

Dejan Zupan

IZPITNE NALOGE IN REŠITVE NALOG S POSTOPKOM IZ PREDMETA STATIKA NA
UNIVERZITETNEM ŠTUDIJU GRADBENIŠTVA

Igor Planinc

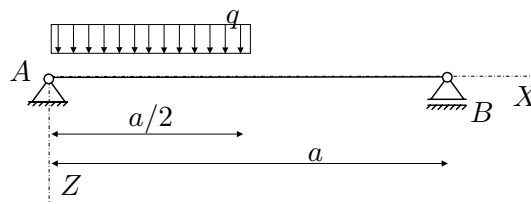
VPRAŠANJA IZ TEORIJE PRI PREDMETU STATIKA NA
UNIVERZITETNEM ŠTUDIJU GRADBENIŠTVA

ŠTUDIJSKO LETO: 2004/05

STATIKA (UNI) - IZPITNI ROK (31. 01. 2005)

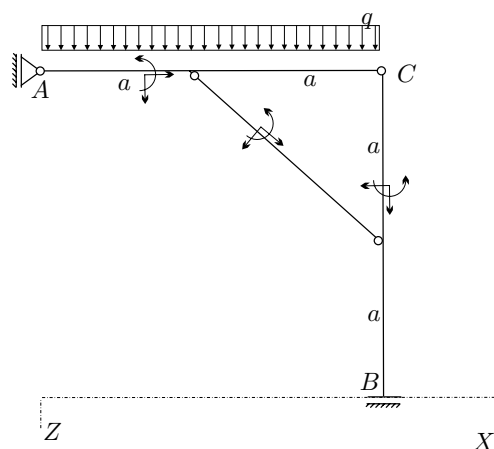
RAČUNSKI DEL IZPITA:

1. Za nosilec na sliki izračunajte in prikažite diagrame notranjih statičnih količin! (OBVEZNA NALOGA! 20%)



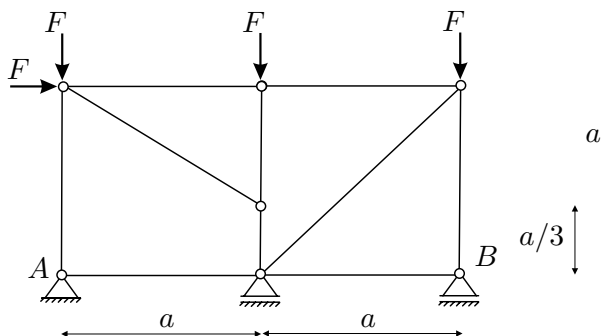
2. Za konstrukcijo na sliki izračunajte stopnjo statične nedoločenosti, reakcije in notranje statične količine (N_x, N_z, M_y)! Rezultate notranjih statičnih količin prikažite z diagrami!

Podatki: $a = 4 \text{ m}$, $q = 2 \text{ kN/m}$
(OBVEZNA NALOGA! 45%)



3. Za palično konstrukcijo na sliki izračunajte stopnjo statične nedoločenosti in osne sile v vseh palicah! (35%)

Podatki: $a = 3 \text{ m}$, $F = 5 \text{ kN}$.

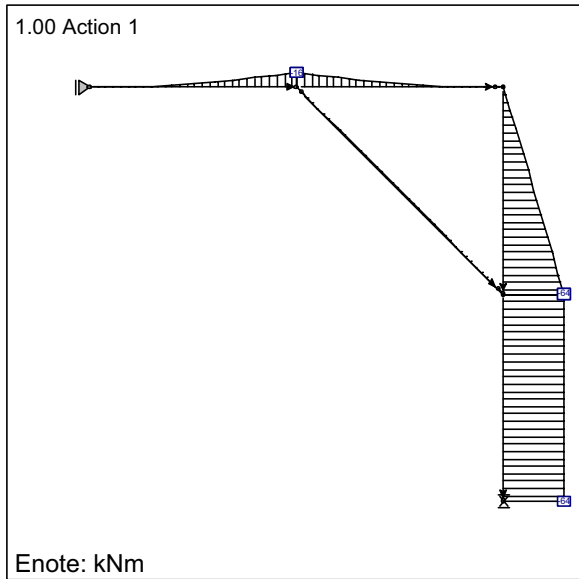


TEORETIČNI DEL IZPITA:

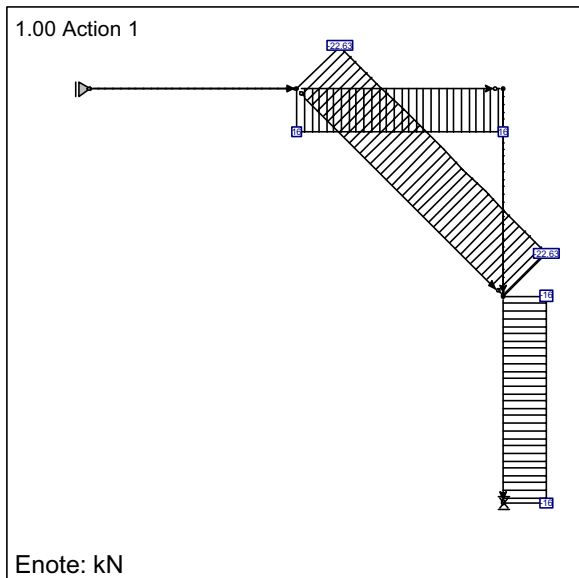
Izmed treh zastavljenih vprašanj si izberete dve, na kateri boste odgovarjali. Izbrani vprašanji jasno označite! Pišite čitljivo.

1. Ravnotežni par sil in dvojica sil!
2. Pomiki in zasuki togega telesa (izpeljava enačb)!
3. Opišite določanje reakcij in notranjih sil statično določenih linijskih konstrukcij z izrekom o virtualnih pomikih (pomagajte si s primerom)!

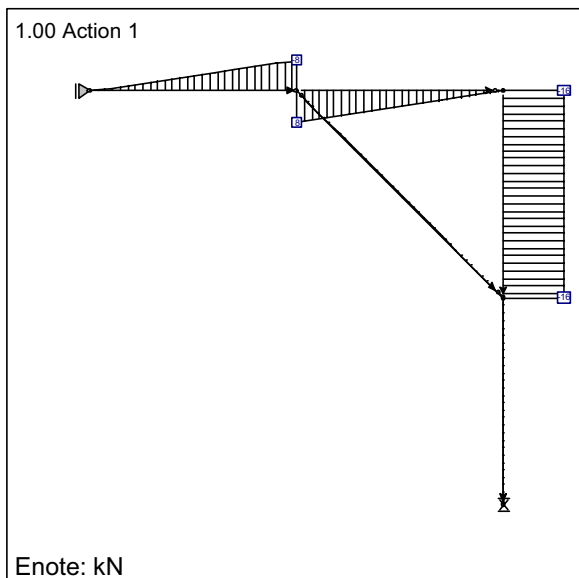
LC1: Load case 2: Upogibni moment My

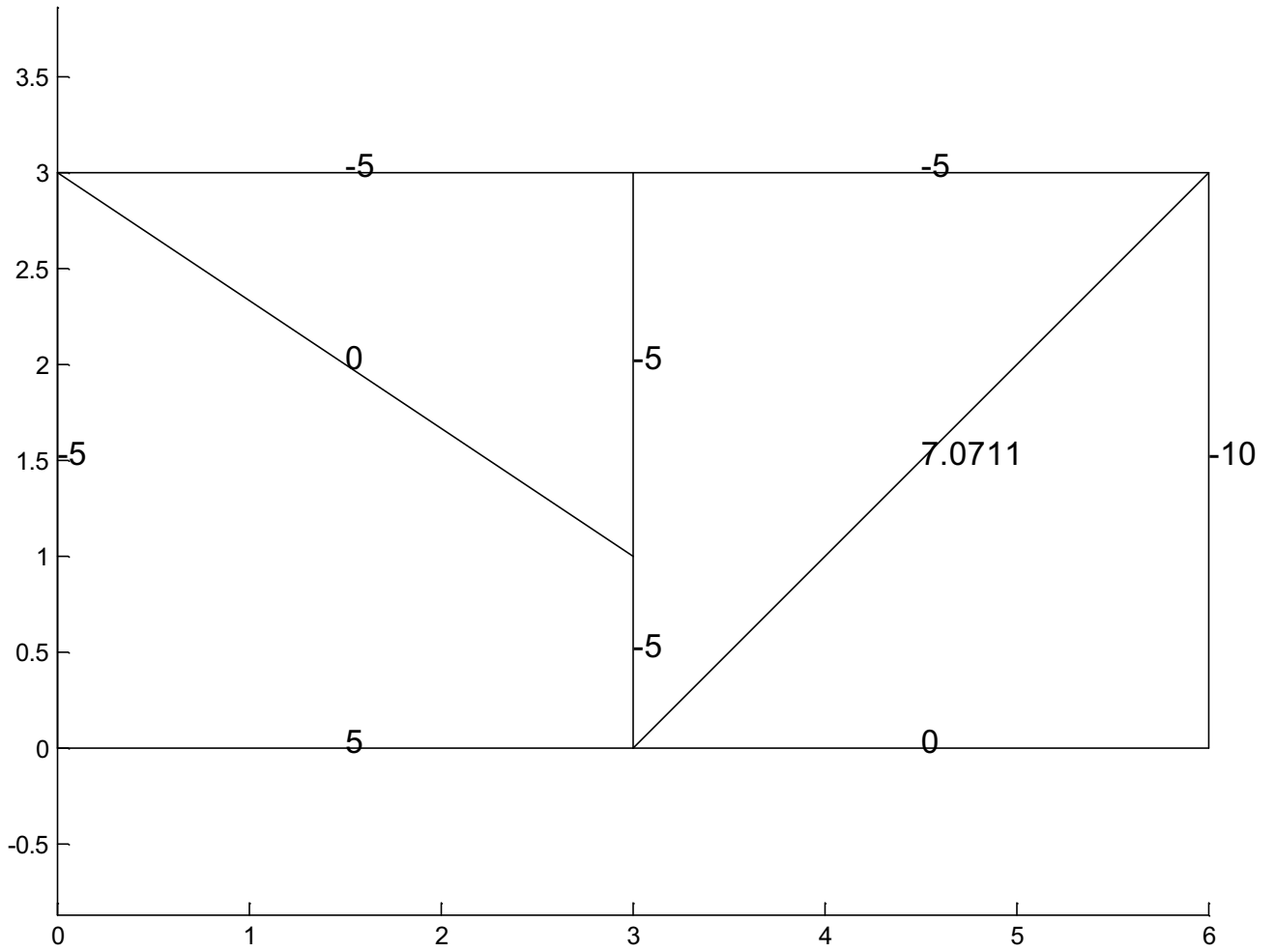


LC1: Load case 2: Osna sila Fx



LC1: Load case 2: Preèna sila Fz

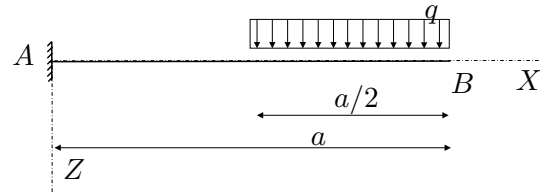




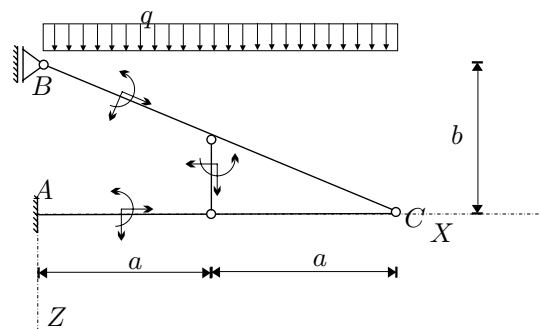
STATIKA (UNI) - 1. IZREDNI IZPITNI ROK (14. 3. 2005)

RAČUNSKI DEL IZPITA:

1. Za nosilec na sliki izračunajte reakcije v vezeh ter izračunajte in prikažite diagrame notranjih statičnih količin! (OBVEZNA NALOGA! 20%)

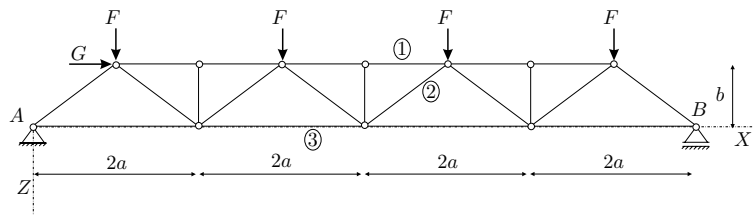


2. Za konstrukcijo na sliki izračunajte stopnjo statične nedoločenosti, reakcije in notranje statične količine (N_x, N_z, M_y)! Rezultate notranjih statičnih količin prikažite z diagrami!



Podatki: $a = 3 \text{ m}$, $b = 2 \text{ m}$,
 $q = 10 \text{ kN/m}$.
 (OBVEZNA NALOGA! 50%)

3. Za palično konstrukcijo na sliki izračunajte stopnjo statične nedoločenosti in osne sile v palicah 1, 2 in 3! (30%)



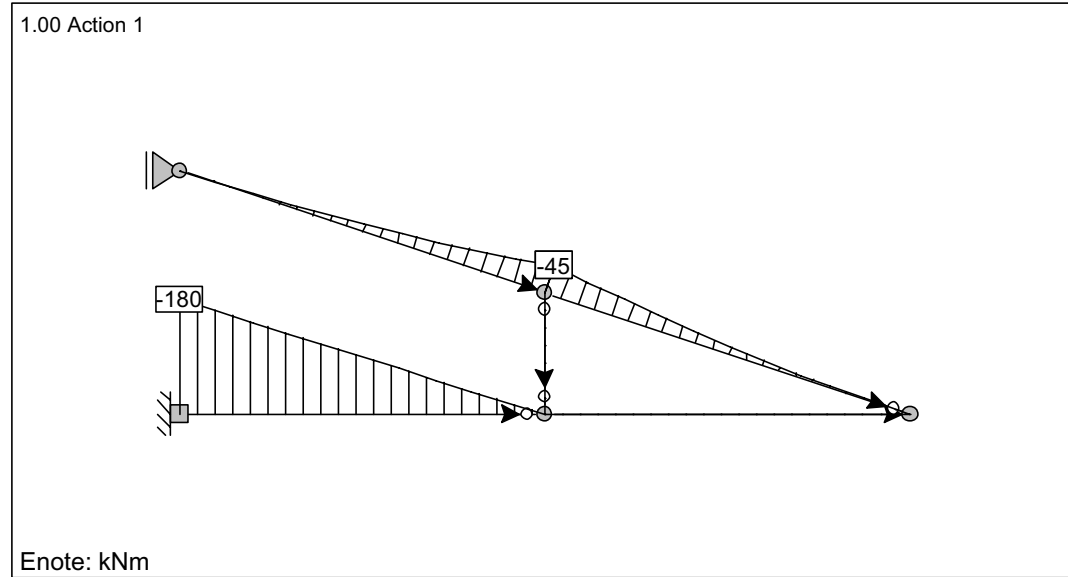
Podatki: $a = 3 \text{ m}$, $b = 2 \text{ m}$,
 $F = 5 \text{ kN}$, $G = 10 \text{ kN}$.

