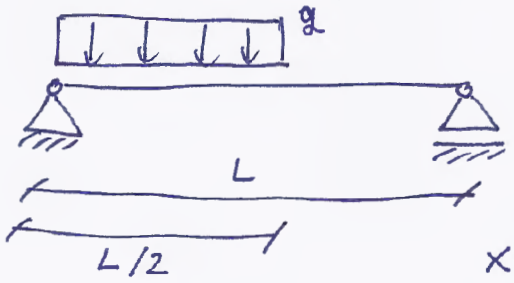
Prostoležeči nosilec s porazdeljeno obtežbo na polovici razpona

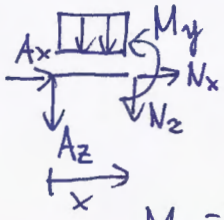


$x \in [0, L]$

uporabimo $\frac{d^2w}{dx^2} = -\frac{M_y}{EI_y}$

2 POLJI

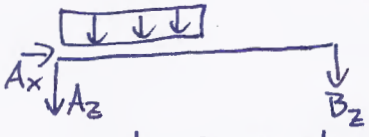
$x \in [0, L/2]$



$M_y = -A_z x - g\frac{x^2}{2}$

1. polje $\boxed{M_y = \frac{3}{8}gLx - g\frac{x^2}{2}}$

$x \in [L/2, L]$



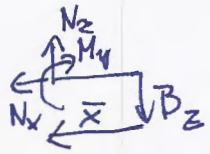
$A_z + B_z = -g\frac{L}{2}$

$-B_z \cdot L - g\frac{L}{2}\cdot\frac{L}{4} = 0$

$\boxed{A_z = -\frac{3gL}{8}}$

$\boxed{B_z = -g\frac{L}{8}}$

$\bar{x} = L - x$



$M_y = -B_z\bar{x}$

2. polje $\boxed{M_y = \frac{gL}{8}(L-x)}$

$$\frac{d^2w}{dx^2} = -\frac{3gL}{8EI_y}x + \frac{g}{2EI_y}x^2$$

$$\frac{dw}{dx} = C_1 - \frac{3gL}{16EI_y}x^2 + \frac{g}{6EI_y}x^3$$

1. polje $\boxed{w(x) = C_2 + C_1 x - \frac{gL}{16EI_y}x^3 + \frac{g}{24EI_y}x^4}$

$$\frac{d^2w}{dx^2} = -\frac{gL^2}{8EI_y} + \frac{gL}{8EI_y}x$$

$$\frac{dw}{dx} = D_1 - \frac{gL^2}{8EI_y}x + \frac{gL}{16EI_y}x^2$$

2. polje $\boxed{w(x) = D_2 + D_1 x - \frac{gL^2}{16EI_y}x^2 + \frac{gL}{48EI_y}x^3}$

ROBNI POGOJI

$w(0) = 0$

$\Rightarrow C_2 = 0$

$w(L) = 0$

$\Rightarrow D_2 + D_1 \cdot L - \frac{gL^4}{16EI_y} + \frac{gL^4}{48EI_y} = 0$

ZVEZNOST

$w^{(1)}(L/2) = w^{(2)}(L/2)$

$w^{(1)}(L/2) = w^{(2)}(L/2)$

$$C_1 - \frac{3gL^3}{54EI_y} + \frac{gL^3}{48EI_y} = D_1 - \frac{gL^3}{16EI_y} + \frac{gL^3}{54EI_y}$$

$$C_1 \cdot \frac{L}{2} - \frac{gLL^3}{8\cdot 16EI_y} + \frac{gL4}{16\cdot 24EI_y} = D_2 + D_1\frac{L}{2} - \frac{gL4}{2\cdot 16EI_y} + \frac{gL4}{8\cdot 48EI_y}$$

pokračování:

$$D_2 + D_1 L = \frac{qL^4}{24EI_y}$$

$$C_1 - D_1 = \frac{qL^3}{48EI_y}$$

$$C_1 \cdot L - D_1 \cdot L - 2D_2 = -\frac{qL^4}{64EI_y}$$

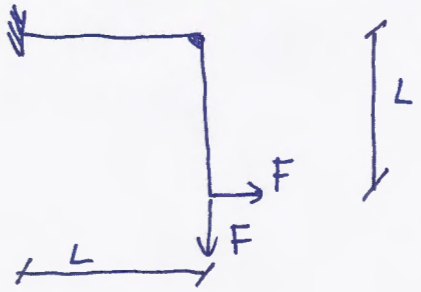
$$C_1 = \frac{3qL^3}{128EI_y} \qquad D_1 = \frac{17qL^3}{384EI_y} \qquad D_2 = -\frac{qL^4}{384EI_y}$$

$$w(x) = \begin{cases} \frac{3qL^3}{128EI_y} x - \frac{qL}{16EI_y} x^3 + \frac{q}{24EI_y} x^4 & x \in [0, L/2] \\ -\frac{qL^4}{384EI_y} + \frac{17qL^3}{384EI_y} x - \frac{qL^2}{16EI_y} x^2 + \frac{qL}{48EI_y} x^3 & x \in [\frac{L}{2}, L] \end{cases}$$

$$w\left(\frac{L}{2}\right) = \frac{3qL^4}{256EI_y} - \frac{qL^4}{48EI_y} + \frac{qL^4}{384EI_y}$$

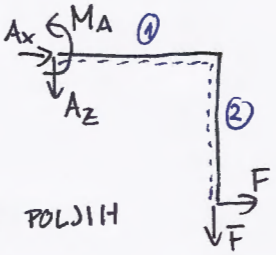
$$\boxed{w\left(\frac{L}{2}\right) = \frac{5qL^4}{768EI_y}}$$

