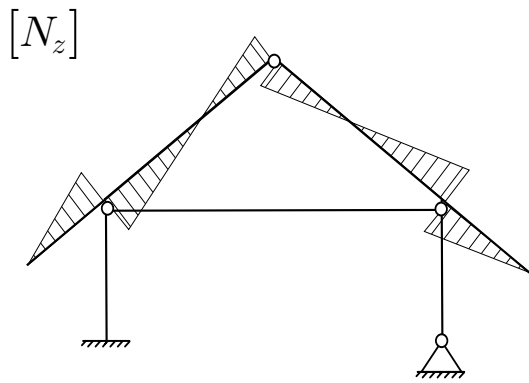
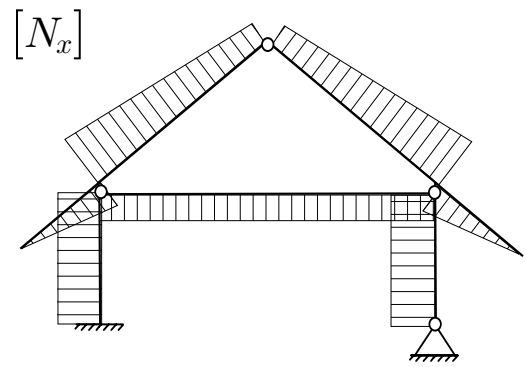
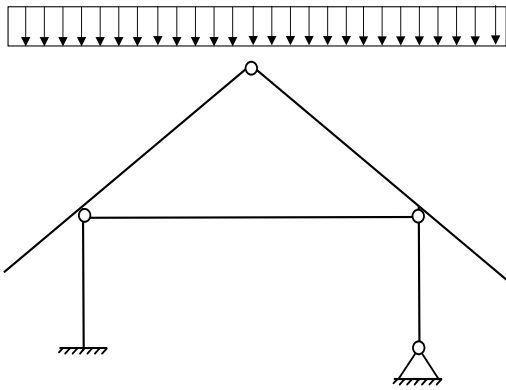


TEORETIČNI DEL IZPITA:

Izmed treh zastavljenih vprašanj si izberete dve, na kateri boste odgovarjali. Izbrani vprašanji jasno označite! Pišite čitljivo.

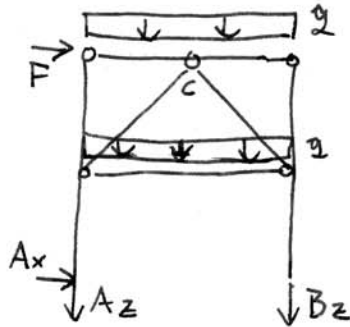
1. Opišite razliko med računskim in dejanskim številom prostostnih stopenj sistema togih teles. Kako izračunamo računsko in kako dejansko število prostostnih stopenj? Kako na podlagi teh pojmov razvrstimo gradbene linijske konstrukcije? Razumevanje podkrepite s preprostimi primeri!
2. Izpeljite in opišite obe nadomestni obliki ravnotežnih pogojev (razumevanje podkrepite s preprostim primerom)!
3. Opišite določanje reakcij in notranjih sil statično določenih linijskih konstrukcij z izrekom o virtualnih pomikih (Razumevanje ilustrirajte na dvostransko previsnem prostoležečem nosilcu s prečno in horizontalno točkovno silo na previsnem robu. Izračunajte vse notranje sile v izbranem značilnem prečnem prerezu nosilca in vse reakcije)!