TEORETIČNI DEL IZPITA:

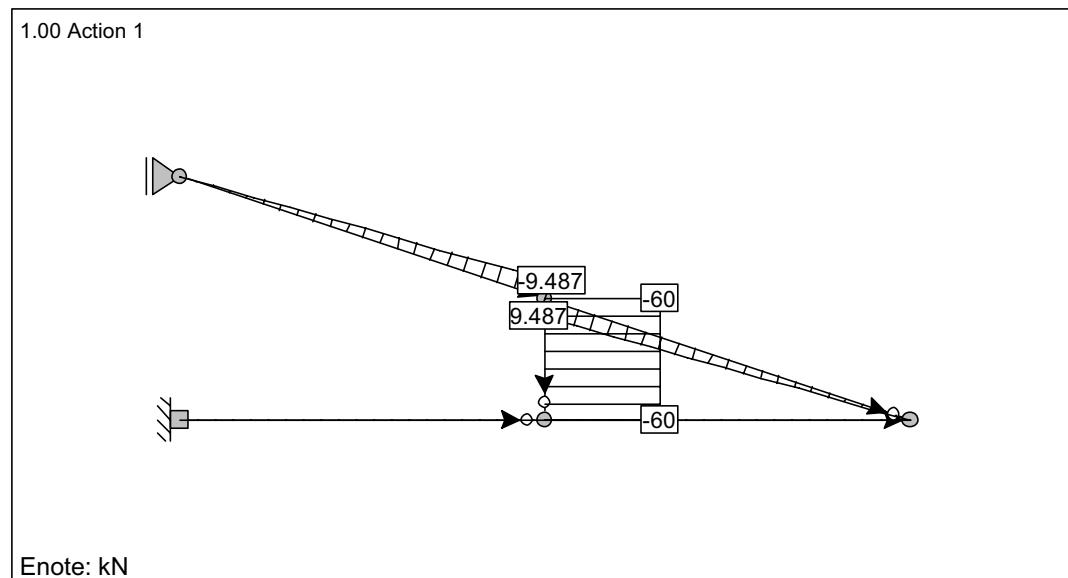
Izmed treh zastavljenih vprašanj si izberete dve, na kateri boste odgovarjali. Izbrani vprašanji jasno označite! Pišite čitljivo.

1. Izpeljite ravnotežne pogoje za sile, ki delujejo na sistemu delcev s togimi vezmi in togem telesu!
2. Opišite kinematične enačbe sistema togih teles ter postopek računanja dejanskega števila prostostnih stopenj sistema togih teles (ilustracija z značilnim primerom)!
3. Opišite določanje reakcij in notranjih sil statično določenih linijskih konstrukcij z izrekom o virtualnih pomikih (pomagajte si s primerom)!

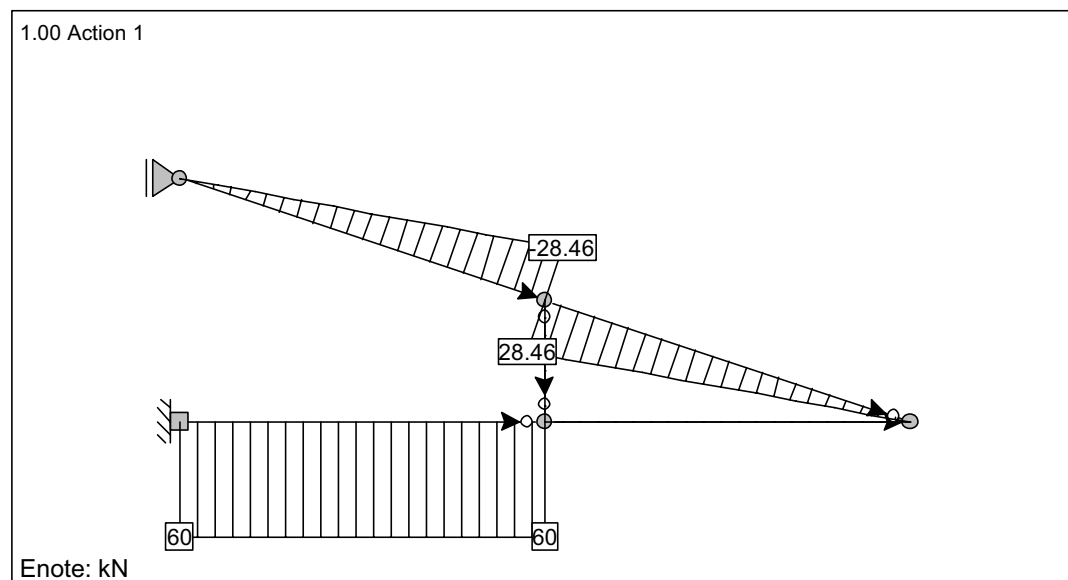
LC1: Load case 2: Upogibni moment My

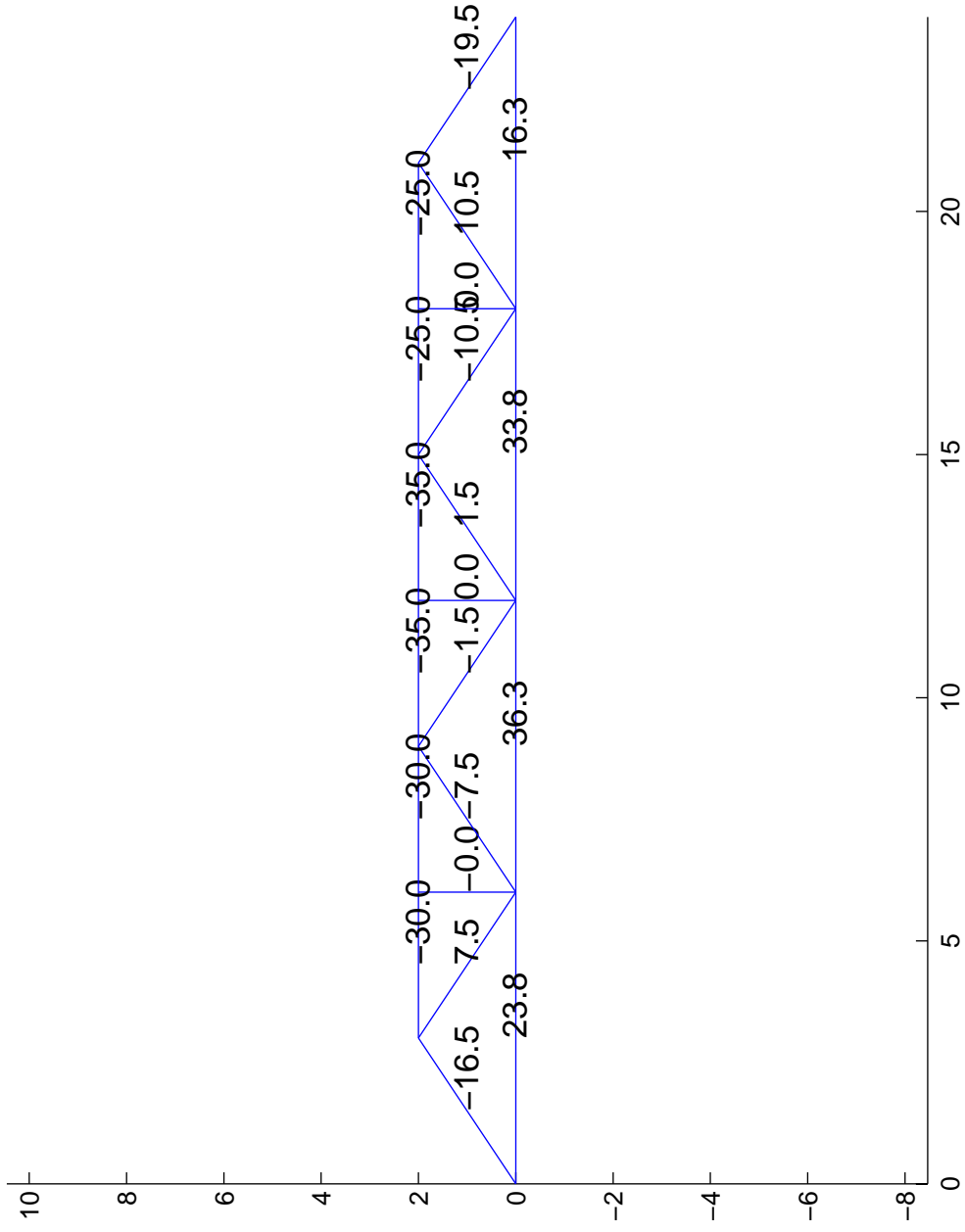


LC1: Load case 2: Osna sila Fx



LC1: Load case 2: Preèna sila Fz



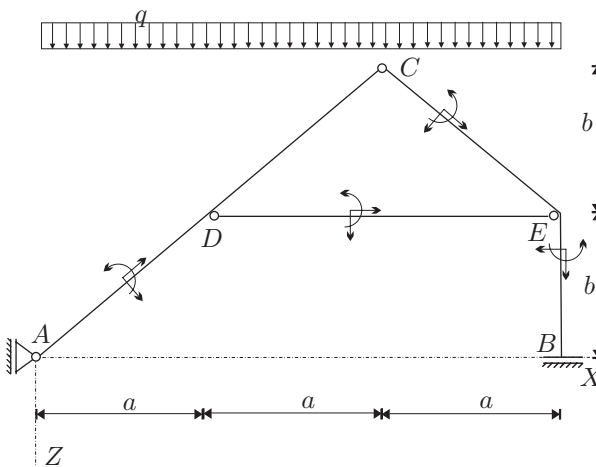


STATIKA (UNI) - 1. IZPITNI ROK (10. 6. 2005)

RAČUNSKI DEL IZPITA:

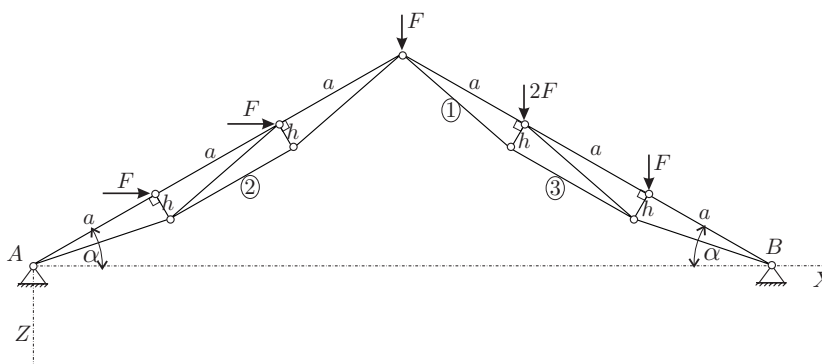
1. Za konstrukcijo na sliki izračunajte stopnjo statične nedoločenosti, reakcije in notranje statične količine (N_x, N_z, M_y)! Rezultate notranjih statičnih količin prikažite z diagrami!

Podatki: $a = 4\text{ m}$, $b = 3\text{ m}$,
 $q = 2\text{ kN/m}$.
 (OBVEZNA NALOGA! 40%)



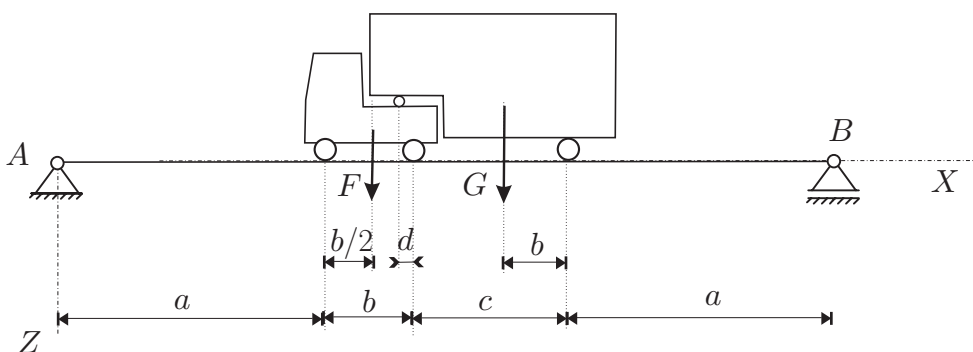
2. Za palično konstrukcijo na sliki izračunajte stopnjo statične nedoločenosti in osne sile v palicah 1, 2 in 3! (30%)

Podatki: $a = 5\text{ m}$,
 $h = 1\text{ m}$,
 $\alpha = 30^\circ$, $F = 5\text{ kN}$.



3. Izračunajte in narišite diagram upogibnih momentov v mostu na sliki! Most (prostoležeči nosilec) je obremenjen le s težo vlačilca. Medosna razdalja vlačilca je b , razdalja od zadnje osi vlačilca do osi prikolice pa c . Prikolica je členkasto pripeta na vlačilec, vez leži za razdaljo d pred drugo osjo vlačilca. F je teža vlačilca, ki ima prijemališče na sredini, G pa teža prikolice, ki ima prijemališče na razdalji b pred osjo prikolice. (30%)

Podatki: $a = 15\text{ m}$, $b = 4\text{ m}$, $c = 10\text{ m}$, $d = 0.5\text{ m}$,
 $F = 30\text{ kN}$, $G = 150\text{ kN}$.



STATIKA (UNI) - 1. IZPITNI ROK (10. 6. 2005)

TEORETIČNI DEL IZPITA:

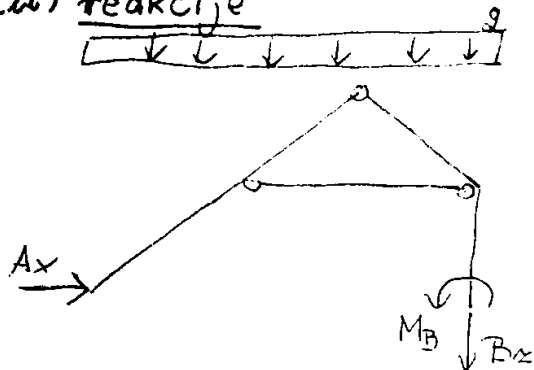
Izmed treh zastavljenih vprašanj si izberete dve, na kateri boste odgovarjali. Izbrani vprašanji jasno označite! Pišite čitljivo.

1. Definicija števila prostostnih stopenj! Koliko prostostnih stopenj ima N masnih delcev, ki se gibljejo po kroglji? Koliko prostostnih stopenj ima N togih teles, ki so povezana s členkom (odgovore utemeljite)?
2. Izpeljite ravnotežne pogoje za sistem delcev z breztežnimi togimi vezmi in togo telo!
3. Opišite določanje reakcij in notranjih sil statično določenih linijskih konstrukcij z izrekom o virtualnih pomikih (pomagajte si s primerom)!