Določi pomike kolenastega nosilca na sliki



$L = 2\,m$
$F = 1\,kN$
$E = 1000\ kN/cm^2$
$A = 400\ cm^2$
$I = 15000\ cm^4$

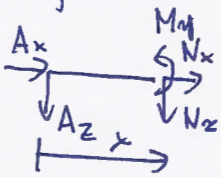
a.) REAKCIJE



$A_x = -F$
$A_z = -F$
$-M_A - F \cdot L + F \cdot L = 0$

b.) RAČUN PO POLJIH

polje ①



$N_x = -A_x = F$

$M_y = -A_z \cdot x = F \cdot x$

$$\frac{du}{dx} = \frac{N_x}{EA_x} = \frac{F}{EA_x}$$

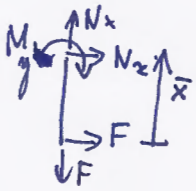
$$u^{①}(x) = C + \frac{Fx}{EA_x}$$

$$\frac{d^2w}{dx^2} = -\frac{M_y}{EI_y} = -\frac{Fx}{EI_y}$$

$$\frac{dw}{dx} = C_1 - F\frac{x^2}{2EI_y}$$

$$w^{①}(x) = C_2 + C_1 x - F\frac{x^3}{6EI_y}$$

polje ②



$N_x = F$

$M_y = F\bar{x}$

$M_y = F(L - x)$

$$\frac{du}{dx} = \frac{N_x}{EA_x} = \frac{F}{EA_x}$$

$$u^{②}(x) = D + \frac{Fx}{EA_x}$$

$$\frac{d^2w}{dx^2} = \frac{Fx - FL}{EI_y}$$

$$\frac{dw}{dx} = D_1 + \frac{Fx^2}{2EI_y} - \frac{FLx}{EI_y}$$

$$w^{②}(x) = D_2 + D_1 x - \frac{FLx^2}{2EI_y} + \frac{Fx^3}{6EI_y}$$

c.) ROBNI POGOJI

$w^{①}(0) = 0$
$w_y^{①}(0) = 0$
$u^{①}(0) = 0$

$u^{①}(L) = w^{②}(0)$! (+ pomik osnog je negativen mimo)
$w^{①}(L) = w^{②}(0)$
$\varphi^{①}(L) = \varphi^{②}(0)$

$w^{(1)}(0) = C_2 = 0$

$w^{(2)}(0) = C = 0$

$\left.\frac{dw}{dx}\right|_{x=0} = C_1 = 0$

$w^{(1)}(x) = -\frac{Fx^3}{6EI_y}$

$u^{(1)}(x) = \frac{Fx}{EA_x}$

$w^{(2)}(0) = -\frac{FL^3}{6EI_y} = D$

$u^{(2)}(0) = \frac{FL}{EA_x} = D_2$

$\left.\frac{dw^{(2)}}{dx}\right|_{x=0} = D_1 = -\frac{FL^2}{2EI_y}$

$w^{(2)}(x) = -\frac{FL^3}{6EI_y} + \frac{Fx}{EA_x}$

$u^{(2)}(x) = +\frac{FL}{EA_x} - \frac{FL^2x}{2EI_y} - \frac{FLx^2}{2EI_y} + \frac{Fx^3}{6EI_y}$

$u^{(1)}(L) = \frac{1\,kN \cdot 200\,cm}{1000\,kN/cm^2 \cdot 400\,cm^2} = 0{,}0005\,cm$

$w^{(1)}(L) = -\frac{1\,kN \cdot 8 \cdot 10^6\,cm^3}{6 \cdot 10^3\,kN/cm^2 \cdot 15 \cdot 10^3\,cm^4} = -0{,}0889\,cm$

$u^{(2)}(L) = -0{,}0889 + \frac{1\,kN \cdot 200\,cm}{1000\,kN/cm^2 \cdot 400\,cm} = -0{,}0884\,cm$

$w^{2}(L) = -0{,}0005 - \frac{FL^3}{2EI_y} - \frac{FL^3}{2EI_y} + \frac{FL^3}{6EI_y} = 0{,}0005 - \frac{5FL^3}{6EI_y}$

$= -0{,}0005 - \frac{5 \cdot 1\,kN \cdot 8 \cdot 10^6\,cm^3}{6 \cdot 1000\,kN/cm^2 \cdot 15 \cdot 10^3\,cm^4} = -0{,}44\,cm$

Določi upogibnico in notranje sile:

$M = 1\,kNm$

$a = 100\,cm$

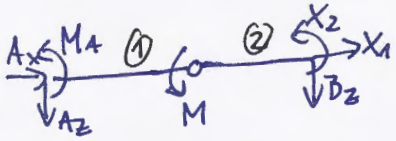
1. NAČIN vpeljemo sprostitve

$\hat{n}_{PS} = 3\cdot 2 - 3 - 3 - 2 = -2$ (2x nedoločena)

(zgrebejev nosilec) $+ u^{②}(L) = 0$

$+ \omega_y^{②}(L) = 0$

polje ①



polje ②

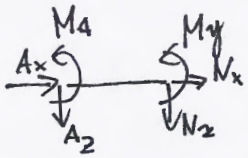
$A_x + X_1 = 0$

$A_z + B_z = 0$

$M_A + M + X_2 - B_z \cdot 2a = 0$

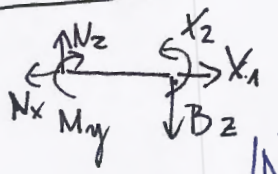
$-B_z \cdot a + X_2 = 0$

$A_x = -X_1$
$A_z = -\frac{X_2}{a}$
$M_A = X_2 - M$
$B_z = \frac{X_2}{a}$



$N_x = -A_x$ $\boxed{N_x = X_1}$

$M_y = -A_z \cdot x - M_A$ $\boxed{M_y = -X_2 + M + \frac{X_2}{a}x}$



$\boxed{N_x = X_1}$

$M_y = X_2 - B_z \cdot x$

$M_y = X_2 - \frac{X_2}{a}(a - x)$

$M_y = X_2 - X_2 + \frac{X_2}{a}x$

$\frac{du}{dx} = \frac{X_1}{EA_x}$ $\boxed{u^{①}(x) = C + \frac{X_1 \cdot x}{EA_x}}$

$\boxed{u^{②}(x) = D + \frac{X_1 x}{EA_x}}$

$\frac{d^2w}{dx^2} = \frac{+X_2 - M}{EI_y} = -\frac{X_2 x}{aEI_y}$

$\frac{dw}{dx} = C_1 + \frac{X_2 - M}{EI_y}x - \frac{X_2 x^2}{2aEI_y}$

$\frac{dw}{dx} = D_1 - \frac{X_2 a - X_2 L}{aEI_y}x - \frac{X_2 x^2}{2aEI_y}$

$\boxed{w^{①}(x) = C_2 + C_1 x + \frac{X_2 - M}{2EI_y}x^2 - \frac{X_2 x^3}{6aEI_y}}$

$\boxed{w^{②}(x) = D_2 + D_1 x + \frac{X_2(x-a)}{2aEI_y}x^2 - \frac{X_2 x^3}{6aEI}}$