1. Naloga: PRAVILNI DIAGRAMI



a.) $\tilde{m}_{ps} = 7 \cdot 3 - 2 - 1 - 2 \cdot 2 - 2 \cdot 4 - 6 = 0$

b.) REAKCIJE



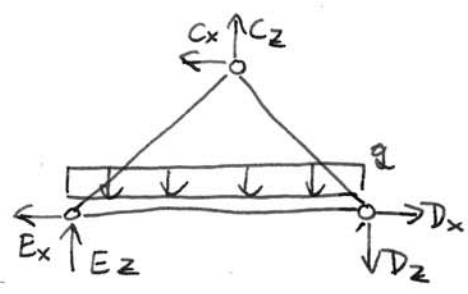
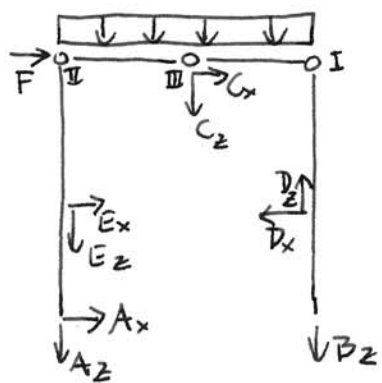
$$\begin{aligned} \sum X: A_x + F &= 0 & \boxed{A_x = -2 \text{ kN}} \\ \sum Z: A_z + B_z + 2q \cdot 2a &= 0 \\ \sum M^A: -B_z \cdot 2a - 2 \cdot q \cdot 2a \cdot a - F \cdot (a+h) &= 0 \end{aligned}$$

$$\boxed{B_z = -22.5 \text{ kN}}$$

$$\boxed{A_z = -17.5 \text{ kN}}$$

KONTROLA: $\sum M^C: A_z \cdot a - B_z \cdot a + A_x(a+h) =$
 $= -17.5 \cdot 2 + 22.5 \cdot 2 - 2 \cdot 5 = 0 \checkmark$

c.) RAZREZ



$\sum M^I: \boxed{D_x = 0}$

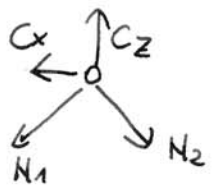
$\sum M^{II} \text{ (at B)}: A_x \cdot (a+h) + E_x \cdot a = 0 \quad \boxed{E_x = 5 \text{ kN}}$

$\sum M^{III} \text{ (at D)}: -B_z \cdot a + D_z \cdot a - q \cdot a \cdot \frac{a}{2} = 0 \quad \boxed{D_z = -17.5 \text{ kN}}$

$\sum M^{III} \text{ (at A)}: A_z \cdot a + E_z \cdot a + A_x \cdot (a+h) + E_x \cdot a + q \cdot a \cdot \frac{a}{2} = 0 \quad \boxed{E_z = 12.5 \text{ kN}}$

$\sum X: C_x + F + E_x + A_x - D_x = 0 \quad \boxed{C_x = -5 \text{ kN}}$

$\sum Z: A_z + B_z + E_z - D_z + C_z + q \cdot 2a = 0 \quad \boxed{C_z = -10 \text{ kN}}$



$$\begin{aligned} N_1 \cdot \frac{\sqrt{2}}{2} + C_x - N_2 \cdot \frac{\sqrt{2}}{2} &= 0 \\ N_1 \cdot \frac{\sqrt{2}}{2} + N_2 \cdot \frac{\sqrt{2}}{2} &= C_z \end{aligned}$$

$$\begin{aligned} N_1 \cdot \sqrt{2} &= C_x + C_z & \boxed{N_1 = -3.5 \text{ kN}} \\ N_2 \cdot \sqrt{2} &= C_z + C_x & \boxed{N_2 = -10.6 \text{ kN}} \end{aligned}$$

d.) NOTRANJE SILE PO POLJIMA

polje AE

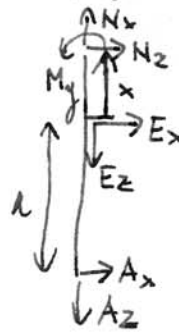


$$\begin{aligned} N_x &= A_x \\ N_z &= -A_z \\ M_y &= -A_x \cdot x \end{aligned}$$

$N_x = -17.5 \text{ kN}$
$N_z = 2 \text{ kN}$
$M_y = 2x$

$M_y(3) = 6 \text{ kNm}$

polje EI

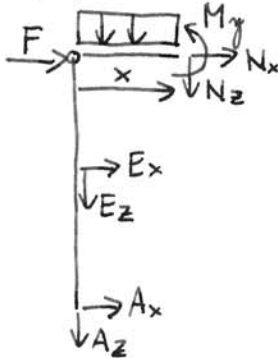


$$\begin{aligned} N_x &= A_x + E_x \\ N_z &= -A_z - E_z \\ M_y &= -E_x \cdot x - A_x(h+x) \end{aligned}$$

$N_x = -5 \text{ kN}$
$N_z = -3 \text{ kN}$
$M_y = 6 - 3x$

$M_y(2) = 0$

polje II



$$\begin{aligned} N_x &= -E_x - A_x - F \\ N_z &= -A_z - E_z - q \cdot x \\ -M_y &= A_x(a+h) + E_x \cdot a + A_z \cdot x + E_z \cdot x + q \cdot x \cdot \frac{x}{2} \end{aligned}$$