1. HALOGA

(i) $m_B = 3 \cdot 3 - 1 \cdot 2 - 3 \cdot 2 = 0$

(ii) reakcije



$\sum X: A_x = 0$

$A_x = 0$

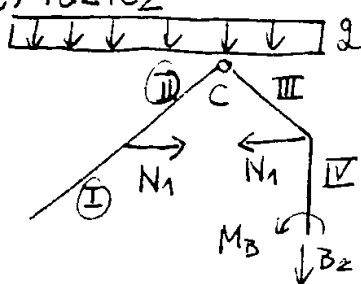
$\sum Z: B_z + g \cdot 3 \cdot 2 = 0$

$B_z = -24 \text{ kN}$

$\sum M^B: M_B + g \cdot 3 \cdot 2 \cdot \frac{3 \cdot 2}{2} = 0$

$M_B = -144 \text{ kNm}$

(iii) razrez



$\sum M^C_{AC}: g \cdot 2a \cdot a + N_1 \cdot b = 0$

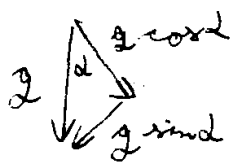
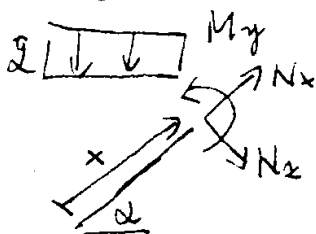
$N_1 = -\frac{g \cdot 2a^2}{b}$

$N_1 = -21.3 \text{ kN}$

$N_1 = -\frac{64}{3}$

(iv) notranje sile

polje I



$\sum \psi: N_x - g \cdot \sin \alpha \cdot x \cdot \cos \alpha = 0$

$\sum \zeta: N_z + g \cdot \cos \alpha \cdot x \cdot \cos \alpha = 0$

$\sum M^I: M_y + g \cdot x \cdot \cos \alpha \cdot \frac{x \cdot \cos \alpha}{2} = 0$

$\tan \alpha = \frac{3}{4}$

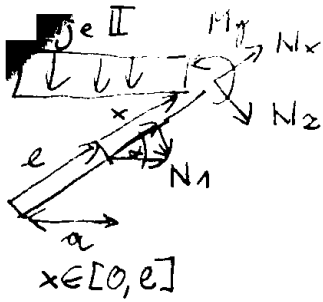
$x \in [0, l]$

$l = \sqrt{4^2 + 3^2} = 5$

$\sin \alpha = \frac{3}{5}$

$\cos \alpha = \frac{4}{5}$

$N_x = \frac{24}{25} x$	$N_x(l) = 4.8$
$N_z = -\frac{32}{25} x$	$N_z(l) = -6.4$
$M_y = -\frac{16}{25} x^2$	$M_y(l) = -16$



$$\sum x: N_x - q \sin(\alpha) x \cos \alpha + N_1 \cos \alpha = 0$$

$$\sum z: N_z + q \cos(\alpha) x \cos \alpha + N_1 \sin \alpha = 0$$

$$\sum M^T: M_y + q(\alpha x) \cos \alpha \frac{(\alpha x) \cos \alpha}{2} + N_1 x \sin \alpha = 0$$

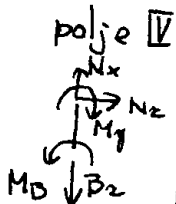
$$N_x = 4.8 + \frac{64}{3} \cdot \frac{4}{5} + \frac{24}{5} x = 21.9 + \frac{24}{25} x$$

$$N_z = -6.4 + \frac{64}{3} \cdot \frac{3}{5} - \frac{32}{5} x = -6.4 - \frac{32}{25} x$$

$$M_y = -16 + \frac{64}{3} \cdot \frac{2}{5} x - \frac{16}{25} x^2 - \frac{32}{25} x$$

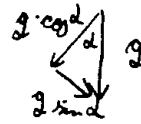
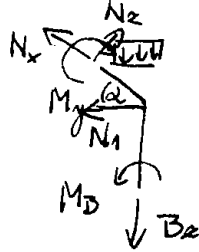
$$= -16 + \frac{32}{5} x - \frac{16}{25} x^2$$

	x=0	x=5
N_x	21.9	26.7
N_z	6.4	0
M_y	-16	0



$N_x = -24 \text{ kN}$
$M_y = -144 \text{ kN}$
$N_z = 0$

polje III



$$\sum x: N_x - q \sin \alpha x \cos \alpha - B_z \sin \alpha = 0$$

$$\sum z: N_z - q \cos \alpha x \cos \alpha - B_z \cos \alpha = 0$$

$$\sum M^T: -M_y + M_D - B_z x \cos \alpha - q x \cos \alpha \frac{x \cos \alpha}{2} = 0$$

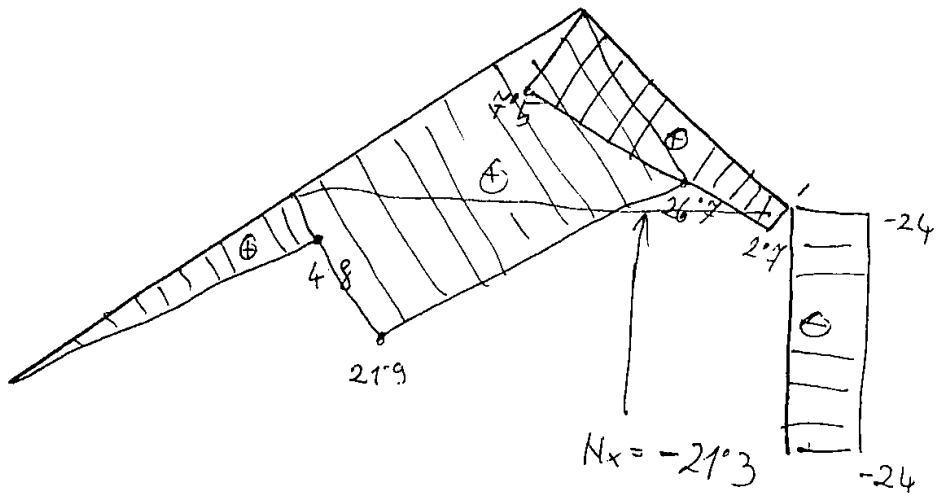
$$N_x = -24 \cdot \frac{3}{5} + \frac{24}{25} x + \frac{64 \cdot 4}{3 \cdot 5} = 2.7 + \frac{24}{25} x$$

$$N_z = -24 \cdot \frac{4}{5} + \frac{32}{25} x - \frac{64 \cdot 3}{3 \cdot 5} = -32 + \frac{32}{25} x$$

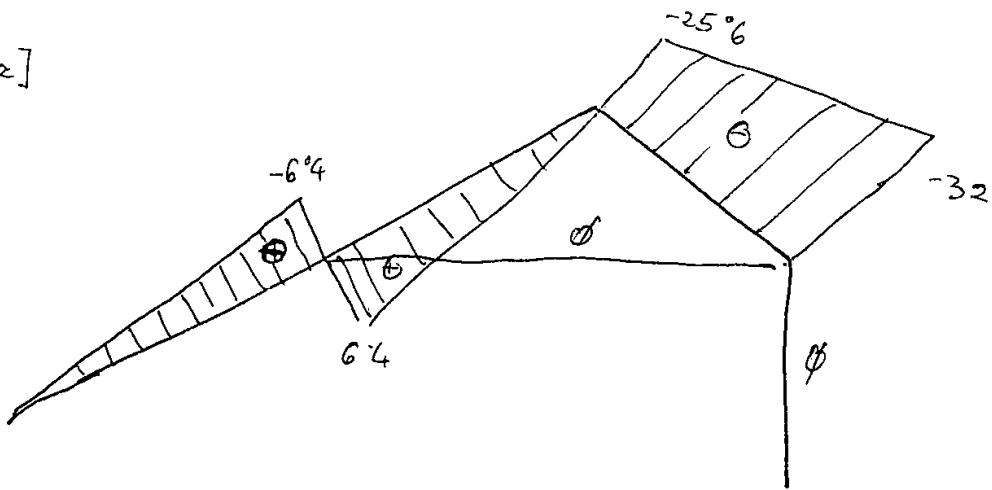
$$M_y = -144 + 24 \cdot \frac{4}{5} x - \frac{16}{25} x^2 + \frac{64}{3} x = -144 + 32x - \frac{16}{25} x^2$$

	x=0	x=2=5
N_x	2.7	7.5
N_z	-32	-25.6
M_y	-144	0

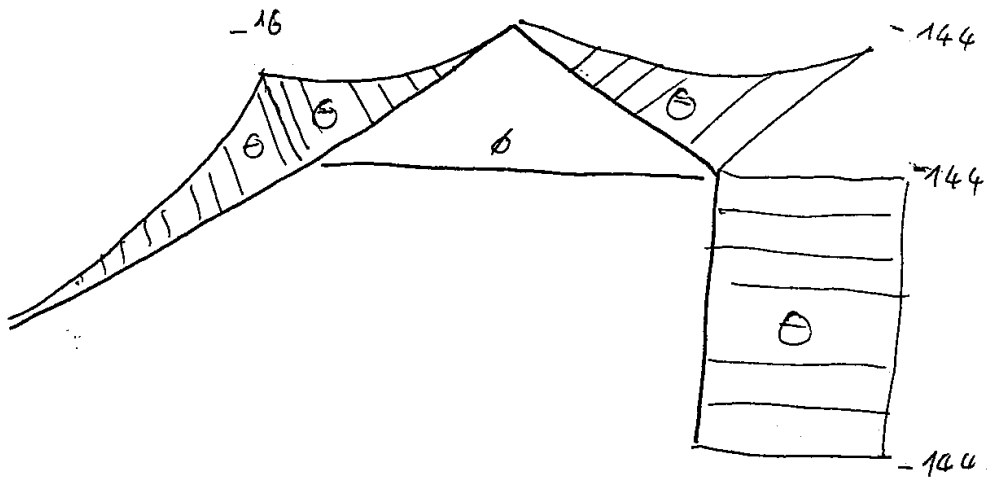
[N_x]



[N_z]



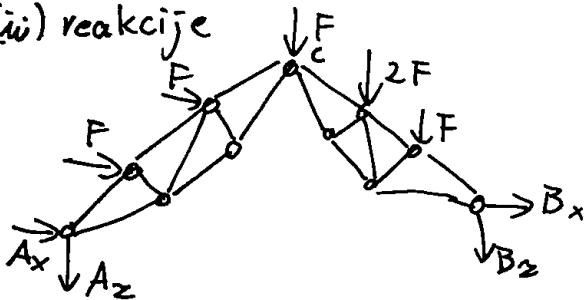
[M_y]



2. NALOGA

(i) $\sum \vec{m}_{P3} = 0$

(ii) reakcije



$$\sum X: A_x + B_x = -2F$$

$$\sum Z: A_z + B_z = -4F$$

$$\sum M^A: B_z \cdot 6a \cos \alpha + F \cdot 3a \cos \alpha + 2F \cdot 4a \cos \alpha + F \cdot 5a \cos \alpha + F \cdot a \sin \alpha + F \cdot 2a \sin \alpha = 0$$

$$6B_z \cos \alpha = -16F \cdot \cos \alpha - 3F \cdot \sin \alpha$$

$$B_z = -\frac{8}{3}F - \frac{1}{2}F \cdot \tan \alpha$$

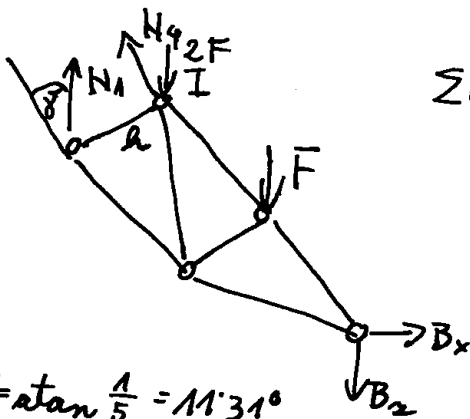
$$B_z = -14.8 \text{ kN} \quad A_z = -5.2 \text{ kN}$$

$$\sum M^C: A_x \cdot 3a \cdot \sin \alpha + A_z \cdot 3a \cos \alpha + F \cdot 2a \sin \alpha + F \cdot a \sin \alpha = 0$$

$$A_x = -(A_z \cot \alpha + F)$$

$$A_x = 4.05 \text{ kN} \quad B_x = -14.05 \text{ kN}$$

(iii) palica ①



$$\gamma = \arctan \frac{1}{5} = 11.31^\circ$$

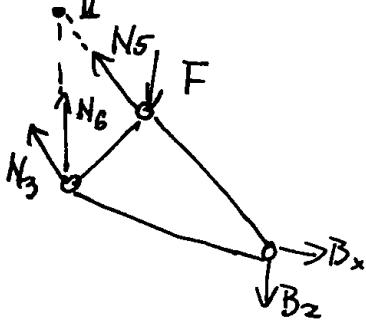
$$-F \cdot a \cdot \cos \alpha$$

$$\sum M^I: B_x \cdot 2a \sin \alpha - B_z \cdot 2a \cos \alpha - N_1 \cdot \cos \gamma \cdot h = 0$$

$$N_1 = \frac{12.5}{\cos \gamma} \left(B_x \cdot \frac{1}{2} - B_z \cdot \frac{\sqrt{3}}{2} - F \cdot \frac{1}{2} \cdot \frac{\sqrt{3}}{2} \right)$$

$$N_1 = 36.9 \text{ kN}$$

(iv) palica (3)

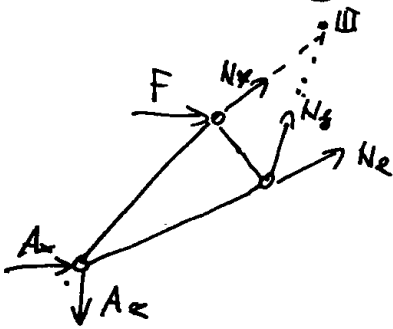


$$\Sigma M^{II}: B_x \cdot 2a \cdot \sin \alpha - B_z \cdot 2a \cdot \cos \alpha - F \cdot a \cdot \cos \alpha - N_3 \cdot h = 0$$

$$N_3 = 5 \cdot (B_x \cdot 2 \cdot \frac{1}{2} - B_z \cdot 2 \cdot \frac{\sqrt{3}}{2} - F \cdot \frac{\sqrt{3}}{2})$$

$$N_3 = 36.3 \text{ KN}$$

(v) palica (2)



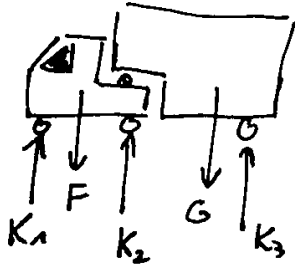
$$\Sigma M^{III}: A_x \cdot 2a \cdot \sin \alpha + A_y \cdot 2a \cdot \cos \alpha + N_2 \cdot h + F \cdot a \cdot \sin \alpha = 0$$

$$N_2 = -5 (A_x \cdot 2 \cdot \frac{1}{2} + A_y \cdot 2 \cdot \frac{\sqrt{3}}{2} + F \cdot \frac{1}{2})$$

$$N_2 = 12.3 \text{ KN}$$

3. NALOGA

a.) silc v kolesih



$$\sum Z: K_1 + K_2 + K_3 = F + G$$

$$\sum M^{\odot}: -F \cdot \frac{b}{2} + K_2 \cdot b - G \cdot c + K_3 \cdot (b+c) = 0$$

$$\sum M^{\vee}: -G(c-b+d) + K_3(c+d) = 0$$

$$K_3 = G \frac{c+d-b}{c+d}$$

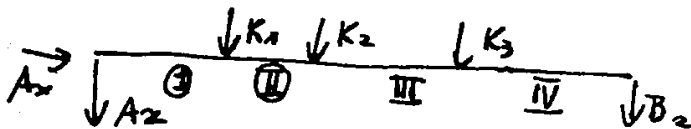
$$K_3 = 150 \cdot \frac{6.5}{10.5} \Rightarrow \boxed{K_3 = 92.9 \text{ kN}}$$

$$K_2 = \frac{1}{b} (F \cdot \frac{b}{2} + G \cdot c - K_3 \cdot (b+c))$$

$$\boxed{K_2 = 65 \text{ kN}}$$

$$\boxed{K_1 = 22.1 \text{ kN}}$$

b.) Obtežba na mostu



$$\sum x \quad \boxed{A_x = 0}$$

$$\sum Z: A_z + B_z = -(K_1 + K_2 + K_3) = -180 \text{ kN}$$

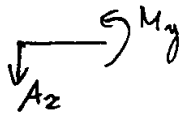
$$\sum M^{\oplus}: K_1 \cdot a + K_2 \cdot (a+b) + K_3 \cdot (a+b+c) + B_z \cdot (a+b+c+a) = 0$$

$$B_z = -\frac{1}{44} (15K_1 + 19K_2 + 29K_3)$$

$$\boxed{B_z = -96.8 \text{ kN}}$$

$$\boxed{A_z = -83.2 \text{ kN}}$$

a.) notranje sile :
polje I

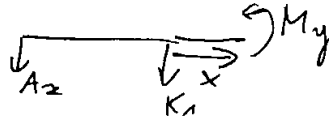


$$M_y = -A_z x$$

$$M_y = 832x$$

$$M_y(15) = 12475$$

polje II

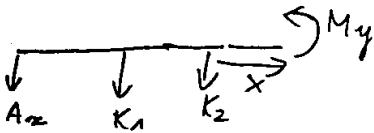


$$M_y = 12475 - K_1 x - A_z x$$

$$M_y = 12475 + 61.1x$$

$$M_y(4) = 14919$$

polje III

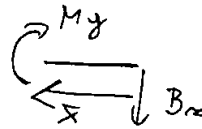


$$M_y = 14919 - (A_z + K_1 + K_2)x$$

$$M_y = 14919 - 39x$$

$$M_y(10) = 14529$$

polje IV

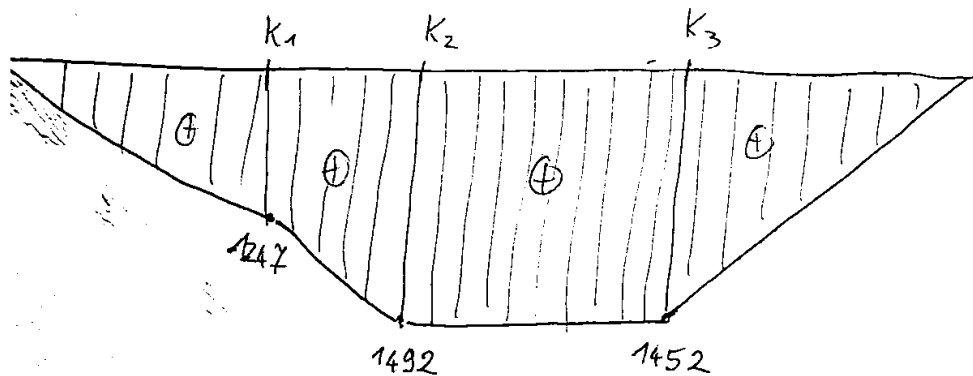


$$M_y = -B_z x$$

$$M_y = 968x$$

$$M_y(15) = 1452 \text{ kNm}$$

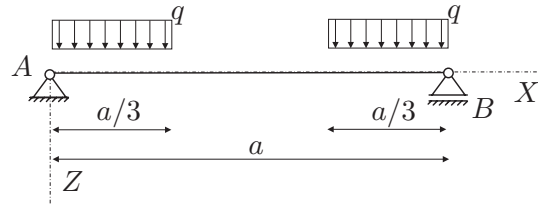
d.) diagrami



STATIKA (UNI) - 2. IZPITNI ROK (27. 6. 2005)