ROBNI POGOJI

$u^{①}(0) = C = 0$

$u^{②}(a) = D + \frac{X_1 a}{EA_x} = 0$

$u^{①}(a) = u^{②}(0)$

$\frac{X_1 \cdot a}{EA_x} = D$ $\Rightarrow$ $\boxed{X_1 = 0}$

$D = 0$

$\boxed{u^{②}(x) = w^{②}(x) = 0}$

levi rob:

$$w^{①}(0)=0 \Rightarrow C_2=0$$

$$\left.\frac{dw^{①}}{dx}\right|_0=0 \Rightarrow C_1=0$$

desni rob:

$$w^{②}(L)=0 \Rightarrow D_2+D_1 a-\frac{X_2 a^2}{6aEI_y}=0$$

$$\left.\frac{dw^{②}}{dx}\right|_a=0 \Rightarrow D_1-\frac{X_2 a^2}{2EI_y}=0$$

$$D_1=\frac{X_2 a}{2EI_y} \qquad D_2=-\frac{X_2 a^2}{3EI_y}$$

zveznost na sredi

$$w^{①}(a)=w^{②}(0)$$

$$\frac{X_2-M}{2EI_y}a^2-\frac{X_2 a^3}{6aEI_y}=D_2 \quad /\cdot 6EI_y:a$$

$$3X_2-3M-X_2=-2X_2 \Rightarrow X_2=\frac{3}{4}M$$

$$w^{①}(x)=\frac{M}{8EI_y}x^2-\frac{Mx^3}{8aEI_y}$$

$$w^{②}(x)=\frac{3Ma}{8EI_y}-\frac{3Ma^2x}{12EI_y}-\frac{Mx^3}{8aEI_y}$$

NOTRANJE SILE

polje ①:

$N_x=0$

$M_y=\frac{M}{4}+\frac{3M}{4a}x$

$M_y(a)=M$

polje ②:

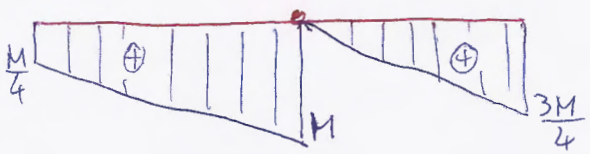
$N_x=0$

$M_y=\frac{3M}{4a}x$

$M_y(a)=\frac{3M}{4}$

$[N_x]$

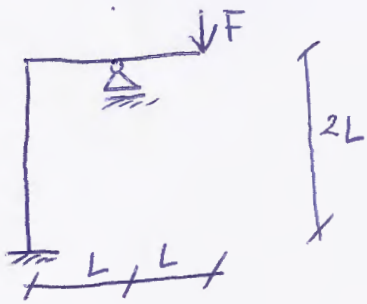
$[M_y]$



$$N_z=\frac{dM_y}{dx}=\frac{3M}{4a}$$

$[N_z]$

Za konstrukcijo na sliki določi pomike in notranje sile!



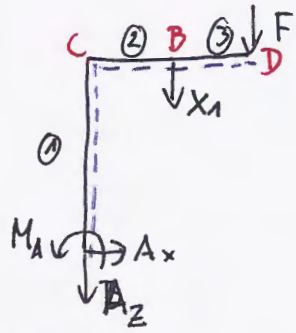
$L = 2\,m$

$F = 15\,kN$

$E = 20000\,kN/cm^2$

$I_y = 4'5 \cdot 10^4\,cm^4$

$A = 160\,cm^2$



REAKCIJE

$$A_x = 0$$

$$A_z = -F - X_1$$

$$M_A = 2FL + X_1 L$$

polje ①

$N_x = A_z$

$M_y = -M_A$

$M_y = 2FL - X_1 L$

$$\frac{du}{dx} = + \frac{N_x}{EA_x}$$

$$\frac{du}{dx} = \frac{-F - X_1}{EA}$$

$$u^{①}(x) = C - \frac{F + X_1}{EA} x$$

$u^{①}(0) = 0 \Rightarrow C = 0$

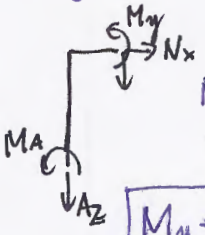
$$\frac{d^2w}{dx^2} = -\frac{M_y}{EI_y} = + \frac{2FL + X_1 L}{EI_y}$$

$$\frac{dw^{①}}{dx} = C_1 + \frac{2FL + X_1 L}{EI_y} x$$

$$w^{①}(x) = C_2 + C_1 x + \frac{2FL + X_1 L}{EI_y} \frac{x^2}{2}$$

$$\left.\begin{aligned} w^{①}(0) &= 0 \\ w^{②}(0) &= 0 \end{aligned}\right\} \Rightarrow C_1 = C_2 = 0$$

polje ②



$N_x = 0$

$M_y = -M_A - A_z x$

$$M_y = -2FL - X_1 L + (F + X_1) x$$

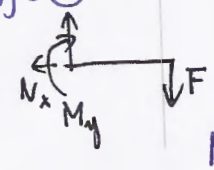
$$u^{②}(x) = D$$

$$\frac{d^2w}{dx^2} = + \frac{2F + X_1}{EI_y} \cdot L - \frac{F + X_1}{EI_y} x$$

$$\frac{dw^{②}}{dx} = D_1 + \frac{2F + X_1}{EI_y} Lx - \frac{F + X_1}{2EI_y} x^2$$

$$w^{②}(x) = D_2 + D_1 x + \frac{2F + X_1}{2EI_y} Lx^2 - \frac{F + X_1}{6EI_y} x^3$$

polje ③



$N_x = 0$

$M_y = -F\bar{x}$

$M_y = -F(L - x)$

$$u^{③}(x) = E$$

$$\frac{d^2w}{dx^2} = \frac{FL}{EI_y} - \frac{Fx}{EI_y}$$

$$\frac{dw^{③}}{dx} = E_1 + \frac{FL}{EI_y} x - \frac{Fx^2}{2EI_y}$$

$$w^{③}(x) = E_2 + E_1 x + \frac{FL}{2EI_y} x^2 - \frac{Fx^3}{6EI_y}$$

ROBNI POGOJI

točka B:

$w^{②}(0)=0 \Rightarrow E_2=0$

$w^{②}(L)=0 \Rightarrow$

$$D_2 + D_1 L + \frac{2F+X_1}{EI_y} L^3 - \frac{F+X_1}{6EI_y} L^3 = 0$$

$$\left.\frac{dw^{②}}{dx}\right|_{x=L} = \left.\frac{dw^{③}}{dx}\right|_{x=0}$$

$$D_1 + \frac{2F+X_1}{EI_y} L^2 - \frac{F+X_1}{2EI_y} L^2 = E_1$$

$$D_1 + \frac{3F+X_1}{2EI_y} L^2 = E_1$$

točka C:

$u^{①}(2L) = -w^{②}(0)$ (pazi predznake!)

$$-\frac{F+X_1}{EA} 2L = -D_2$$

$$D_2 = \frac{F+X_1}{EA} 2L$$

$w^{①}(2L) = u^{②}(0)$

$$+\frac{2F+X_1}{2EI_y} 4L^3 = D$$

$$D = +\frac{2F+X_1}{EI_y} 2L^3$$

$\left.\frac{dw^{①}}{dx}\right|_{2L} = \left.\frac{dw^{②}}{dx}\right|_{0}$

$$+\frac{2F+X_1}{EI_y} 2L^2 = D_1$$

$$D_1 = \frac{2F+X_1}{EI_y} 2L^2$$

rešimo enačbo $w^{②}(L)=0$

$$\frac{F+X_1}{EA} 2L + \frac{2F+X_1}{EI_y} 2L^3 + \frac{11F+5X_1}{6EI_y} L^3 = 0$$

$$X_1\left(\frac{2L}{EA} + \frac{2L^3}{EI_y} + \frac{5L^3}{6EI_y}\right) = -F\left(\frac{2L}{EA} + \frac{4L^3}{EI_y} + \frac{11L^3}{6EI_y}\right)$$

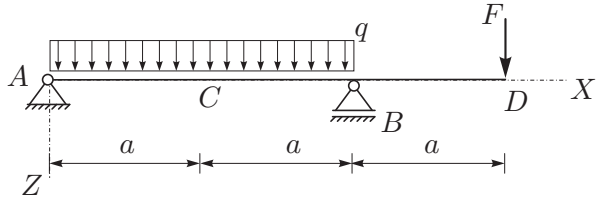
$$X_1\left(\frac{2L}{EA} + \frac{17L^3}{6EI_y}\right) = -F\left(\frac{2L}{EA} + \frac{35L^3}{6EI_y}\right)$$

$$X_1 = -15 \cdot \frac{\frac{400}{20000 \cdot 160} + \frac{35 \cdot 8 \cdot 10^6}{6 \cdot 2 \cdot 10^4 \cdot 45 \cdot 10^4 \cdot 100}}{\frac{400}{20000 \cdot 160} + \frac{17 \cdot 80 \cdot 10^6}{6 \cdot 2 \cdot 10^4 \cdot 45 \cdot 10^4 \cdot 100}}$$

$$X_1 = -15 \frac{0.05198}{0.0253}$$

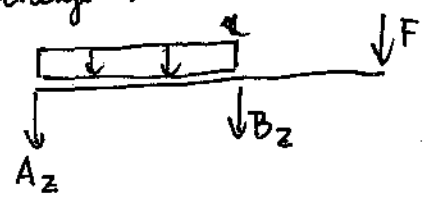
$$\boxed{X_1 = -30.8\ \text{kN}}$$

4. Za konstrukcijo na sliki izrazite upogibnico in izvrednotite vertikalna pomika v točkah C in D! Podatki: $a = 3\,\mathrm{m}$, $q = 2\,\mathrm{kN/m}$, $F = 10\,\mathrm{kN}$, $E = 2500\,\mathrm{kN/cm^2}$, $A = 400\,\mathrm{cm^2}$, $I_y = 200000\,\mathrm{cm^4}$.



4. NALOGA

a.) notranje sile



$A_x = 0$

$A_z + B_z = -F - q \cdot 2a = -10 - 2 \cdot 6 = -22\,kN$

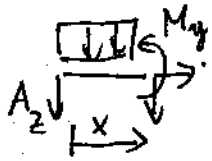
$-B_z \cdot 6 - F \cdot 9 - q \cdot 6 \cdot 3 = 0$

$B_z = -F \cdot \frac{3}{2} - q \cdot 3$

$B_z = -21\,kN$

$A_z = -1\,kN$

polje ①



$M_y = -A_z x - q\frac{x^2}{2}$

$\boxed{M_y = x - x^2}$

$M_y(6) = 6 - 36 = -30\,kNm$

polje ②

$M_y = -F\bar{x}$

$\bar{x} = 3 - x$

~~$M_y = +10x - 20$~~

$\boxed{M_y = -30 + 10x}$

$M_y(0) = -30\,kNm$

kontrola zveznost momentov O.K.

b.) upogibnica

$$\frac{d^2w}{dx^2} = -\frac{x}{EI_y} + \frac{x^2}{EI_y}$$

$$\frac{dw}{dx} = C_1 - \frac{x^2}{2EI_y} + \frac{x^3}{3EI_y}$$

$$w^{①}(x) = C_2 + C_1 x - \frac{x^3}{6EI_y} + \frac{x^4}{12EI_y}$$

$$\frac{d^2w}{dx^2} = \frac{30}{EI_y} - \frac{10x}{EI_y}$$

$$\frac{dw}{dx} = D_1 + \frac{30x}{EI_y} - \frac{10x^2}{2EI_y}$$

$$w^{②}(x) = D_2 + D_1 x + \frac{15x^2}{EI_y} - \frac{5x^3}{3EI_y}$$

c.) robni pogoji

$w^{①}(0) = 0 \Rightarrow C_2 = 0$

$w^{①}(6) = 0 \Rightarrow C_1 \cdot 6 - \frac{6^3}{6EI_y} + \frac{6^4}{12EI_y} = 0$

$C_1 = -\frac{12}{EI_y}$

$w^{②}(0) = 0 \Rightarrow D_2 = 0$

$\left.\frac{dw^{①}}{dx}\right|_{x=6} = \left.\frac{dw^{②}}{dx}\right|_{x=0}$

$-\frac{12}{EI_y} - \frac{6^2}{2EI_y} + \frac{6^3}{3EI_y} = D_1$

$D_1 = \frac{42}{EI_y}$

$$w^{①}(x) = \frac{1}{EI_y}\left(-12x - \frac{x^3}{6} + \frac{x^4}{12}\right)$$

$$w^{②}(x) = \frac{1}{EI_y}\left(42x + 15x^2 - \frac{5}{3}x^3\right)$$

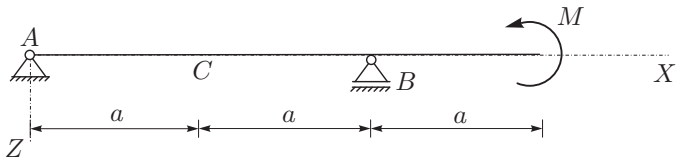
$$w_C = w^{①}(3) = \frac{1}{20\cdot10^4\,cm^4 \cdot 2500\,kN/cm^2}\left(-12\cdot 3 - \frac{3^3}{6} + \frac{3^4}{12}\right) kN\,m^3$$

$$= -\frac{33{,}75\cdot 10^6\,cm}{500\cdot 10^6} = \underline{\underline{-0{,}0675\,cm}}$$