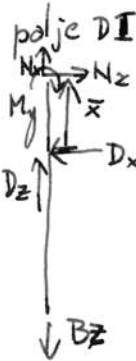
$N_x = -5 \text{ kN}$
$N_z = 5 - 5x$
$M_y = 5x - 2.5x^2$

$N_z(2) = -5 \text{ kN}$
 $M_y(0) = 0$ $M_y(2) = 0$ $M_y(1) = 2.5$ (ekstremum)

polje BD



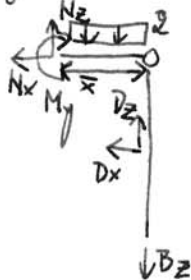
$N_x = B_z$
$N_z = M_y = 0$
$N_x = -22.5 \text{ kN}$



$$\begin{aligned} N_x &= B_z - D_x \\ N_z &= D_x \\ M_y &= -D_x \cdot x \end{aligned}$$

$N_x = -5 \text{ kN}$
$N_z = 0 \mid M_y = 0$

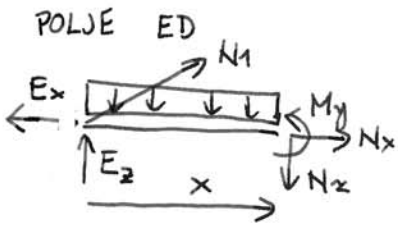
polje III



$$\begin{aligned} N_x &= -D_x \\ N_z &= B_z - D_z + q \cdot x \\ M_y &= -B_z \cdot x + D_z \cdot x - q \cdot x \cdot \frac{x}{2} - D_x \cdot a \end{aligned}$$

$N_x = 0$
$N_z = -5 + 5x$
$M_y = +5x - 2.5x^2$

$M_y(1) = 2.5$ (ekstremum)



$$N_x = E_x - N_1 \cdot \frac{\sqrt{2}}{2}$$

$$N_z = +E_z + N_1 \cdot \frac{\sqrt{2}}{2} - q \cdot x$$

$$M_y = E_z \cdot x + N_1 \cdot \frac{\sqrt{2}}{2} \cdot x - q \cdot x \cdot \frac{x}{2}$$