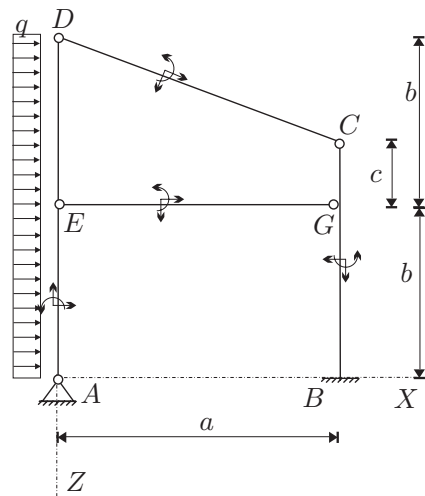
RAČUNSKI DEL IZPITA:

1. Za nosilec na sliki izračunajte reakcije v podporah ter izračunajte in prikažite diagrame notranjih statičnih količin! (OBVEZNA NALOGA! 20%)



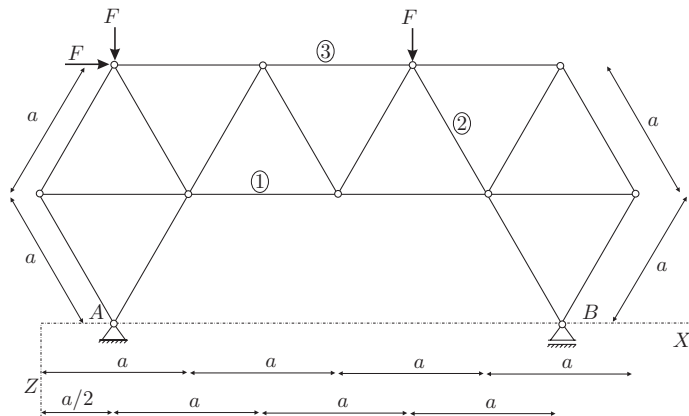
2. Za konstrukcijo na sliki izračunajte stopnjo statične nedoločenosti, reakcije in notranje statične količine (N_x, N_z, M_y)! Rezultate notranjih statičnih količin prikažite z diagrami!

Podatki: $a = 4\text{ m}$, $b = 3\text{ m}$, $c = 1\text{ m}$,
 $q = 10\text{ kN/m}$.
 (OBVEZNA NALOGA! 50%)



3. Za palično konstrukcijo na sliki izračunajte stopnjo statične nedoločenosti in osne sile v palicah 1, 2 in 3! (30%)

Podatki: $a = 2\text{ m}$, $F = 5\text{ kN}$.



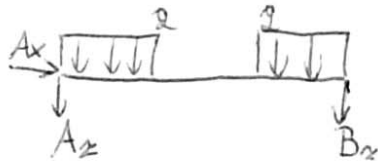
TEORETIČNI DEL IZPITA:

Izmed treh zastavljenih vprašanj si izberete dve, na kateri boste odgovarjali. Izbrani vprašanja jasno označite! Pišite čitljivo.

1. Izpeljite in opišite nadomestne ravnotežne pogoje (razumevanje podkrepite s preprostim primerom)!
2. Zapišite izraz za računsko število prostostnih stopenj sistema togih teles (razumevanje podkrepite s preprostim primerom)!
3. Opišite določanje reakcij in notranjih sil statično določenih linijskih konstrukcij z izrekom o virtualnih pomikih (pomagajte si s primerom)!

1. NALOGA

a.) reakcije

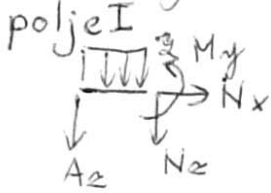


$$\Sigma X: A_x = 0$$

$$\Sigma Z: A_z + B_z = -\frac{q \cdot 2a}{3}$$

$$\Sigma M^A: -B_z \cdot a - q \cdot \frac{a}{3} \cdot \frac{a}{6} - q \cdot \frac{a}{3} \cdot \frac{5a}{6} = 0$$

b.) notranje sile



$$N_x = 0$$

$$N_z = -A_z - q \cdot x \Rightarrow N_z = \frac{q \cdot a}{3} - q \cdot x \quad N_z(0) = \frac{q \cdot a}{3} \quad N_z\left(\frac{a}{3}\right) = 0$$

$$M_y = -A_z \cdot x - q \cdot x \cdot \frac{x}{2} \Rightarrow M_y = \frac{q \cdot a}{3} \cdot x - \frac{q \cdot x^2}{2} \quad M_y(0) = 0$$

$$M_y\left(\frac{a}{3}\right) = \frac{q \cdot a^2}{18}$$

$$B_z = -\frac{q \cdot a}{3} \quad A_z = -\frac{q \cdot a}{3}$$

polje II



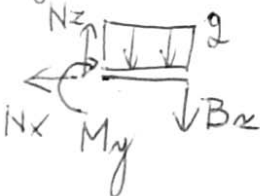
$$N_x = 0$$

$$N_z = -A_z - q \cdot \frac{a}{3} \quad N_z = 0$$

$$M_y = -A_z \left(\frac{a}{3} + x\right) - q \cdot \frac{a}{3} \cdot \left(\frac{a}{3} + x\right) = \frac{q \cdot a^2}{18}$$

$$M_y = \frac{q \cdot a^2}{18}$$

polje III



$$N_x = 0$$

$$N_z = B_z + q \cdot \bar{x} \quad N_z = -\frac{q \cdot a}{3} + q \cdot \bar{x}$$

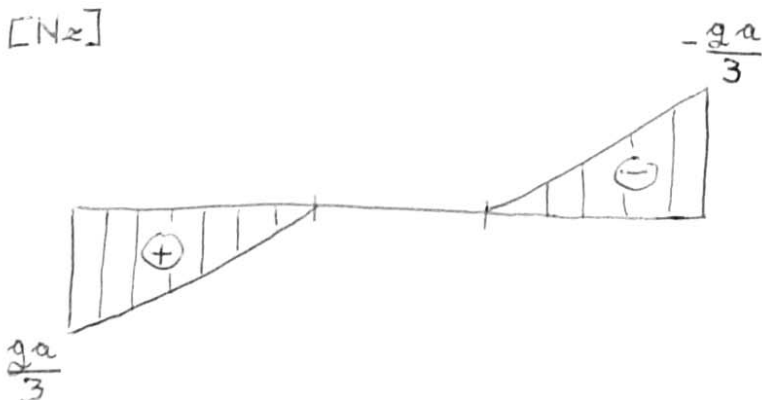
$$M_y = -B_z \cdot \bar{x} - q \cdot \bar{x} \cdot \frac{\bar{x}}{2} \quad M_y = \frac{q \cdot a}{3} \cdot \bar{x} - \frac{q \cdot \bar{x}^2}{2} \quad M_y(0) = 0$$

$$M_y\left(\frac{a}{3}\right) = \frac{q \cdot a^2}{18}$$

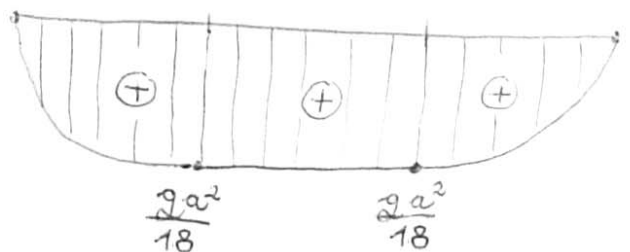
c.) diagrami

[Nz]

[Nz]



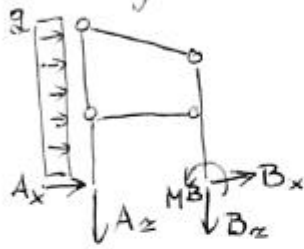
[My]



2. NALOGA

a.) $\tilde{m}_{ps} = 5 \cdot 3 - 2 - 3 - 3 \cdot 2 - 4 = 0$

b.) reakcije



$\sum X: A_x + B_x = -q \cdot 2b$

$B_x = -45 \text{ kN}$

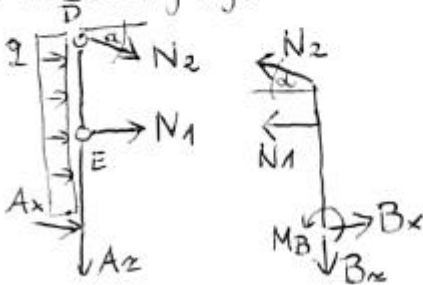
$\sum Z: A_z + B_z = 0$

$\sum M^A: M_B - B_z \cdot a - q \cdot 2b \cdot b = 0$

$\sum M^E_{AE}: A_x \cdot b + q \cdot b \cdot \frac{b}{2} = 0$

$A_x = -q \frac{b}{2} = -15 \text{ kN}$

c.) razstavljanje



$\sum X: -N_1 - N_2 \cos \alpha + B_x = 0$
desni del

$\sum Z: -N_2 \sin \alpha + B_z = 0$
desni del

$\sum M^B: M_B + N_1 \cdot b + N_2 (b+c) \cdot \cos \alpha = 0$
desni del

$\sum M^E_{DE}: -N_2 \cos \alpha \cdot b - q \cdot b \cdot \frac{b}{2} = 0$
levi del

$N_2 = -\frac{qb}{2 \cos \alpha}$

$\tan \alpha = \frac{2}{4} \Rightarrow \alpha = 26.565^\circ$

$N_2 = -16.8 \text{ kN}$

$N_1 = -30 \text{ kN}$

$B_z = -7.5 \text{ kN}$

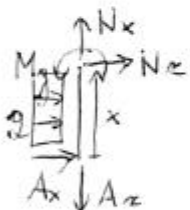
$M_B = 150 \text{ kNm}$

$A_z = 7.5 \text{ kN}$

KONTROLA: $\sum M^A: 150 + 7.5 \cdot 4 - 10 \cdot 6 \cdot 3 = 0 \checkmark$

d) notranje sile

polje I



$N_x = A_z = 7.5 \text{ kN}$

$N_z = -A_x - qx$

$N_z = 15 - 10x$

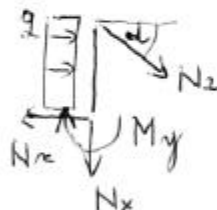
$M_y = -A_x x - q x \cdot \frac{x}{2}$

$M_y = 15x - 5x^2$

$M_y(0) = M_y(3) = 0$

$M_y(\frac{3}{2}) = 11.25 \text{ kNm}$
(ekstrem)

polje II



$N_x = -N_2 \sin \alpha = 7.5 \text{ kN}$

$N_z = N_2 \cos \alpha + qx$

$N_z = -15 + 10x$

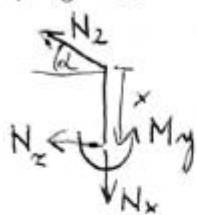
$M_y = -N_2 \cos \alpha x - qx \cdot \frac{x}{2}$

$M_y = 15x - 5x^2$

$M_y(0) = M_y(3) = 0$

$M_y(\frac{3}{2}) = 11.25 \text{ kNm}$ (ekstrem)

polje III



$$N_x = N_2 \sin \alpha$$

$$N_x = -7.5 \text{ kN}$$

$$N_z = -N_2 \cos \alpha$$

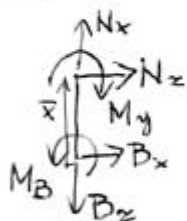
$$N_z = 15 \text{ kN}$$

$$M_y = -N_2 \cos \alpha \cdot x$$

$$M_y = 15x$$

$$M_y(0) = 0 \quad M_y(1) = 15 \text{ kNm}$$

polje IV



$$N_x = B_z$$

$$N_x = -7.5 \text{ kN}$$

$$N_z = -B_x$$

$$N_z = 45 \text{ kN}$$

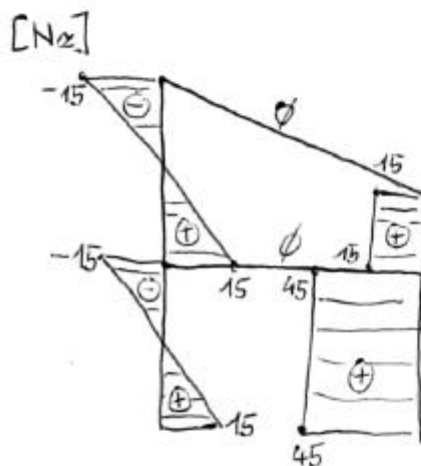
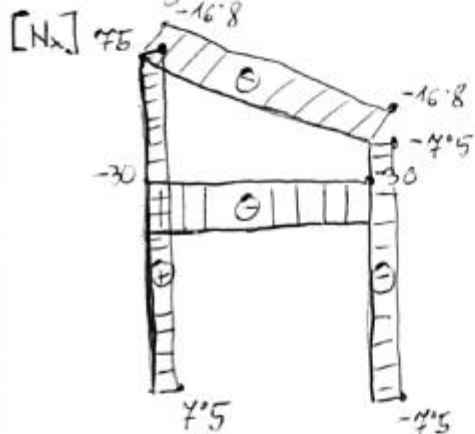
$$M_y = B_x x + M_B$$

$$M_y = 150 - 45x$$

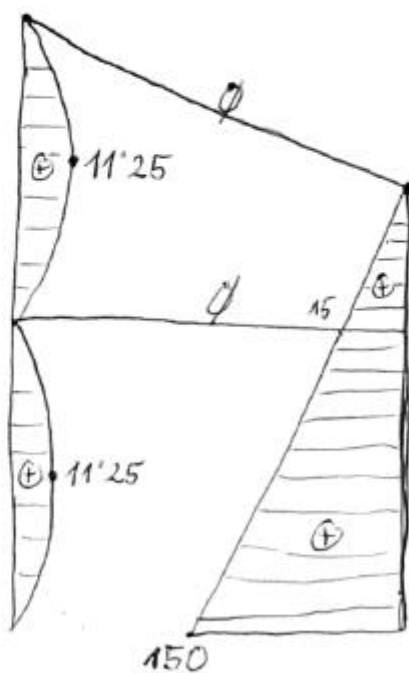
$$M_y(0) = 150 \text{ kNm}$$

$$M_y(3) = 15 \text{ kNm}$$

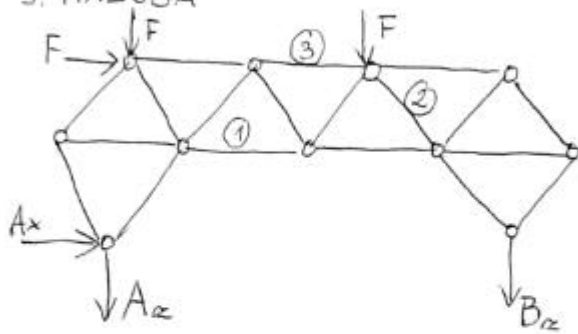
e) diagrami



[My]