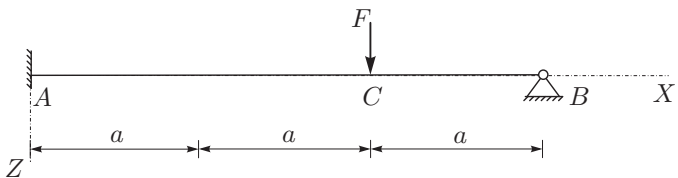
$$w_D = w^{②}(3) = \frac{1}{20\cdot10^4\,cm^4 \cdot 2500\,kN/cm^2}\left(42\cdot 3 + 15\cdot 9 - 5\cdot 9\right) kN\cdot m^3$$

$$= \frac{216\cdot 10^6\,cm}{500\cdot 10^6} = \underline{\underline{0{,}432\,cm}}$$

3. Za konstrukcijo na sliki izrazite prečne pomike in določite navpična pomika v točki C in na prostem koncu! (15%)
Podatki: $a = 1.5\,\mathrm{m}$, $M = 10\,\mathrm{kNm}$,
$E = 20000\,\mathrm{kN/cm^2}$,
$A = 100\,\mathrm{cm^2}$, $I_y = 10000\,\mathrm{cm^4}$.



4. Za statično nedoločeno konstrukcijo na sliki izrazite upogibnico, notranje sile in vertikalni pomik v prijemališču sile! Rezultate notranjih statičnih količin prikažite z diagrami! (30%)
Podatki: $a = 1\,\mathrm{m}$, $F = 15\,\mathrm{kN}$,
$E = 21000\,\mathrm{kN/cm^2}$,
$A = 100\,\mathrm{cm^2}$, $I_y = 1800\,\mathrm{cm^4}$.



3. NALOGA

$A_x = 0$

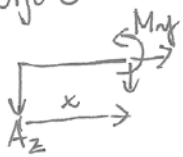
$A_z + B_z = 0$

$-B_z \cdot 3 + M = 0$

$A_z = -\frac{10}{3}\,kN$

$B_z = \frac{10}{3}\,kN$

polje ①



$M_y + A_z \cdot x = 0$

$M_y = \frac{10}{3}x$

$$\frac{d^2w^{①}}{dx^2} = -\frac{M_y}{EI_y} = \frac{1}{EI_y}\left(-\frac{10}{3}x\right)$$

$$\frac{dw^{①}}{dx} = \frac{1}{EI_y}\left(-\frac{5}{3}x^2 + C_1\right)$$

$$w^{①}(x) = \frac{1}{EI_y}\left(-\frac{5}{9}x^3 + C_1x + C_2\right)$$

polje ②

$M_y = M = 10\,kNm$

$$\frac{d^2w^{②}}{dx^2} = -\frac{M_y}{EI_y} = \frac{1}{EI_y}(-10)$$

$$\frac{dw^{②}}{dx} = \frac{1}{EI_y}(-10x + D_1)$$

$$w^{②}(x) = \frac{1}{EI_y}(-5x^2 + D_1x + D_2)$$

ROBNI POGOJI

$w^{①}(0) = 0 \Rightarrow C_2 = 0$

$w^{②}(0) = 0 \Rightarrow D_2 = 0$

$w^{①}(3) = w^{②}(0) = 0 \Rightarrow -\frac{5}{9}\cdot 3^3 + C_1 \cdot 3 = 0$

$\boxed{C_1 = 5}$

$\varphi^{①}(3) = \varphi^{②}(0) \Rightarrow -\frac{5}{3}3^2 + C_1 = D_1$

$\boxed{D_1 = -10}$

REŠITEV

$$w^{①}(x) = \frac{1}{EI_y}\left(-\frac{5}{9}x^3 + 5x\right)$$

$$w^{②}(x) = \frac{1}{EI_y}(-5x^2 - 10x)$$

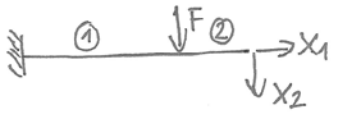
$w_C = w^{①}(1{,}5\,m) = 0{,}028\,cm$

$w_D = w^{②}(1{,}5\,m) = -0{,}131\,cm$

4. NALOGA

2013

2x nedoločena konstrukcija:

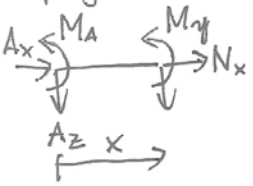


$A_x = -X_1$

$A_z = -15 - X_2$

$M_A = 30 + 3X_2$

polje ①



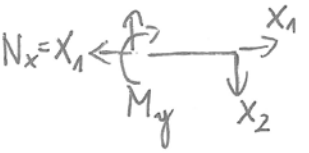
$N_x = X_1$

$M_y = -M_A - A_z \cdot x$

$\boxed{M_y = -30 - 3X_2 + 15x + X_2 x}$

polje ②

$\bar{x} = 1 - x$



$\boxed{N_x = X_1}$

$M_y = -X_2 \bar{x}$

$\boxed{M_y = X_2 x - X_2}$

osni pomiki

$\frac{du^{①}}{dx} = \frac{X_1}{EA_x}$

$u^{①}(x) = \frac{1}{EA_x}(X_1 x + C)$

robni pogoji

$u^{①}(0) = 0 \Rightarrow \boxed{C = 0}$

$\frac{du^{②}}{dx} = \frac{X_1}{EA_x}$

$u^{②}(x) = \frac{1}{EA_x}(X_1 x + D)$

$u^{②}(1) = 0 \quad X_1 + D = 0 \Rightarrow 3X_1 = 0$

$u^{①}(2) = u^{②}(0) \Rightarrow X_1 \cdot 2 = D$

$\boxed{X_1 = 0}$

$\boxed{D = 0}$

$u^{①}(x) = u^{②}(x) = 0$

$X_1 = 0$

prečni pomiki

$\frac{d^2 w^{①}}{dx^2} = \frac{1}{EI_y}(30 + 3X_2 - 15x - X_2 x)$

$\frac{dw^{①}}{dx} = \frac{1}{EI_y}(30x + 3X_2 x - \frac{15}{2}x^2 - X_2 \frac{x^2}{2} + C_1)$