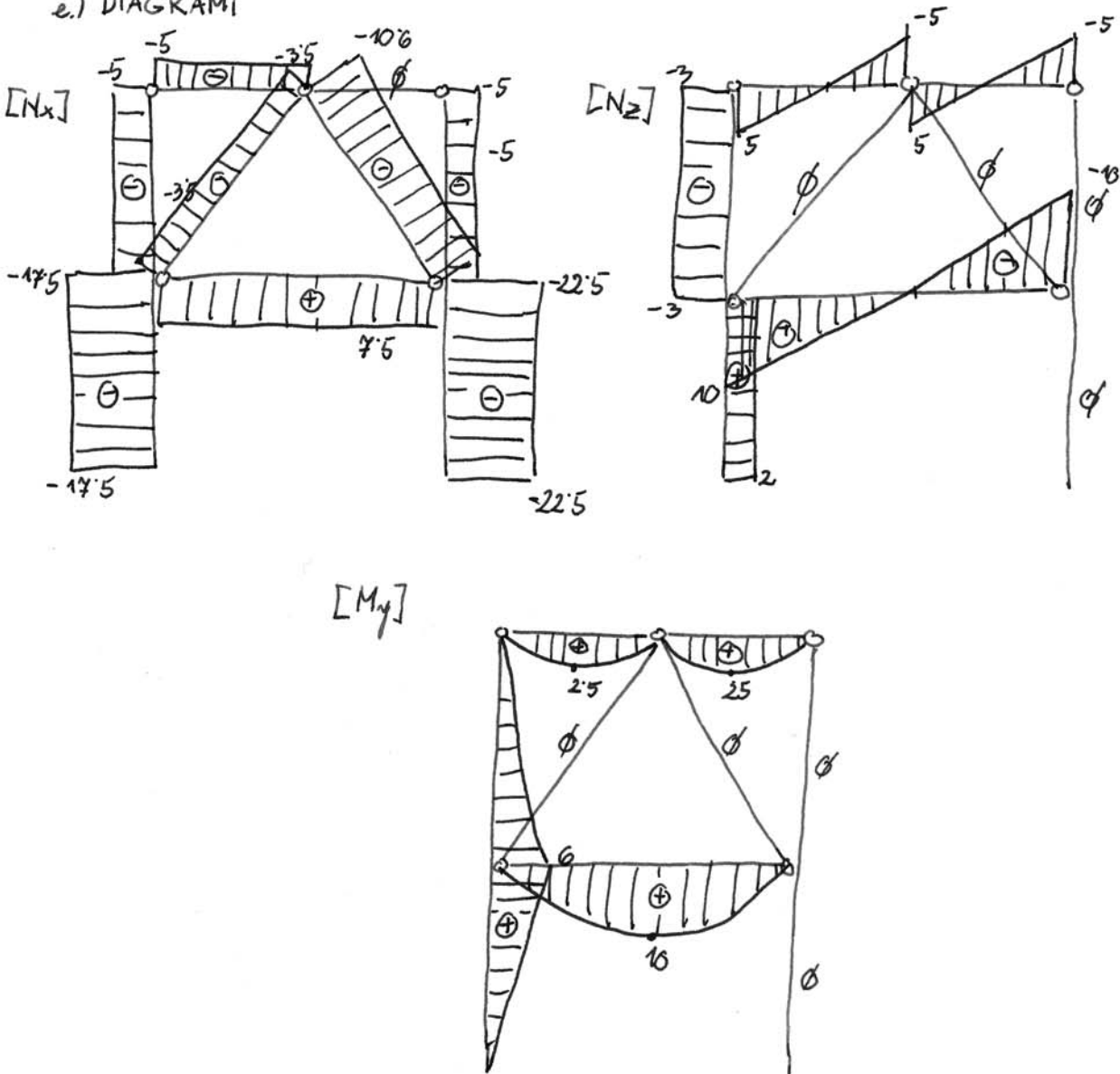
$$N_x = 7.5 \text{ kN}$$

$$N_z = 10 - 5x$$

$$M_y = 10x - 2.5x^2$$

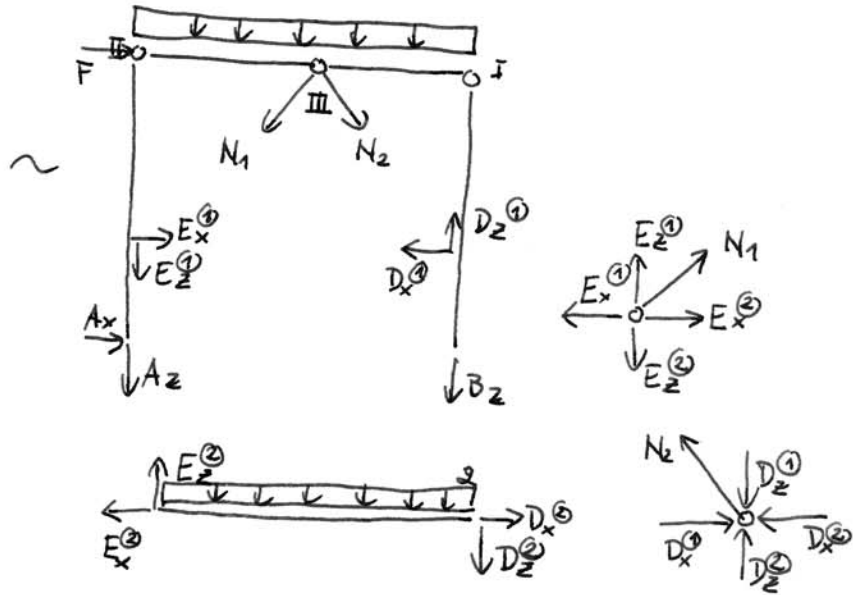
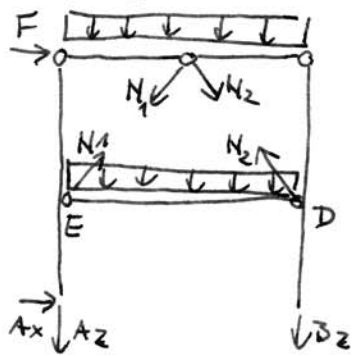
$$M_y(2) = 10 \text{ kNm (obstoj)}$$

e.) DIAGRAMI



PRILOGA:

RAZREZ 2. NAČIN



$$\sum M_{BC}^I: D_x^{(1)} = 0$$

$$\sum M_{AB}^{II}: E_x^{(1)} \cdot a + A_x \cdot (a+h) = 0$$

$$E_x^{(1)} = +5 \text{ kN}$$

$$\sum M_{DB}^{III}: -B_z \cdot a + D_z^{(1)} \cdot a - q \cdot a \cdot \frac{a}{2} = 0$$

$$D_z^{(1)} = -17.5 \text{ kN}$$

$$\text{VEZ D: } N_2 \cdot \frac{\sqrt{2}}{2} + D_x^{(2)} - D_x^{(1)} = 0$$

$$N_2 \cdot \frac{\sqrt{2}}{2} + D_z^{(2)} - D_z^{(1)} = 0$$

$$\text{VEZ E: } N_1 \cdot \frac{\sqrt{2}}{2} + E_x^{(2)} - E_x^{(1)} = 0$$

$$N_1 \cdot \frac{\sqrt{2}}{2} + E_z^{(1)} - E_z^{(2)} = 0$$

$$\sum M_{DE}^{ME}: -D_z^{(2)} \cdot 2a - q \cdot 2a \cdot a = 0$$

$$D_z^{(2)} = -10 \text{ kN}$$

$$\sum Z_{ED}: -E_z^{(2)} + D_z^{(2)} + q \cdot 2a = 0$$

$$E_z^{(2)} = 10 \text{ kN}$$

$$N_2 = -10.6 \text{ kN}$$

$$D_x^{(2)} = 7.5 \text{ kN}$$

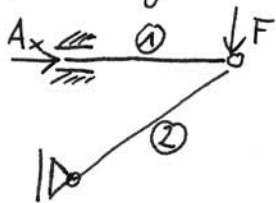
$$E_x^{(2)} = D_x^{(2)}$$

$$N_1 = -3.5 \text{ kN}$$

$$E_z^{(1)} = 12.5 \text{ kN}$$

3. NALOGA

a.) reakcija A_x



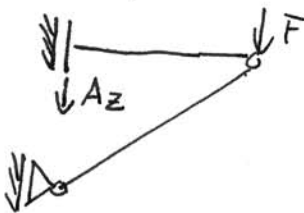
$$A_x \cdot \delta u_A + F \delta w_C = 0$$

$$\delta w_C = \delta w_A = 0$$

$$\Rightarrow A_x \cdot \delta u_A = 0$$

$$\boxed{A_x = 0}$$

b.) reakcija A_z



$$A_z \cdot \delta w_A + F \delta w_C = 0$$

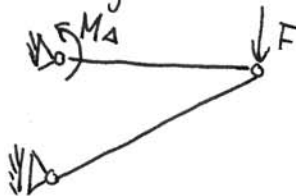
$$\delta w_C = \delta w_A - 2l \delta \varphi_A$$