3. NALOGA



reakcije

$$\sum X: A_x + F = 0$$

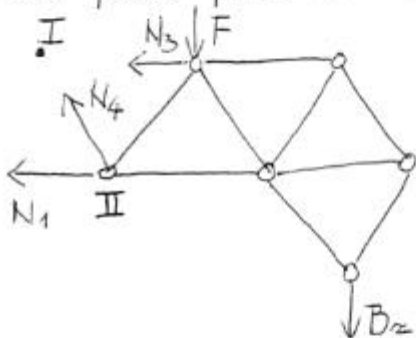
$$A_x = -5 \text{ kN}$$

$$\sum Z: A_z + B_z + 2F = 0$$

$$\sum M^A: -B_z \cdot 3a - F \cdot 2a - F \cdot 2a \frac{\sqrt{3}}{2} = 0$$

$$B_z = -6.22 \text{ kN} \quad A_z = -3.78 \text{ kN}$$

rez preko palice ① in ③



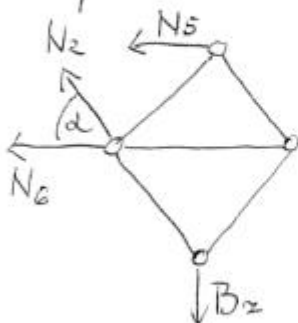
$$\sum M^I: -F a - B_z \cdot 2a - N_1 \cdot a \cdot \frac{\sqrt{3}}{2} = 0$$

$$N_1 = \frac{2}{\sqrt{3}} (-F - 2B_z) \quad N_1 = 8.6 \text{ kN}$$

$$\sum M^{II}: -F \frac{a}{2} - B_z \cdot \frac{3a}{2} + N_3 \cdot a \frac{\sqrt{3}}{2} = 0$$

$$N_3 = \frac{1}{\sqrt{3}} (F + 3B_z) \quad N_3 = -7.9 \text{ kN}$$

rez palice ②



$$\sum Z: -N_2 \sin \alpha + B_z = 0$$

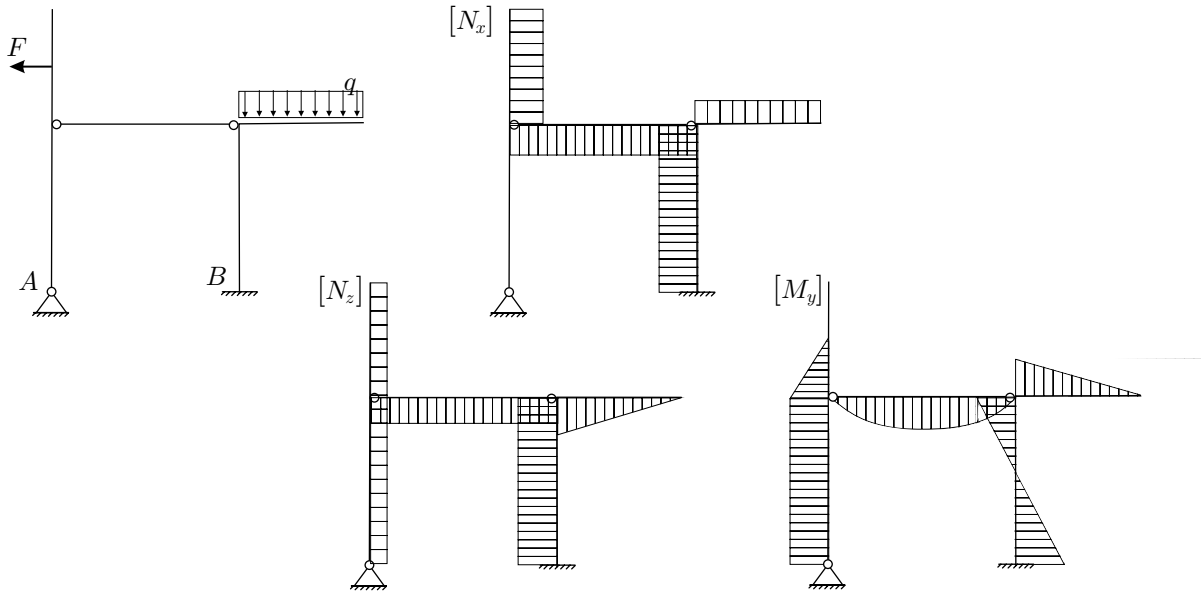
$$N_2 = \frac{B_z}{\sin 60^\circ}$$

$$N_2 = -7.2 \text{ kN}$$

STATIKA (UNI) - 3. IZPITNI ROK (1. 9. 2005)

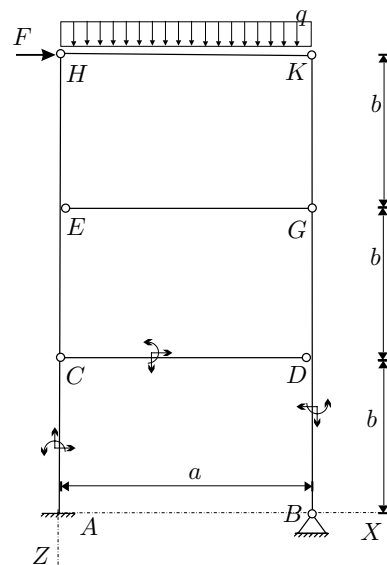
RAČUNSKI DEL IZPITA:

1. Janezek je na izpitu iz statike padel. Njegovi diagrami so polni napak. Pomagaj Janezku in poišči (BREZ RAČUNANJA) vse napake v spodnjih diagramih ! (OBVEZNA NALOGA! 25%)

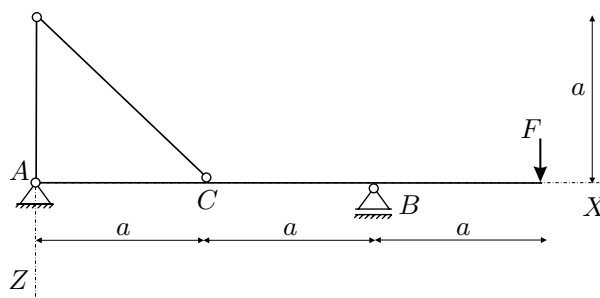


2. Za konstrukcijo na sliki izračunajte stopnjo statične nedoločeniosti, reakcije in notranje statične količine (N_x, N_z, M_y)! Rezultate notranjih statičnih količin prikažite z diagrami!

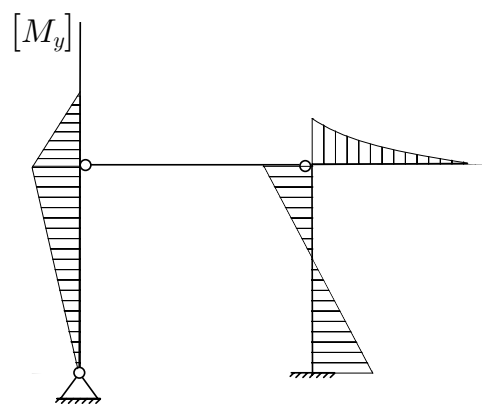
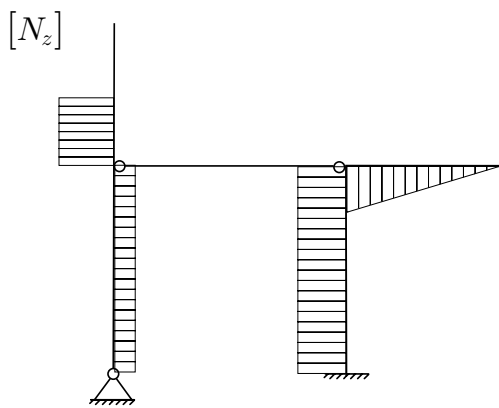
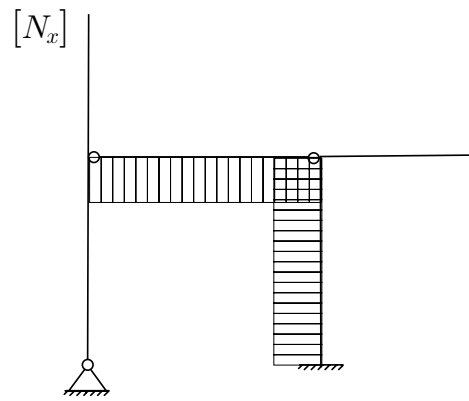
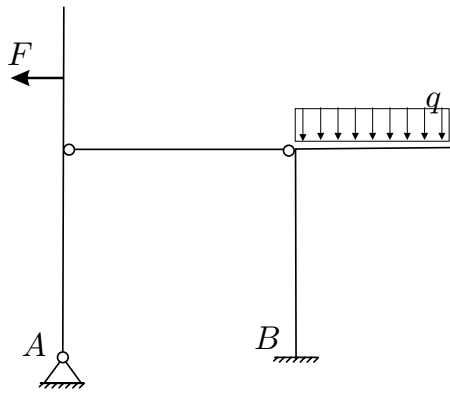
Podatki: $a = 4\text{ m}$, $b = 3\text{ m}$,
 $q = 15\text{ kN/m}$, $F = 10\text{ kN}$.
 (OBVEZNA NALOGA! 50%)



3. S principom virtualnega dela za konstrukcijo na sliki izračunajte reakcije v podporah in notranji moment v točki C – M_C ! (25%)



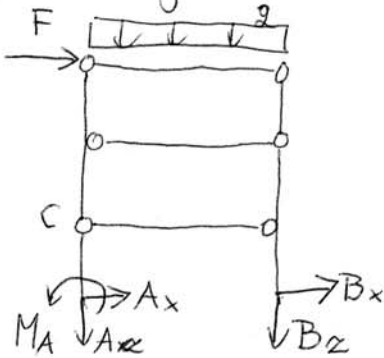
1. Naloga: PRAVILNI DIAGRAMI



2. NALOGA

a.) $\tilde{m}_{PS} = 7 \cdot 3 - 3 \cdot 2 - 4 \cdot 2 - 2 \cdot 2 \cdot (3-1) = 0$

b.) reakcije



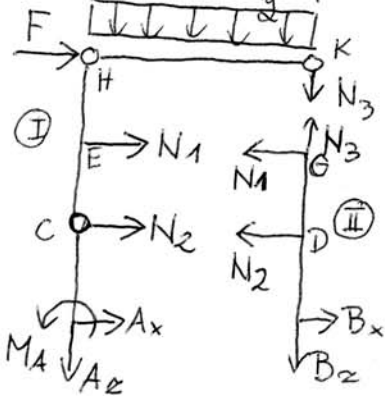
$\Sigma X: A_x + B_x + F = 0$

$\Sigma Z: A_z + B_z + q \cdot a = 0$

$\Sigma M^A: M_A - B_z \cdot a - F \cdot 3b - q \cdot a \cdot \frac{a}{2} = 0$

$\Sigma M^C: M_A + A_x \cdot b = 0$

izrežemo vse palice



$\Sigma M^H: -N_3 \cdot a - q \cdot a \cdot \frac{a}{2} = 0$

$N_3 = -q \frac{a^2}{2}$ $N_3 = -30 \text{ kN}$

$\Sigma Z^{\oplus}: -N_3 + B_z = 0$

$B_z = -30 \text{ kN}$

$A_z = -30 \text{ kN}$

$M_A = 90 \text{ kNm}$

$A_x = -30 \text{ kN}$ $B_x = 20 \text{ kN}$

$\Sigma X^{\oplus}: -N_1 - N_2 + B_x = 0$

$\Sigma M^{B^{\oplus}}: N_1 \cdot 2b + N_2 \cdot b = 0$

$N_1 = -\frac{N_2}{2}$

$-\frac{N_2}{2} = -B_x$

$N_2 = 40 \text{ kN}$

$N_1 = -20 \text{ kN}$

c.) notranje sile

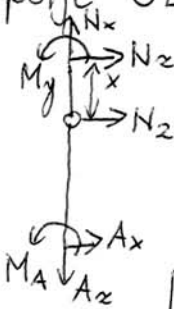
⊕ polje AC



$N_x = A_z$
 $N_z = -A_x$
 $M_y = -M_A - A_x \cdot x$

$N_x = -30 \text{ kN}$
 $N_z = 30 \text{ kN}$
 $M_y = -90 + 30x$

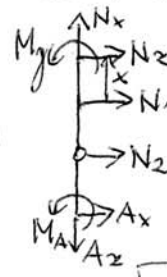
⊕ polje CE



$N_x = A_z$
 $N_z = -A_x - N_2$
 $M_y = -M_A - A_x(b+x) - N_2x$

$N_x = -30 \text{ kN}$
 $N_z = -10 \text{ kN}$
 $M_y = -10x$

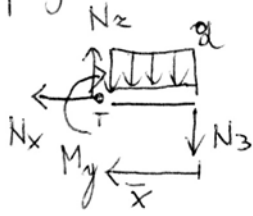
⊕ polje EH



$N_x = A_z$
 $N_z = -A_x - N_1 - N_2$
 $M_y = -M_A - A_x(2b+x) - N_1x - N_2(b+x)$