$w^{①}(x) = \frac{1}{EI_y}(15x^2 + \frac{3}{2}X_2 x^2 - \frac{15}{6}x^3 - X_2 \frac{x^3}{6} + C_1 x + C_2)$

$\frac{d^2 w^{②}}{dx^2} = \frac{1}{EI_y}(X_2 - X_2 x)$

$\frac{dw^{②}}{dx} = \frac{1}{EI_y}(X_2 x - X_2 \frac{x^2}{2} + D_1)$

$w^{②}(x) = \frac{1}{EI_y}(X_2 \frac{x^2}{2} - X_2 \frac{x^3}{6} + D_1 x + D_2)$

robni pogoji

$w^{①}(0) = 0 \Rightarrow C_2 = 0$

$\varphi^{①}(0) = 0 \Rightarrow \frac{dw^{①}}{dx}\Big|_{x=0} = 0 \Rightarrow C_1 = 0$

$w^{②}(1) = 0 \Rightarrow X_2 \frac{1}{2} - X_2 \frac{1}{6} + D_1 + D_2 = 0$

$w^{①}(2) = w^{②}(0) \Rightarrow \quad 60 + 6X_2 - 20 - \frac{4}{3}X_2 = D_2 \Rightarrow D_2 = 40 + \frac{14}{3}X_2$

$\varphi^{①}(2) = \varphi^{②}(0) \Rightarrow \frac{dw^{①}}{dx}\Big|_{x=2} = \frac{dw^{②}}{dx}\Big|_{x=0} \Rightarrow 60 + 6X_2 - 30 - 2X_2 = D_1 \Rightarrow D_1 = 30 + 4X_2$

$\frac{1}{3}X_2 + 30 + 4X_2 + 40 + \frac{14}{3}X_2 = 0$

$\frac{27}{3}X_2 = -70$

$\boxed{X_2 = -\frac{210}{27} = -7{,}8\ kN}$

4. NALOGA, dalje

notranje sile nedoločene konstrukcije

polje ①

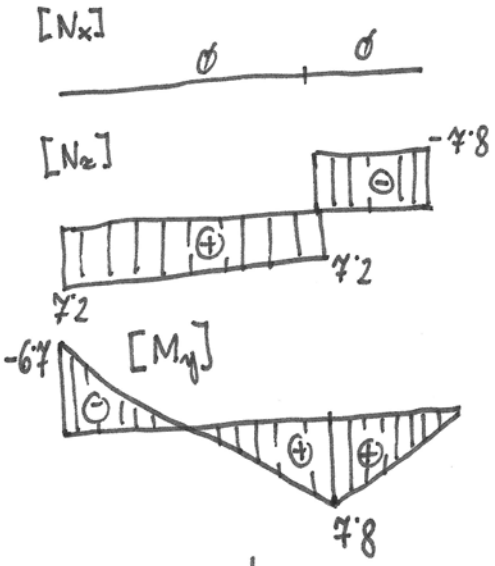
$$\boxed{M_y^{①} = -6{\cdot}7 + 7{\cdot}2x} \quad [kNm]$$

$$N_z^{①} = \frac{dM_y^{①}}{dx} = 7{\cdot}2 \text{ kN}$$

polje ②

$$\boxed{M_y^{②} = 7{\cdot}8 - 7{\cdot}8x} \quad kNm$$

$$N_z^{②} = \frac{dM_y^{②}}{dx} = -7{\cdot}8 \text{ kN}$$



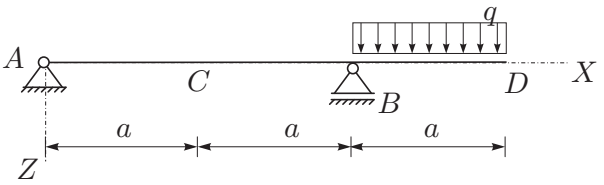
pomik v prijemališču sile

$$w_F = w^{②}(0) = \frac{1}{EI_y} D_2 = \frac{1}{EI_y}\left(40 - \frac{14}{3}\,7{\cdot}8\right) = \frac{3{\cdot}7 \text{ kN m}^3}{21 \cdot 10^3 \cdot 18 \cdot 10^2 \text{ kN m}^2}$$

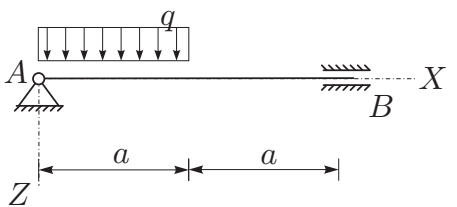
$$= 0{\cdot}00098 m = 0{\cdot}098 cm$$

$$\boxed{w_F = 0{\cdot}98 mm}$$

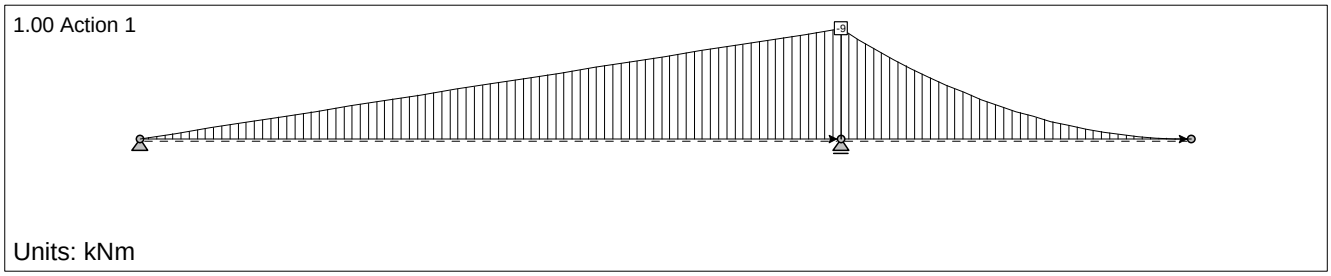
2. Za konstrukcijo na sliki izrazite upogibnico in izvrednotite vertikalna pomika v točkah C in D! (30%)
Podatki: $a = 3\,\mathrm{m}$, $q = 2\,\mathrm{kN/m}$,
$E = 20000\,\mathrm{kN/cm^2}$, $A = 100\,\mathrm{cm^2}$,
$I_y = 20000\,\mathrm{cm^4}$.