$$\delta \varphi_A = 0$$

$$(A_z + F) \delta w_A = 0$$

$$\boxed{A_z = -F}$$

c.) reakcija M_A



$$M_A \cdot \delta \varphi_A + F \delta w_C = 0$$

$$\delta w_C = \delta w_A - 2l \delta \varphi_A$$

$$(M_A - 2lF) \delta \varphi_A = 0 \Rightarrow \boxed{M_A = 2lF}$$

d.) reakcija B_x

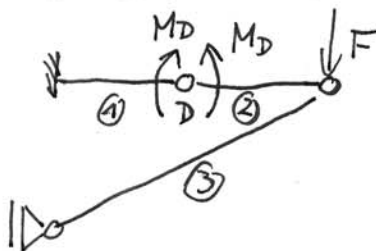


$$B_x \delta u_B + F \delta w_C = 0$$

$$\delta w_C = \delta w_A = 0$$

$$\Rightarrow \boxed{B_x = 0}$$

e.) moment M_D



$$-M_D \delta \varphi_D^{(1)} + M_D \delta \varphi_D^{(2)} + F \delta w_C = 0$$

$$\delta \varphi_D^{(1)} = \delta \varphi_A = 0$$

$$\delta w_D = \delta w_A - l \delta \varphi_A = 0$$

$$\delta w_C = \delta w_D - l \delta \varphi_D^{(2)}$$

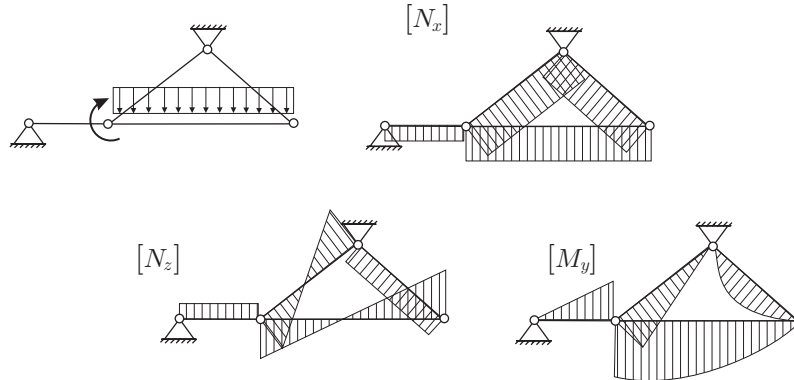
$$= 0$$

$$(M_D - lF) \delta \varphi_D^{(2)} = 0$$

$$\boxed{M_D = lF}$$

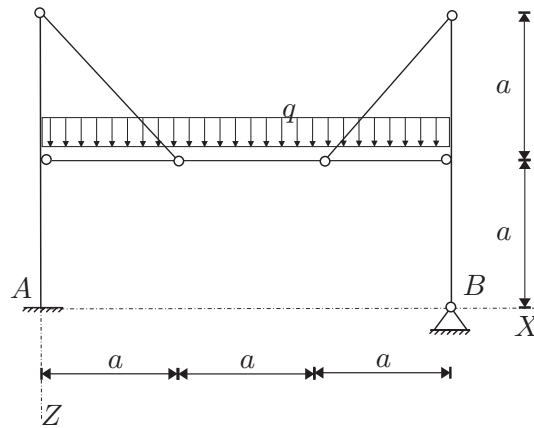
RAČUNSKI DEL IZPITA:

1. Janezek je na izpitu iz statike padel. Njegovi diagrami so polni napak. Pomagaj Janezku in poišči (BREZ RAČUNANJA) vse napake v spodnjih diagramih ! (OBVEZNA NALOGA! 25%)

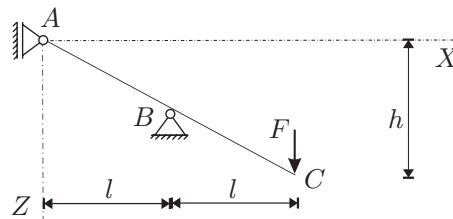


2. Za konstrukcijo na sliki izračunajte stopnjo statične nedoločnosti, reakcije in notranje statične količine (N_x , N_z , M_y)! Rezultate notranjih statičnih količin prikažite z diagrami!

Podatki: $a = 3 \text{ m}$, $q = 4 \text{ kN/m}$.
(OBVEZNA NALOGA! 50%)



3. S principom virtualnega dela za konstrukcijo na sliki izračunajte reakcije v podporah in notranji moment v točki B – M_B ! (25%)

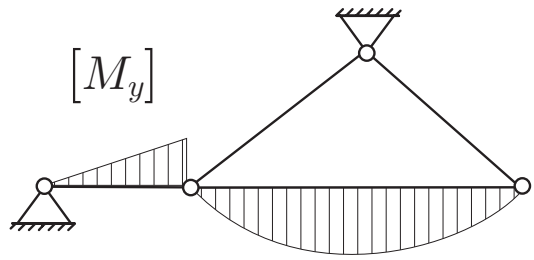
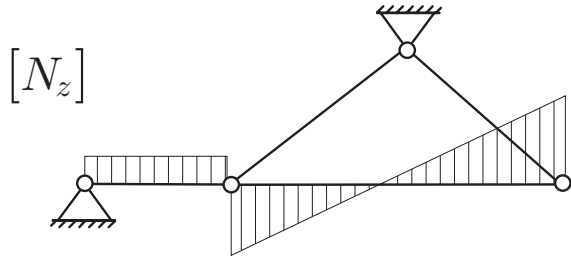
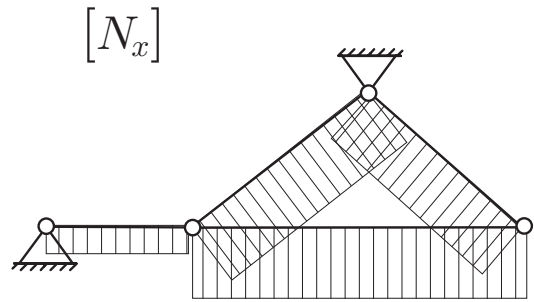
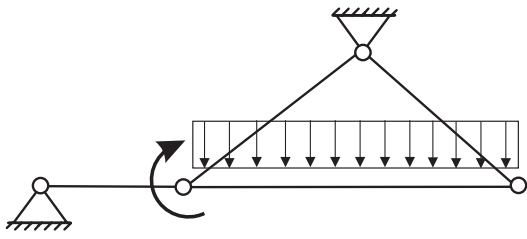