$N_x = -30 \text{ kN}$
 $N_z = 10 \text{ kN}$
 $M_y = -30 + 10x$

Ⓓ polje HK



$$\sum x \quad N_x = 0$$

$$\sum z: -N_z + N_3 + q\bar{x} = 0$$

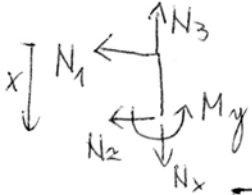
$$\sum M^T: -M_y - N_3 \cdot \bar{x} - q\bar{x} \frac{\bar{x}}{2} = 0$$

$$N_z = -30 + 15\bar{x}$$

$$M_y = 30\bar{x} - 7.5\bar{x}^2$$

$$M_y(2) = 30 \text{ kNm [Extrem]}$$

Ⓔ polje GD



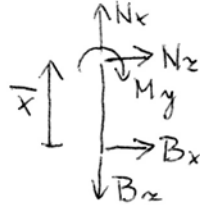
$$N_x = N_3$$

$$N_z = -N_1$$

$$M_y = -N_1 x$$

$N_x = -30 \text{ kN}$
$N_z = 20 \text{ kN}$
$M_y = 20x$

Ⓕ polje BD



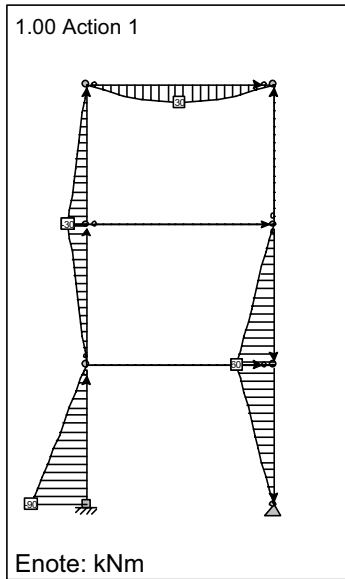
$$N_x = B_z$$

$$N_z = -B_x$$

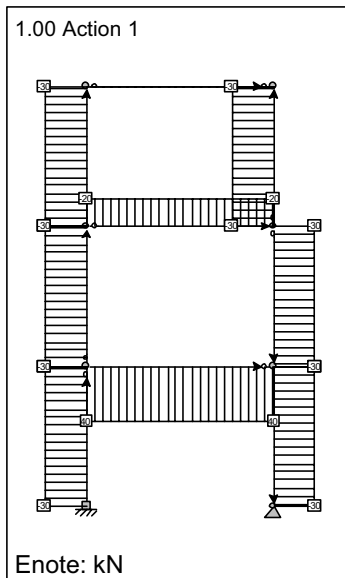
$$M_y = B_x \cdot \bar{x}$$

$N_x = -30 \text{ kN}$
$N_z = -20 \text{ kN}$
$M_y = 20\bar{x}$

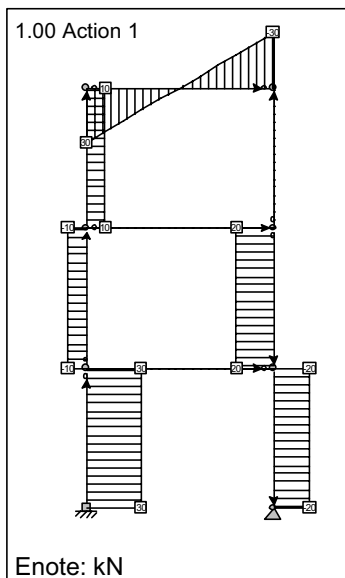
LC1: Load case 2: Upogibni moment M_y



LC1: Load case 2: Osna sila F_x

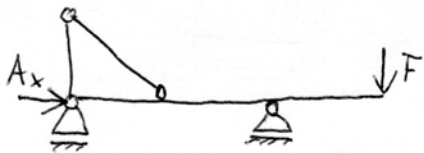


LC1: Load case 2: Preèna sila F_z



3. NALOGA

a.) Reakcija A_x



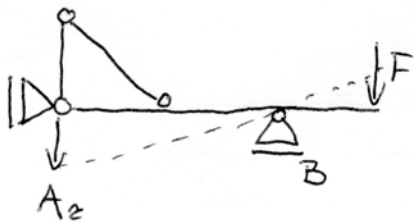
$$\delta W = A_x \delta u_A + F \delta w_F = 0$$

$$\delta w_F = 0$$

$$A_x \delta u_A = 0$$

$$\boxed{A_x = 0}$$

b.) Reakcija A_z



$$\delta W = A_z \delta w_A + F \delta w_F = 0$$

$$\delta w_F = \delta w_A = 0$$

$$\delta w_F = \delta w_A - 3a \delta \varphi_A$$

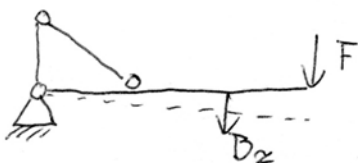
$$\delta w_B = \delta w_A - 2a \delta \varphi_A \Rightarrow \delta w_A = 2a \delta \varphi_A$$

$$\boxed{\delta w_F = -\frac{1}{2} \delta w_A}$$

$$\delta W = A_z \delta w_A - \frac{F}{2} \delta w_A = 0$$

$$\boxed{A_z = \frac{F}{2}}$$

c.) Reakcija B_z



$$\delta W = B_z \delta w_B + F \delta w_F = 0$$

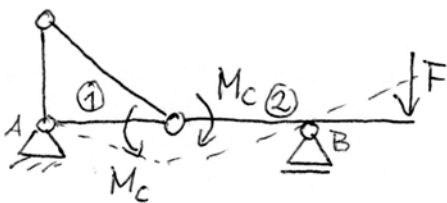
$$\delta w_B = \delta w_A - 2a \delta \varphi_A$$

$$\delta w_F = \delta w_A - 3a \delta \varphi_A$$

$$\delta w_F = \frac{3}{2} \delta w_B$$

$$B_z \delta w_B + \frac{3}{2} F \delta w_B = 0 \Rightarrow \boxed{B_z = -\frac{3}{2} F}$$

d.) Notranji moment M_c



$$\delta W = M_c \delta \varphi_c^{(1)} - M_c \delta \varphi_c^{(2)} + F \delta w_F = 0$$

$$\delta \varphi_c^{(1)} = \delta \varphi_A$$

$$\delta u_c = \delta u_A = 0$$

$$\delta w_c = \delta w_A - a \delta \varphi_A$$

$$\delta u_B = \delta u_c = 0$$

$$\delta w_B = \delta w_c - a \delta \varphi_c^{(2)}$$

$$\boxed{\delta \varphi_c^{(2)} = -\delta \varphi_c^{(1)}}$$

$$\delta w_F = \delta w_B - a \delta \varphi_B$$

$$\delta \varphi_B = \delta \varphi_c^{(2)}$$

$$\boxed{\delta w_F = -a \delta \varphi_c^{(2)}}$$

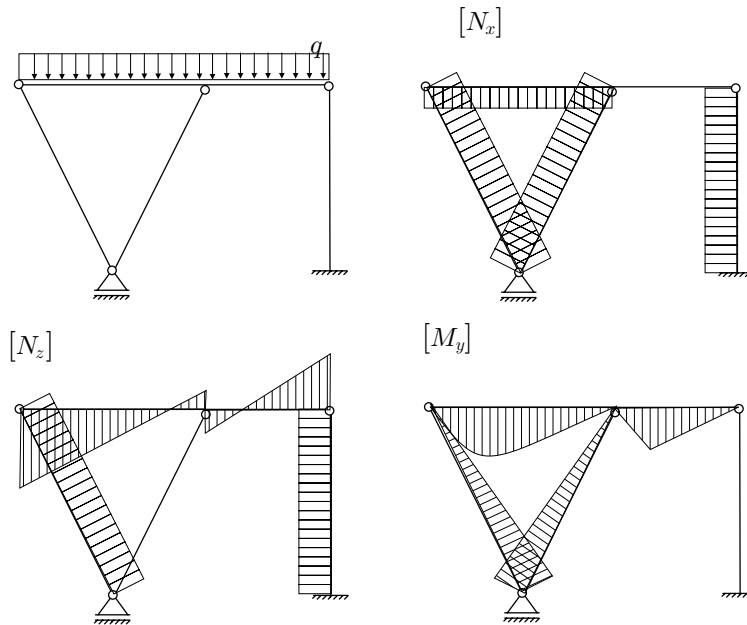
$$M_c \delta \varphi_c^{(1)} + M_c \delta \varphi_c^{(1)} + a F \delta \varphi_c^{(1)} = 0$$

$$\boxed{M_c = -\frac{aF}{2}}$$

STATIKA (UNI) - 4. IZPITNI ROK (15. 9. 2005)

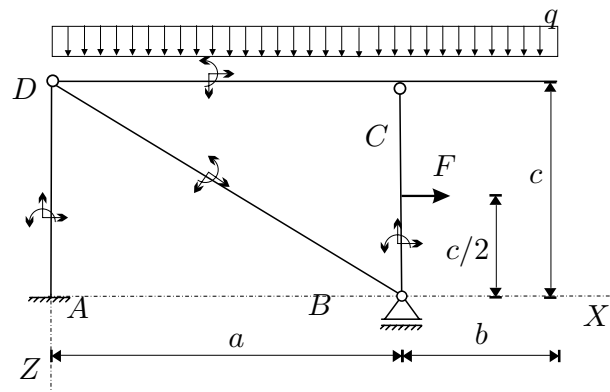
RAČUNSKI DEL IZPITA:

1. Janezek je na izpitu iz statike padel. Njegovi diagrami so polni napak. Pomagaj Janezku in poišči (BREZ RAČUNANJA) vse napake v spodnjih diagramih ! (OBVEZNA NALOGA! 25%)

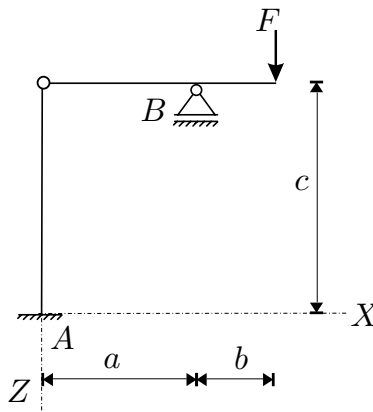


2. Za konstrukcijo na sliki izračunajte stopnjo statične nedoločенosti, reakcije in notranje statične količine (N_x, N_z, M_y)! Rezultate notranjih statičnih količin prikažite z diagrami!

Podatki: $a = 5 \text{ m}$, $b = 2 \text{ m}$,
 $c = 3 \text{ m}$,
 $q = 2 \text{ kN/m}$, $F = 10 \text{ kN}$.
 (OBVEZNA NALOGA! 50%)



3. S principom virtualnega dela za konstrukcijo na sliki izračunajte reakcije v podporah in notranji moment v točki B – M_B ! (25%)

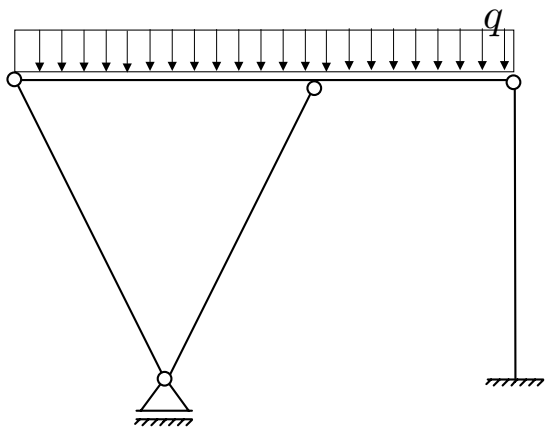


TEORETIČNI DEL IZPITA:

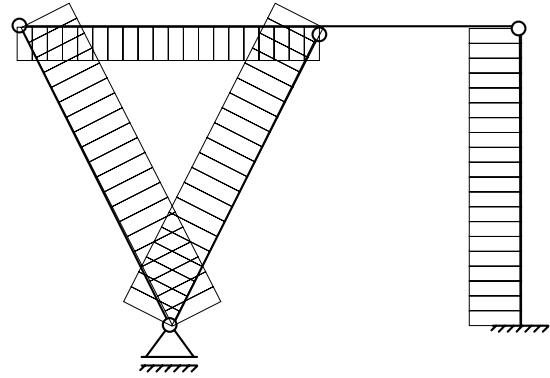
Izmed treh zastavljenih vprašanj si izberete dve, na kateri boste odgovarjali. Izbrani vprašanji jasno označite! Pišite čitljivo.

1. Vzporedna prestavitev sile! Kako lahko nadomestimo silo in moment, ki sta med seboj pravokotna? Dokaz!
2. Ravnotežni pogoji za linijski element z ravno osjo (izpeljava diferencialnih enačb)! Ravnotežne pogoje izpeljite za raven ravninski nosilec, ki je obtežen samo z linijsko obtežbo prečno na os nosilca! Kaj so statični robni pogoji pri prostoležečem nosilcu, ki je obtežen samo z linijsko obtežbo?
3. Opišite določanje reakcij in notranjih sil statično določenih linijskih konstrukcij z izrekom o virtualnih pomikih! Kot ilustracijo, izračunajte reakcije in notranje statične količine v enem prečnem prerezu na Gerberjevem nosilcu s konstantno prečno linijsko obtežbo!

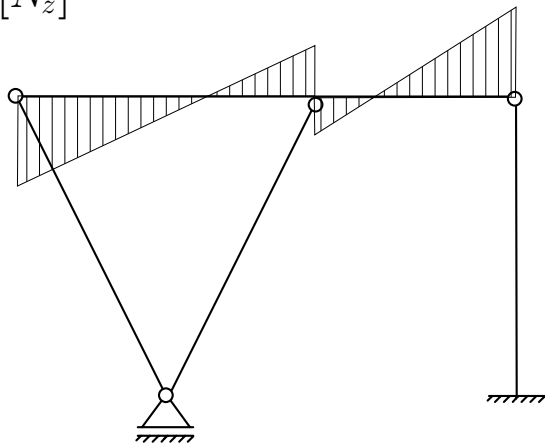
1. Naloga: PRAVILNI DIAGRAMI



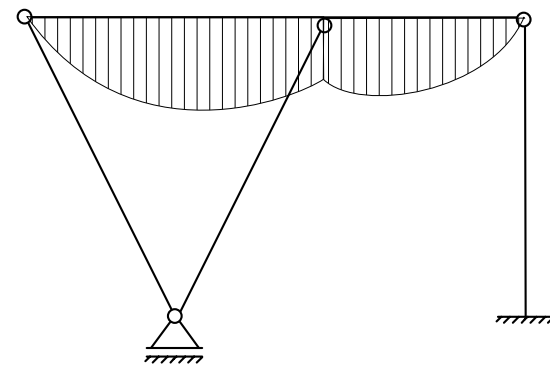
$[N_x]$



$[N_z]$



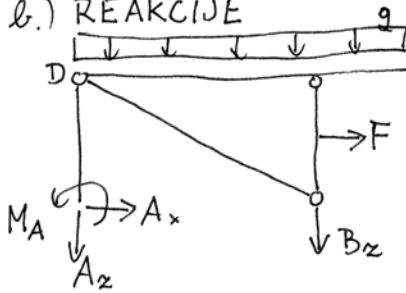
$[M_y]$



2. NALOGA

a.) $\tilde{n}_{ps} = 3 \cdot 4 - 3 - 1 - 2 \cdot 2 - 2 \cdot (3 - 1) = 0$

b.) REAKCIJE



$\sum X: A_x + F = 0 \quad \boxed{A_x = -10 \text{ kN}}$

$\sum Z: A_z + B_z + g(a+b) = 0$

$\sum M^A: -B_z \cdot a - F \cdot \frac{c}{2} - g(a+b) \frac{a+b}{2} + M_A = 0$

DODATNA ENAČBA

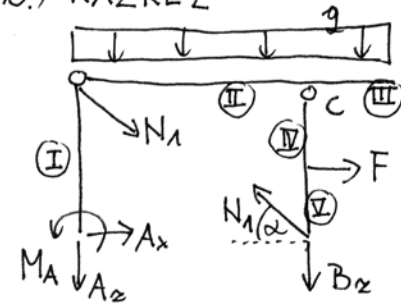
$\sum M^D: A_x \cdot c + M_A = 0$

$\boxed{M_A = 30 \text{ kNm}}$

$\boxed{B_z = -6.8 \text{ kN}}$

$\boxed{A_z = -7.2 \text{ kN}}$

c.) RAZREZ



$\sum M^C: F \cdot \frac{c}{2} - N_1 \cos \alpha \cdot c = 0$

BC

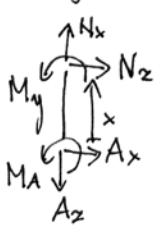
$N_1 = \frac{F}{2 \cos \alpha}$

$\boxed{N_1 = 5.83 \text{ kN}}$

$\tan \alpha = \frac{3}{5}$

d.) NOTRANJE SILE PO POLJIH

polje I $x \in [0, 3]$



$\sum X: N_x = A_z$
 $\sum Z: N_z = -A_x$
 $\sum M: M_y = -M_A - A_x \cdot x$

$\boxed{N_x = -7.2 \text{ kN}} \quad \boxed{N_z = 10 \text{ kN}}$
 $\boxed{M_y = -30 + 10x}$

polje II $x \in [0, 5]$

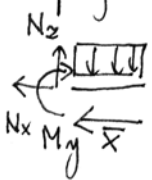


$\sum X: N_x = -A_x - N_1 \cos \alpha$
 $\sum Z: N_z = -A_z - g \cdot x - N_1 \sin \alpha$
 $\sum M: M_y = -M_A - g \cdot x \cdot \frac{x}{2} - A_z \cdot x - N_1 \sin \alpha \cdot x - A_x \cdot c$

$\boxed{N_x = 5 \text{ kN}} \quad \boxed{N_z = 4.2 - 2x}$
 $\boxed{M_y = 4.2x - x^2}$