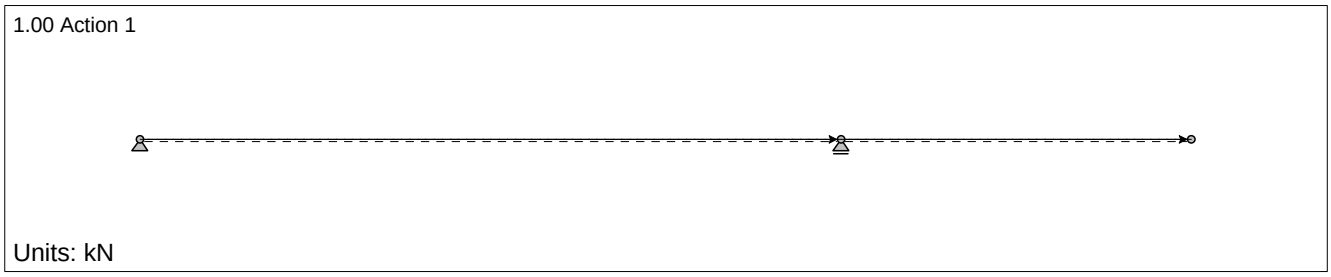
3. Za konstrukcijo na sliki izrazite upogibnico, notranje sile in določite zasuk v točki A! Rezultate notranjih statičnih količin prikažite z diagrami! (40%)
Podatki: $a = 2\,\mathrm{m}$, $q = 10\,\mathrm{kN/m}$,
$E = 3000\,\mathrm{kN/cm^2}$,
$A = 1000\,\mathrm{cm^2}$, $I_y = 200000\,\mathrm{cm^4}$.



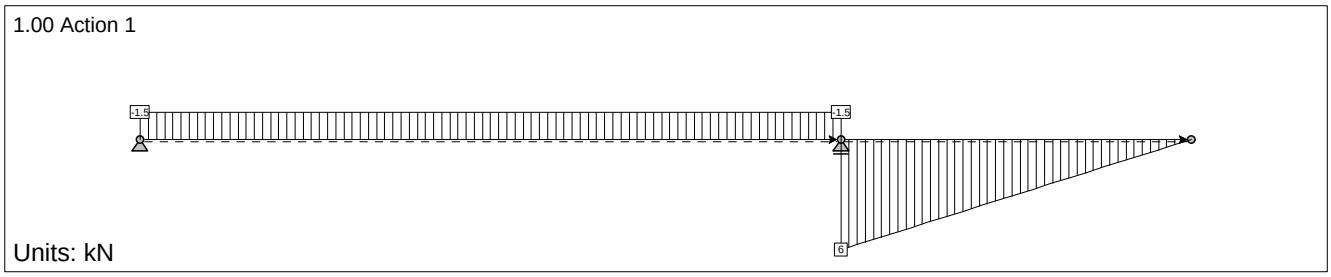
LC1: Load case 2: Bending Moments My



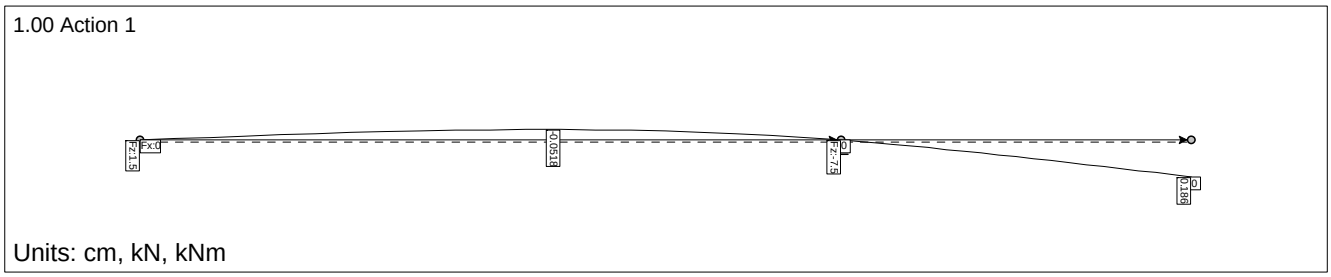
LC1: Load case 2: Axial Forces Fx



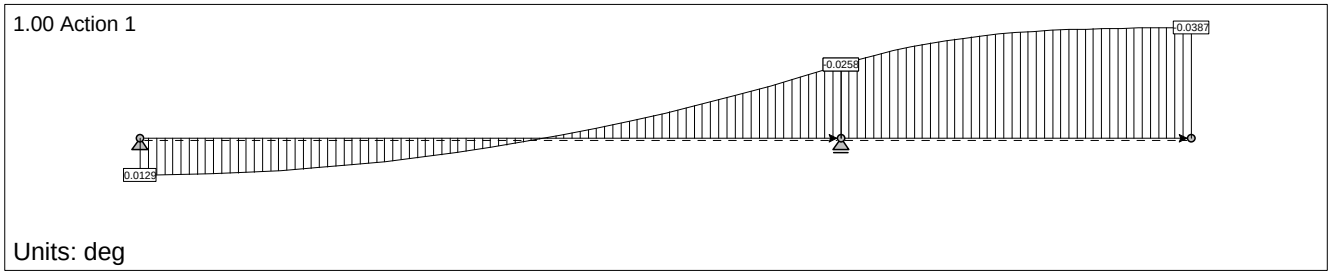
LC1: Load case 2: Shear Forces Fz



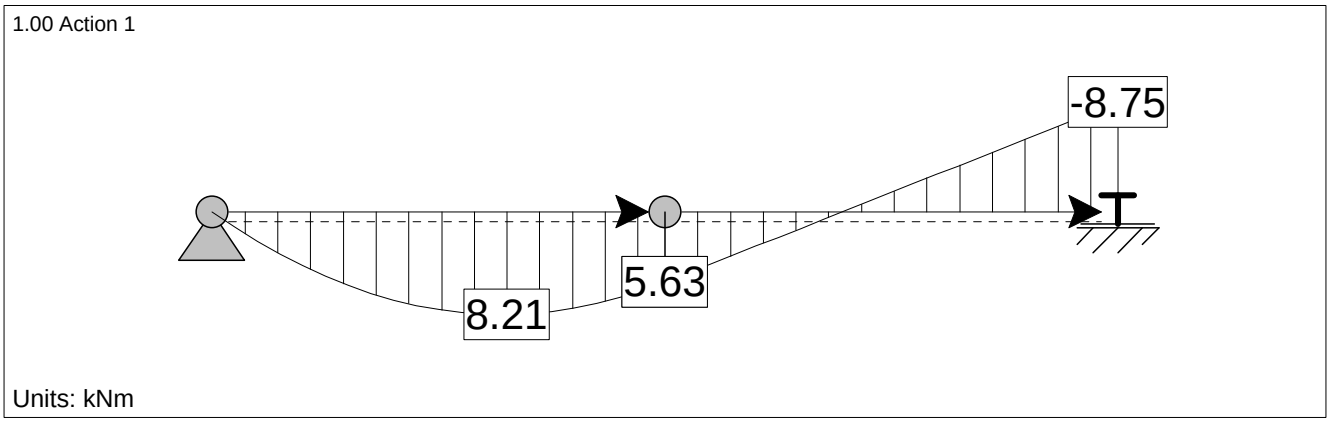
LC1: Load case 2: Displacements and Reactions