TEORETIČNI DEL IZPITA:

Izmed treh zastavljenih vprašanj si izberete dve, na kateri boste odgovarjali. Izbrani vprašanji jasno označite! Pišite čitljivo.

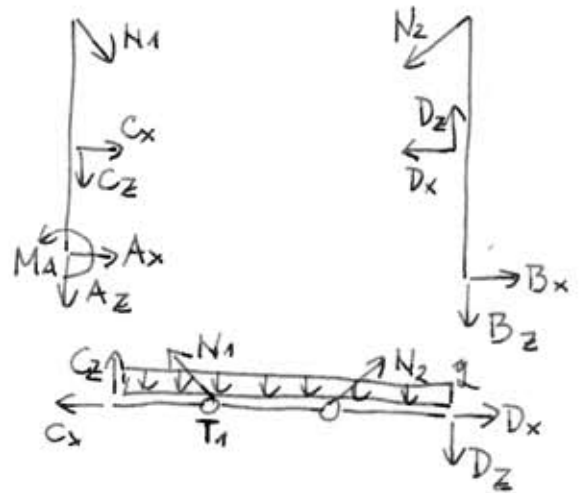
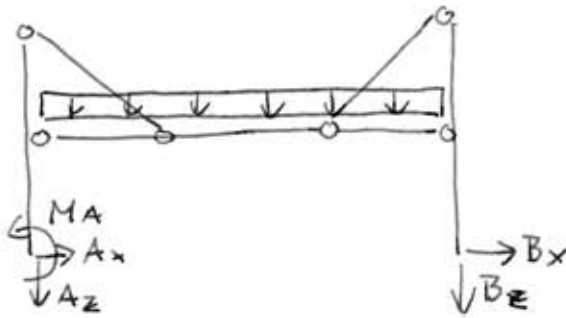
1. Izpeljite in opišite nadomestne ravnotežne pogoje (razumevanje podkrepite z računom reakcij za previsni nosilec s enakomerno prečno linijsko obtežbo)!
2. Zapišite izraz za računsko število prostostnih stopenj sistema togih teles (razumevanje podkrepite s preprostim primerom)! Opišite razliko med računskim in dejanskim številom prostostnih stopenj!
3. Opišite določanje reakcij in notranjih sil statično določenih linijskih konstrukcij z izrekom o virtualnih (razumevanje podkrepite z računom reakcij in notranjih sil v izbrani točki previsnega nosilca, ki je obtežen s prečno in horizontalno točkovno silo na prostem robu)!

1. Naloga: PRAVILNI DIAGRAMI



$$a.) \tilde{n}_{P_0} = 7 \cdot 3 - 3 - 2 - 4 \cdot 2 - 2 \cdot 4 = 0$$

b.1 REAKCIJE



$$\sum M_{CT_1}^{T_1}: -C_z \cdot a + g \cdot a \cdot \frac{a}{2} = 0 \Rightarrow C_z = g \frac{a}{2}$$

$$\sum M_{DT_2}^{T_2}: -D_z \cdot a - g \cdot a \cdot \frac{a}{2} = 0 \Rightarrow D_z = -g \frac{a}{2}$$

$$\boxed{C_z = 6 \text{ kN}}$$

$$\boxed{D_z = -6 \text{ kN}}$$

$$\sum M_{DT_1}^{T_1}: -D_z \cdot 2a - g \cdot 2a \cdot a + N_2 \cdot a \cdot \frac{\sqrt{2}}{2} = 0$$

$$N_2 = \frac{2}{\sqrt{2}} (2D_z + 2ga)$$

$$\boxed{N_2 = 16 \cdot 9.81} \quad \left(\frac{24}{\sqrt{2}} \right)$$

$$\sum Z: C_z + N_1 \cdot \frac{\sqrt{2}}{2} + N_2 \cdot \frac{\sqrt{2}}{2} - D_z - g \cdot 3a = 0$$