$x = 2.1$ (extrem)
 $\boxed{M_y(2.1) = 4.4 \text{ kNm}}$

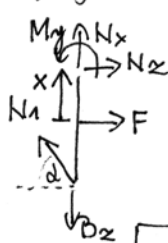
polje III $\bar{x} \in [0, 2]$



$\sum X: N_x = 0$
 $\sum Z: N_z = g \cdot \bar{x}$
 $\sum M: M_y = -g \cdot \frac{\bar{x}^2}{2}$

$\boxed{N_x = 0} \quad \boxed{N_z = 2 \bar{x}}$
 $\boxed{M_y = -\bar{x}^2}$
 $\boxed{M_y(2) = -4 \text{ kNm}}$

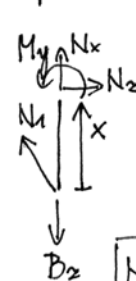
polje IV $x \in [0, 1.5]$



$\sum X: N_x = B_z - N_1 \sin \alpha$
 $\sum Z: N_z = -F + N_1 \cos \alpha$
 $\sum M: M_y = -F \cdot x + N_1 \cos \alpha \cdot (\frac{c}{2} + x)$

$\boxed{N_x = -9.8 \text{ kN}}$
 $\boxed{N_z = -5 \text{ kN}}$
 $\boxed{M_y = 7.5 - 5x}$
 $\boxed{M_y(1.5) = 0}$

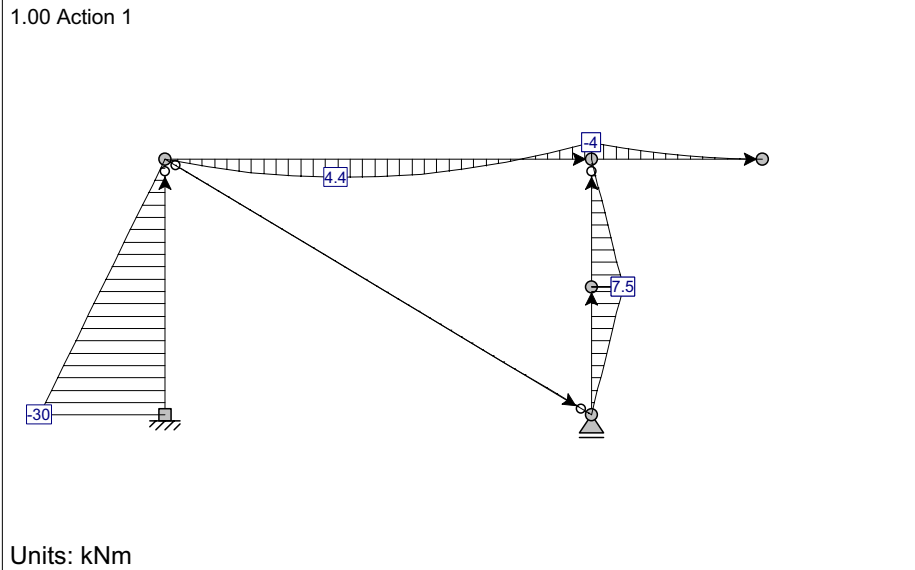
polje V $x \in [0, 1.5]$



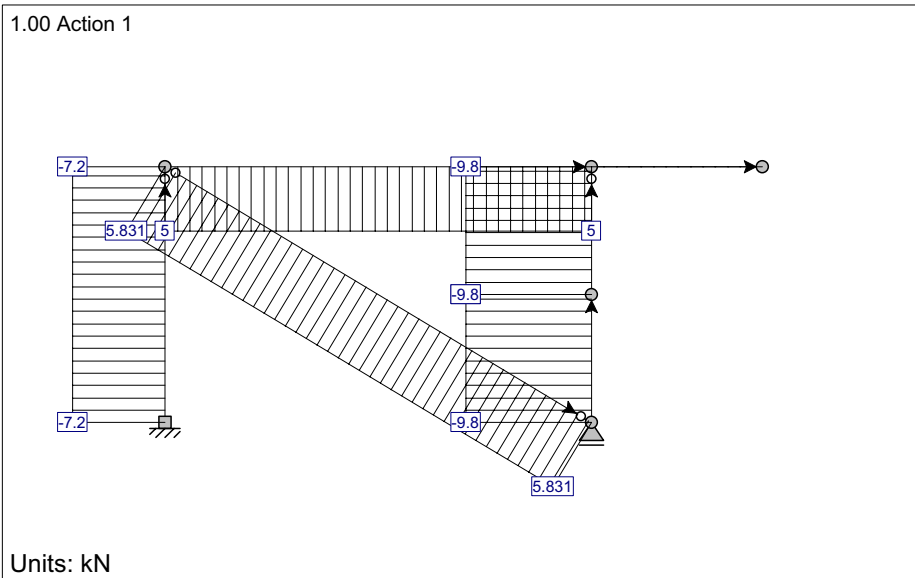
$\sum X: N_x = B_z - N_1 \sin \alpha$
 $\sum Z: N_z = +N_1 \cos \alpha$
 $\sum M: M_y = N_1 \cos \alpha \cdot x$

$\boxed{N_x = -9.8 \text{ kN}}$
 $\boxed{N_z = 5 \text{ kN}}$
 $\boxed{M_y = 5x}$
 $\boxed{M_y(1.5) = 7.5 \text{ kNm}}$

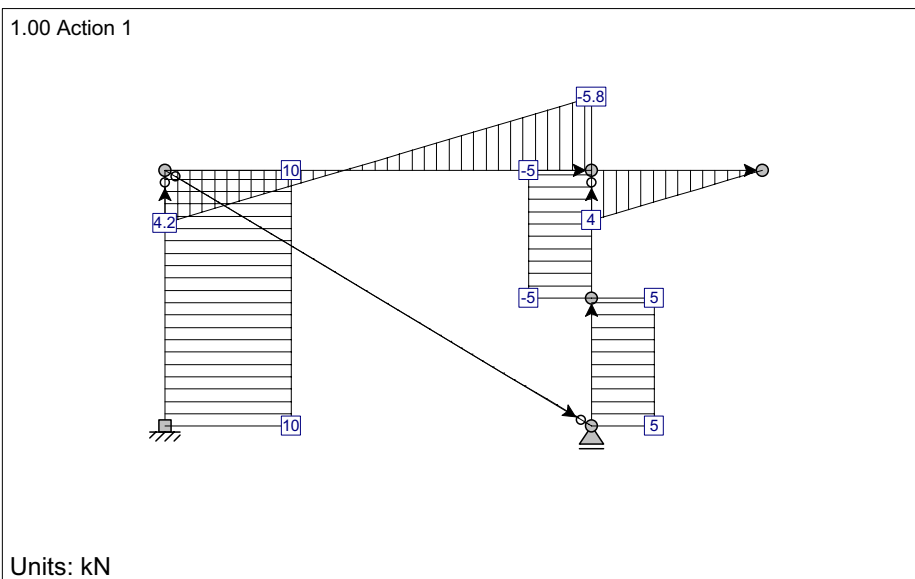
LC1: Load case 2: Bending Moments My



LC1: Load case 2: Axial Forces Fx

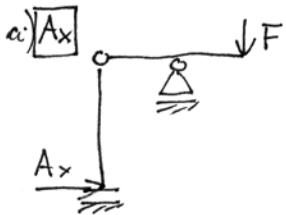


LC1: Load case 2: Shear Forces Fz



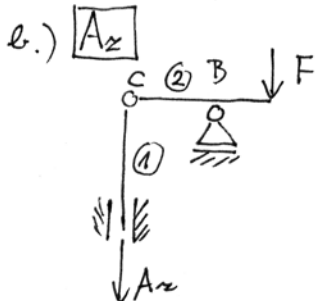
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3. NALOGA



$$\delta W = A_x \delta u_A + F \delta w_F = 0$$

$$\delta w_F = 0 \Rightarrow A_x \delta u_A = 0 \Rightarrow \boxed{A_x = 0}$$



$$\delta W = A_z \delta w_A + F \delta w_F = 0$$

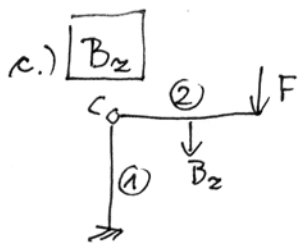
$$\delta w_c = \delta w_A$$

$$\delta w_F = \delta w_c - (a+b) \delta \varphi_c^{(2)}$$

$$\delta w_B = 0 \quad \delta w_B = \delta w_c - a \delta \varphi_c^{(2)} \quad \parallel \quad 0$$

$$\left. \begin{array}{l} \delta w_F = \delta w_A - \frac{a+b}{a} \delta w_A \\ \delta w_F = -\frac{b}{a} \delta w_A \end{array} \right\}$$

$$A_z \delta w_A - \frac{b}{a} \delta w_A \cdot F = 0 \Rightarrow \boxed{A_z = \frac{b}{a} F}$$



$$\delta W = B_z \delta w_B + F \delta w_F = 0$$

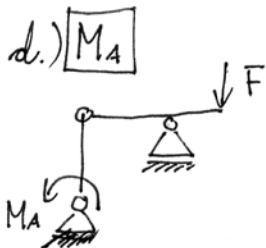
$$\delta w_c = 0$$

$$\delta w_B = \delta w_c - a \delta \varphi_c^{(2)}$$

$$\delta w_F = \delta w_c - (a+b) \delta \varphi_c^{(2)}$$

$$\left. \begin{array}{l} \delta w_F = +\frac{a+b}{a} \delta w_B \end{array} \right\}$$

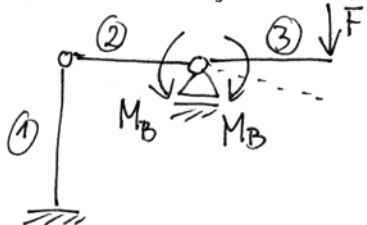
$$(B_z + F \frac{a+b}{a}) \delta w_B = 0 \Rightarrow \boxed{B_z = -\frac{a+b}{a} F}$$



$$\delta W = M_A \delta \varphi^{(1)} + F \delta w_F = 0$$

$$\delta w_F = 0 \Rightarrow \boxed{M_A = 0}$$

e) M_B (notranji moment)



$$\delta W = M_B \delta \varphi_B^{(2)} - M_B \delta \varphi_B^{(3)} + F \delta w_F = 0$$

$$\delta \varphi_B^{(2)} = 0$$

$$\delta w_F = \delta w_B - b \delta \varphi_B^{(3)} \quad \parallel \quad 0$$

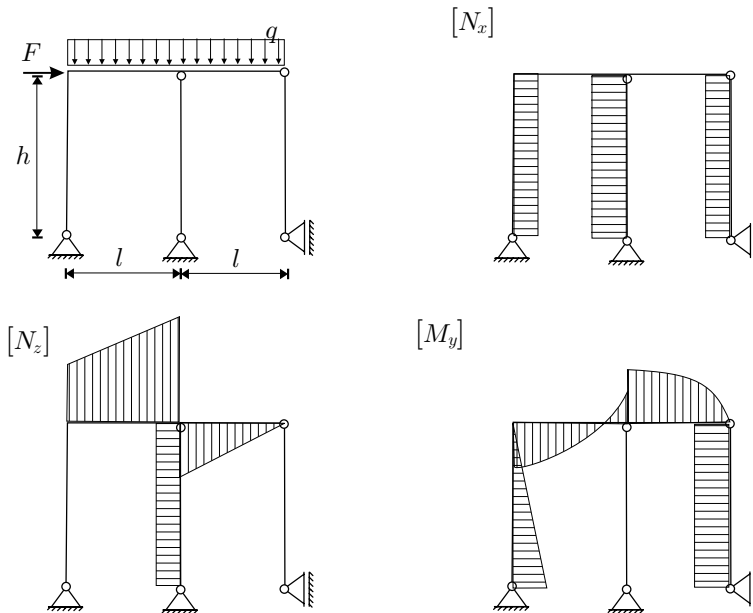
$$\boxed{-M_B \delta \varphi_B^{(3)} - F b \delta \varphi_B^{(3)} = 0}$$

$$\Rightarrow \boxed{M_B = -F b}$$

STATIKA (UNI) - 2. IZREDNI IZPITNI ROK (7. 12. 2005)

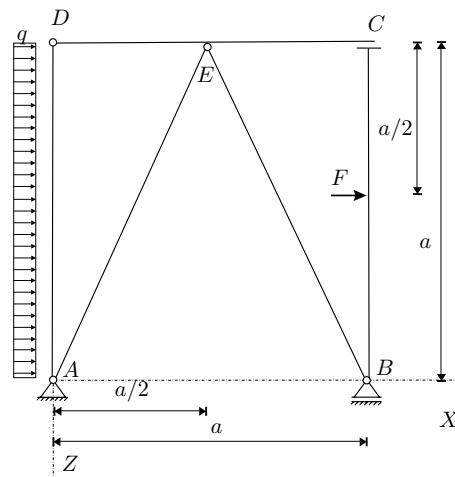
RAČUNSKI DEL IZPITA:

1. Janezek je na izpitu iz statike padel. Njegovi diagrami so polni napak. Pomagaj Janezku in poišči (BREZ RAČUNANJA) vse napake v spodnjih diagramih ! (OBVEZNA NALOGA! 25%)

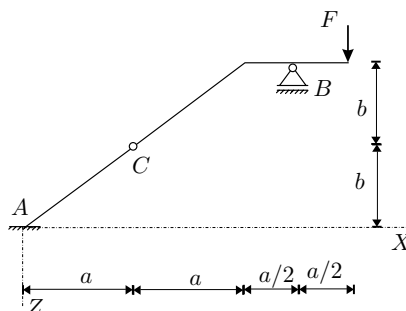


2. Za konstrukcijo na sliki izračunajte stopnjo statične nedoločenosti, reakcije in notranje statične količine (N_x, N_z, M_y)! Rezultate notranjih statičnih količin prikažite z diagrami!

Podatki: $a = 3 \text{ m}$, $q = 5 \text{ kN/m}$,
 $F = 10 \text{ kN}$.
 (OBVEZNA NALOGA! 50%)



3. S principom virtualnega dela za konstrukcijo na sliki izračunajte reakcije v podporah in notranji moment v točki B – M_B ! (25%)

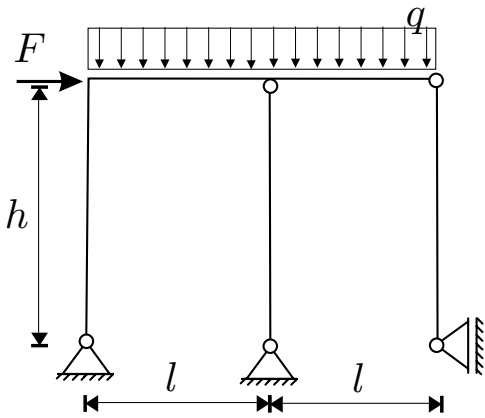


TEORETIČNI DEL IZPITA:

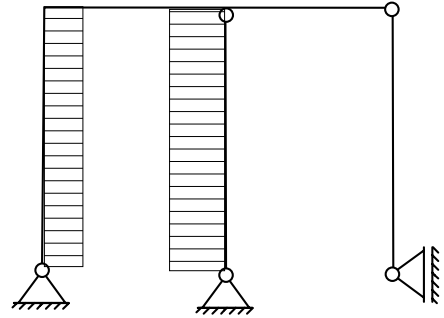
Izmed treh zastavljenih vprašanj si izberete dve, na kateri boste odgovarjali. Izbrani vprašanji jasno označite! Pišite čitljivo.

1. Računski modeli za opis medsebojnega vpliva med telesi!
2. Pomiki in zasuki togega telesa (izpeljava enačb za ravninsko gibanje togega telesa)!
3. Izpeljite in opišite izraz za število odvzetih prostostnih stopenj, ki jih vez odvzame k nepovezanim telesom! Obravnavajte tudi primer, ko imajo vsa telesa na mestu vezi enake nekatere kinematične količine, preostale kinematične količine pa so možne za vsa telesa! (ilustracija s karakterističnimi primeri)

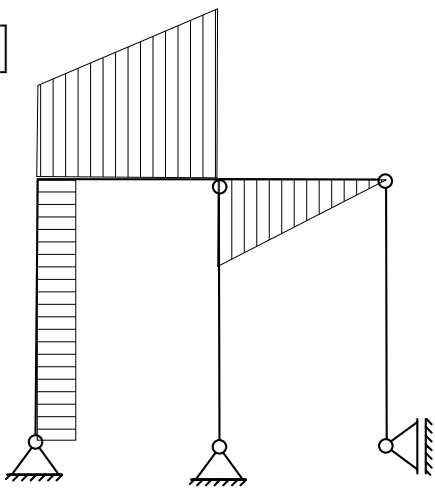
1. Naloga: PRAVILNI DIAGRAMI



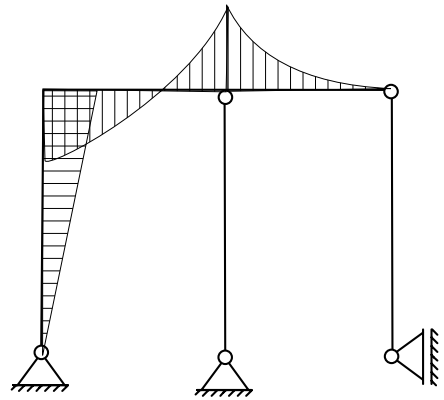
$[N_x]$



$[N_z]$



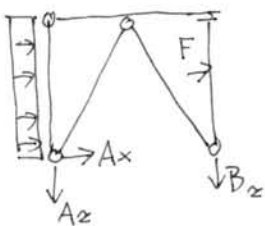
$[M_y]$



2. HALOGA

a.) $\tilde{m}_{ps} = 5 \cdot 3 - 2 - 1 - 3 \cdot 2 - 2 \cdot 2 \cdot 2 = 0$

b.) REAKCIJE



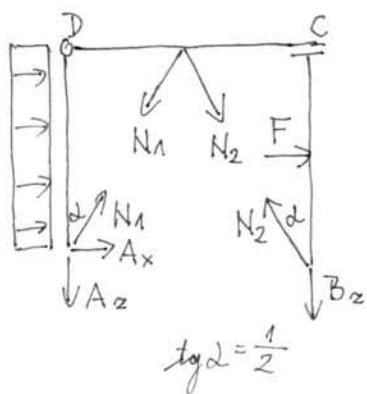
$\Sigma X: A_x + F + g \cdot a = 0$

$\Sigma Z: A_z + B_z = 0$

$\Sigma M_A: -B_z \cdot a - F \cdot \frac{a}{2} - g \cdot a \cdot \frac{a}{2} = 0$

$B_z = -12.5 \text{ kN}$
 $A_z = 12.5 \text{ kN}$
 $A_x = -25 \text{ kN}$

c.) IZREŽEMO PALICE



$\Sigma M_{AD}^D: N_1 \cdot \sin \alpha \cdot a + A_x \cdot a + g \cdot a \cdot \frac{a}{2} = 0$

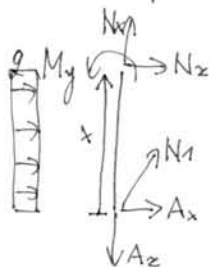
$N_1 = \frac{17.5}{\sin \alpha} = 39.13 \text{ kN}$

$\Sigma X: F - N_2 \cdot \sin \alpha = 0$

$N_2 = \frac{F}{\sin \alpha} = 22.36 \text{ kN}$

d.) NOTRANJE SILE PO POLJIH

d.1) polje AD



$N_x = A_x - N_1 \cdot \cos \alpha$

$N_z = -A_z - N_1 \cdot \sin \alpha - g \cdot x$

$M_y = -A_x \cdot x - N_1 \cdot \sin \alpha \cdot x - g \cdot \frac{x^2}{2}$

$N_x = -22.5 \text{ kN}$
 $N_z = 7.5 - 5x \quad N_z(3) = -7.5 \text{ kN}$
 $M_y = 17.5x - 2.5x^2$
 $M_y(1.5) = 5.6 \text{ kNm (desetrem)}$

sile v vezi C:



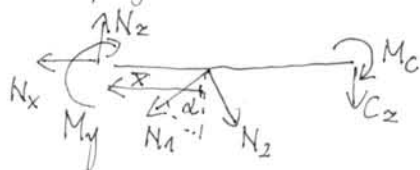
$C_z = B_z - N_2 \cdot \cos \alpha$

$C_z = -32.5 \text{ kN}$

$M_c = -F \cdot \frac{a}{2} + N_2 \cdot \sin \alpha \cdot a$

$M_c = 15 \text{ kNm}$

d.2) polje DE



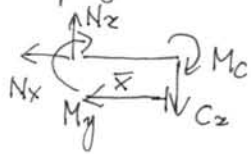
$\Sigma X: -N_x - N_1 \cdot \sin \alpha + N_2 \cdot \sin \alpha = 0$

$\Sigma Z: N_z = C_z + N_1 \cdot \cos \alpha + N_2 \cdot \cos \alpha$

$\Sigma M^T: M_y = M_c - C_z \cdot (\frac{a}{2} + \bar{x}) - N_1 \cdot \cos \alpha \cdot \bar{x} - N_2 \cdot \cos \alpha \cdot \bar{x}$

$N_x = -7.5 \text{ kN}$
 $N_z = 22.5 \text{ kN}$
 $M_y = 33.75 - 22.5\bar{x} \quad M_y(1.5) = 0$

d3.) polje CE

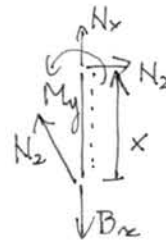


$$\begin{aligned} \sum X: N_x &= 0 \\ \sum Z: N_z &= C_z \\ \sum M: M_y &= -M_c - C_z x \end{aligned}$$

$N_x = 0$
$N_z = -32.5 \text{ kN}$
$M_y = -15 + 32.5x$

$$M_y(1.5) = 33.75 \text{ kNm}$$

d4.) polje BF

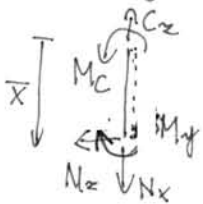


$$\begin{aligned} N_x &= B_z - N_2 \cdot \cos \alpha \\ N_z &= N_2 \cdot \sin \alpha \\ M_y &= N_2 \cdot \sin \alpha \cdot x \end{aligned}$$

$N_x = -32.5 \text{ kN}$
$N_z = 10 \text{ kN}$
$M_y = 10x$

$$M_y(1.5) = 15 \text{ kNm}$$

d5.) polje EF



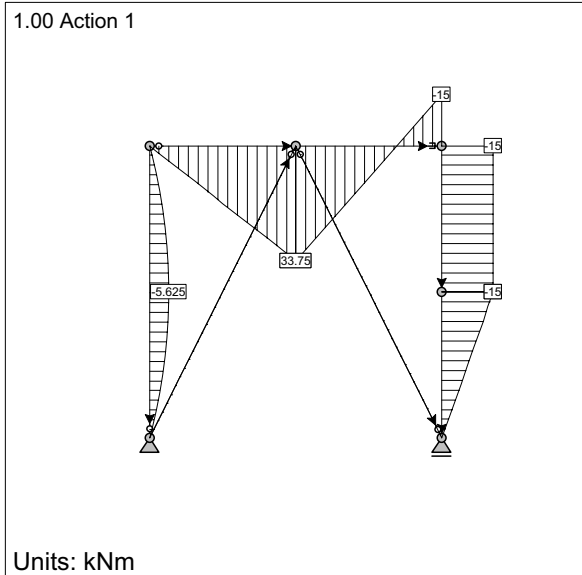
$$\begin{aligned} N_x &= C_x \\ N_z &= 0 \\ M_y &= +M_c \end{aligned}$$

$N_x = -32.5 \text{ kN}$
$N_z = 0$
$M_y = +15 \text{ kNm}$

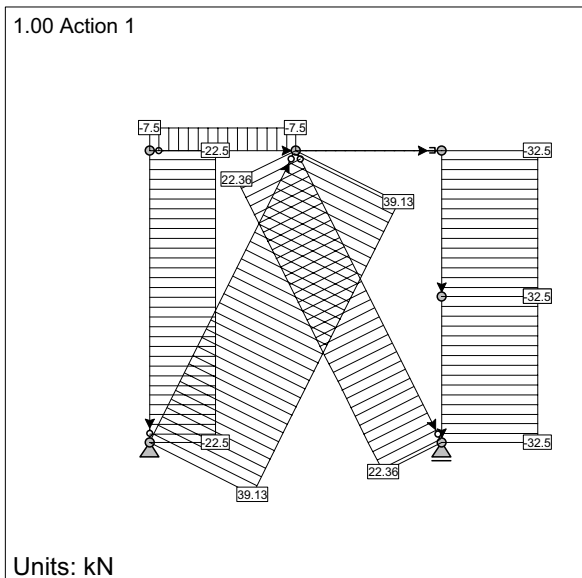
e.) DIAGRAMI

glej naslednjo stran

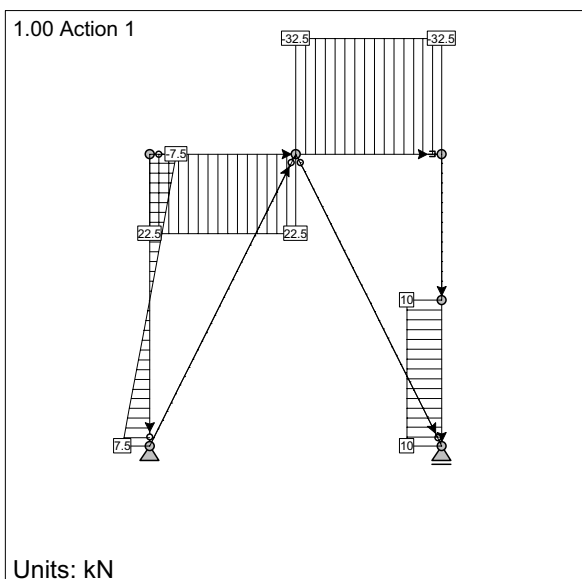
LC1: Load case 2: Bending Moments My



LC1: Load case 2: Axial Forces Fx



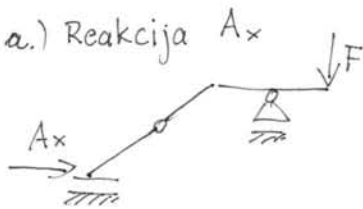
LC1: Load case 2: Shear Forces Fz



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3. NALOGA

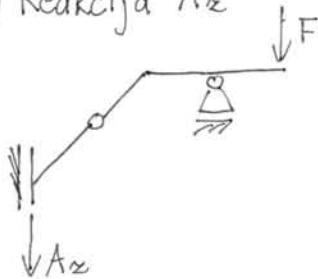
a.) Reakcija A_x



$$\delta W = A_x \delta u_A + F \delta w_F = 0$$

$$\begin{aligned} \delta w_F &= 0 \\ \delta u_A \text{ je poljuben} &\Rightarrow \boxed{A_x = 0} \end{aligned}$$

b.) Reakcija A_z



$$\delta W = A_z \delta w_A + F \delta w_F = 0$$

$$\delta w_c = \delta w_A - a \delta \varphi_A \stackrel{=0}{\parallel}$$

$$\boxed{\delta w_c = \delta w_A}$$

$$\delta u_c = \delta w_A - b \delta \varphi_A \stackrel{=0}{\parallel}$$

$$\delta u_c = 0$$

$$\delta w_F = \delta w_c - 2a \delta \varphi_c^{(2)}$$

$$\delta w_B = \delta w_c - \frac{3}{2} a \delta \varphi_c^{(2)}$$

$$\delta u_F = \delta u_c - b \delta \varphi_c^{(2)} \stackrel{=0}{\parallel}$$

$$\stackrel{=0}{\parallel} \delta w_A$$

$$\Rightarrow \boxed{\delta \varphi_c^{(2)} = \frac{2}{3a} \delta w_A}$$

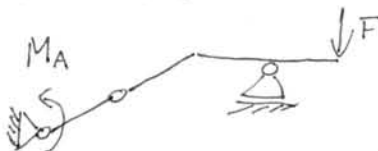
$$\delta w_F = \delta w_A - \frac{4}{3} \delta w_A$$

$$\boxed{\delta w_F = -\frac{1}{3} \delta w_A}$$

$$\Rightarrow (A_z - \frac{1}{3}F) \delta w_A = 0$$

$$\boxed{A_z = \frac{F}{3}}$$

c.) Moment M_A



$$\delta W = M_A \delta \varphi_A + F \delta w_F = 0$$

$$\delta u_c = \delta u_A - b \delta \varphi_A \stackrel{=0}{\parallel}$$

$$\delta w_F = \delta w_c - b \delta \varphi_c^{(2)}$$

$$\delta w_c = \delta w_A - a \delta \varphi_A \stackrel{=0}{\parallel}$$

$$\delta w_F = \delta w_c - 2a \delta \varphi_c^{(2)}$$

$$\delta w_B = \delta w_c - \frac{3a}{2} \delta \varphi_c^{(2)} \stackrel{=0}{\parallel}$$

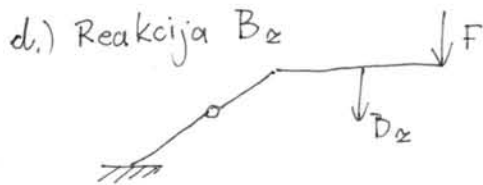
$$-a \delta \varphi_A - \frac{3a}{2} \delta \varphi_c^{(2)} = 0$$

$$\boxed{\delta \varphi_c^{(2)} = -\frac{2}{3} \delta \varphi_A}$$

$$\delta w_F = -a \delta \varphi_A + \frac{4a}{3} \delta \varphi_A = \frac{1a}{3} \delta \varphi_A$$

$$\boxed{M_A \delta \varphi_A + \frac{F \cdot a}{3} \delta \varphi_A = 0}$$

$$\Rightarrow \boxed{M_A = -\frac{F \cdot a}{3}}$$



$$\delta W = B_z \delta w_B + F \delta w_F = 0$$

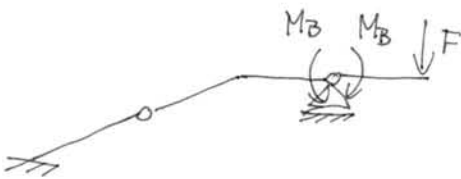
$$\begin{aligned} \delta w_B &= \delta w_C - \frac{3a}{2} \delta \varphi_C^{(2)} \\ \delta u_B &= \delta u_C - b \delta \varphi_C^{(2)} \end{aligned}$$

$$\begin{aligned} \delta w_F &= \delta w_C - 2a \delta \varphi_C^{(2)} \\ \delta u_F &= \delta u_C - b \delta \varphi_C^{(2)} \end{aligned}$$

$$\delta W = B_z \cdot \left(-\frac{3a}{2}\right) \delta \varphi_C^{(2)} - 2aF \delta \varphi_C^{(2)} = 0$$

$$\Rightarrow \boxed{B_z = -\frac{4}{3} F}$$

e.) Notranji moment v B



$$\delta W = M_B \delta \varphi_B^{(1)} - M_B \delta \varphi_B^{(2)} + F \delta w_F = 0$$

$$\delta \varphi_B^{(1)} = 0$$

$$\delta w_F = \delta w_B - \frac{a}{2} \delta \varphi_B^{(2)}$$

$$-M_B \delta \varphi_B^{(2)} - \frac{F}{2} a \delta \varphi_B^{(2)} = 0$$

$$\boxed{M_B = -\frac{F}{2} a}$$