$$12 - 4 \cdot 3 \cdot 3 + 12 + N_1 \frac{\sqrt{2}}{2} = 0$$

$$N_1 = + \frac{12 \cdot 2}{\sqrt{2}}$$

$$\boxed{N_1 = 16 \cdot 9.81}$$

$$\sum M^{T_3}: B_x \cdot 2a - D_x \cdot a = 0 \quad B_x = \frac{D_x}{2}$$

$$\sum X: N_2 \cdot \frac{\sqrt{2}}{2} + D_x - B_x = 0$$

$$N_2 \frac{\sqrt{2}}{2} + B_x = 0$$

$$\boxed{B_x = -12 \text{ kN}}$$

$$\boxed{A_x = 12 \text{ kN}}$$

$$\boxed{D_x = -24 \text{ kN}}$$

$$\boxed{C_x = -24 \text{ kN}}$$

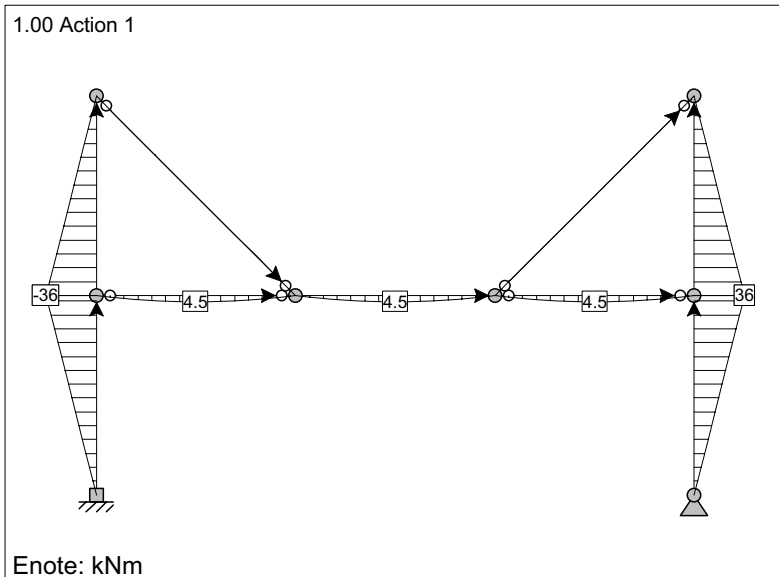
$$\sum Z: N_2 \frac{\sqrt{2}}{2} - D_z + B_z = 0$$

$$B_z = D_z - N_2 \frac{\sqrt{2}}{2} = -6 - 12 = -18 \text{ kN}$$

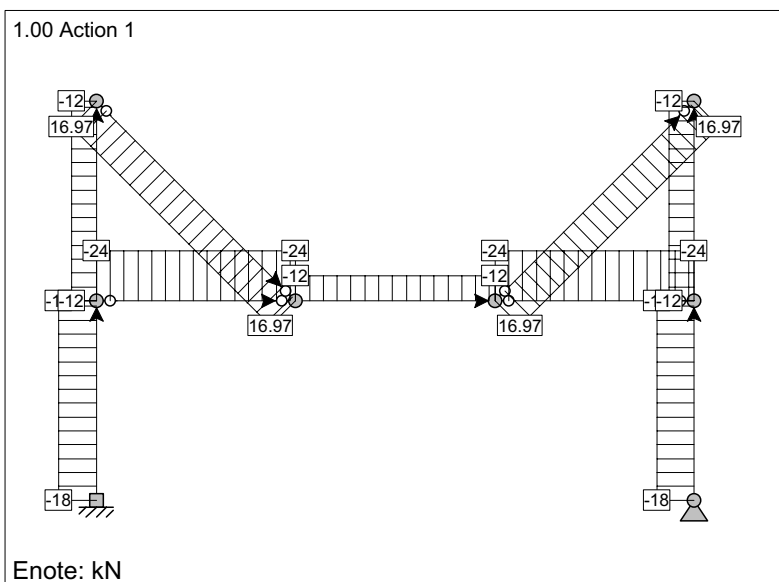
$$\Rightarrow M_A = 0$$

$$\boxed{A_z = -18 \text{ kN}}$$

LC1: Load case 1: Upogibni moment My



LC1: Load case 1: Osna sila Fx



LC1: Load case 1: Preèna sila Fz