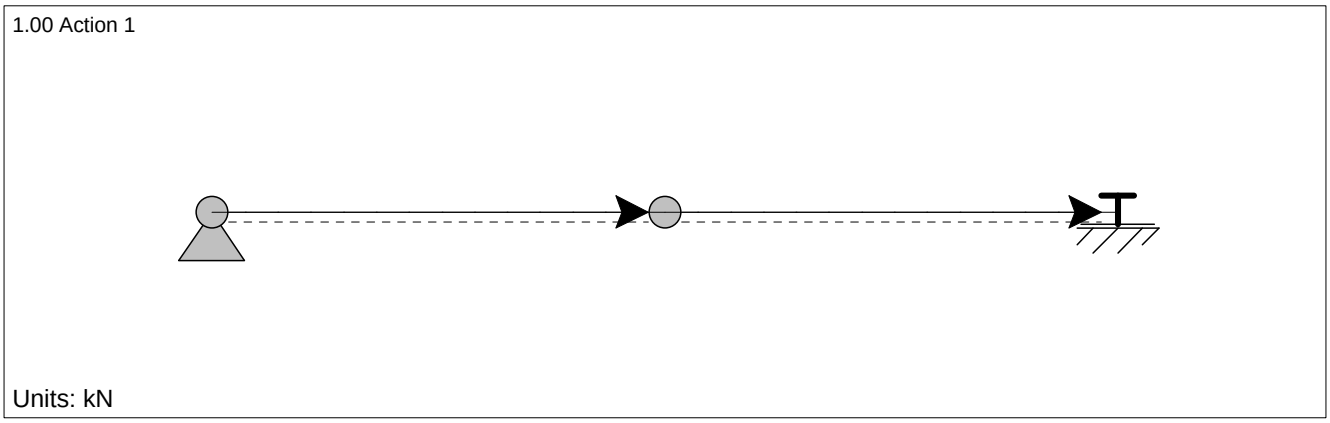
LC1: Load case 2: Rotations



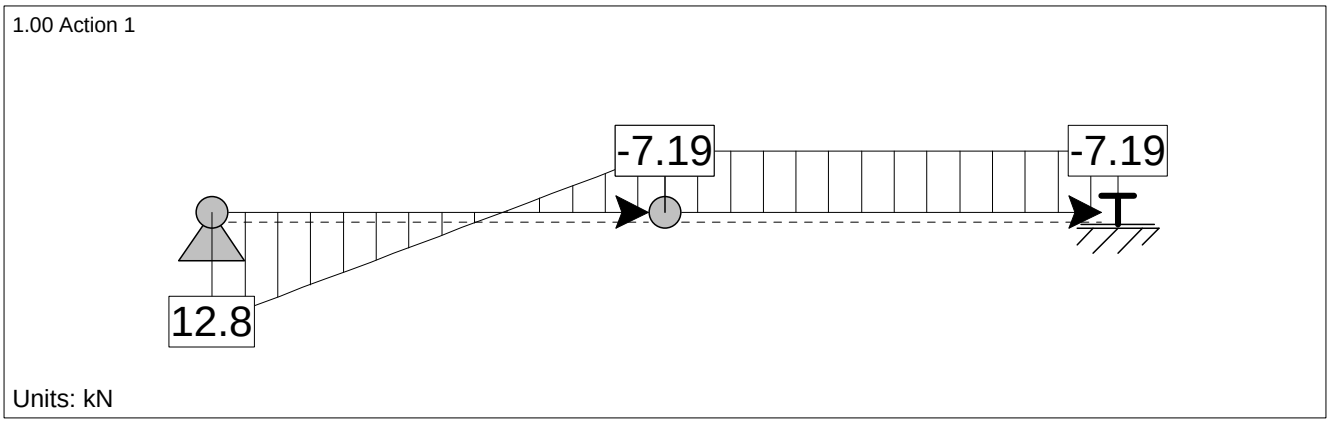
LC1: Load case 1: Bending Moments My



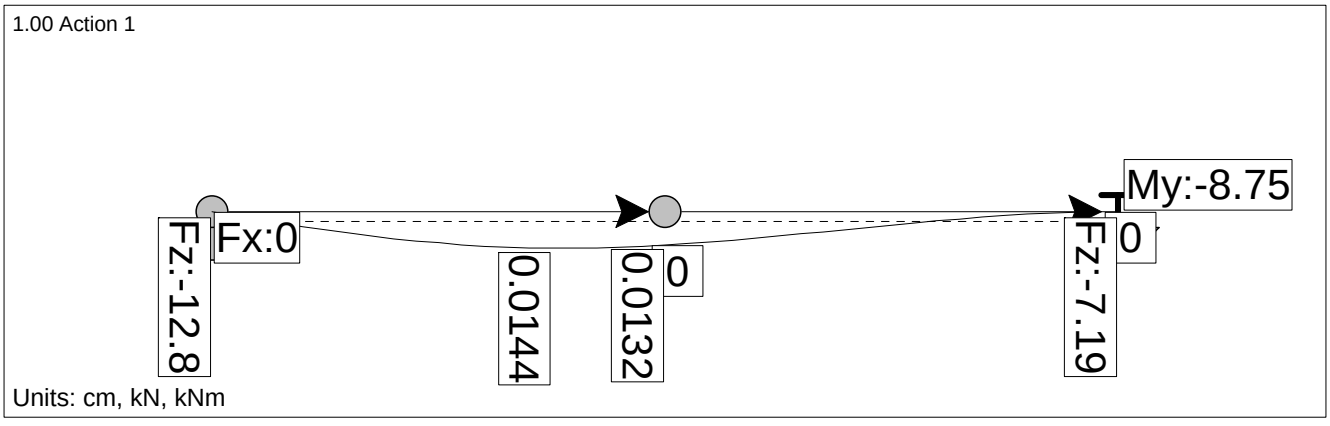
LC1: Load case 1: Axial Forces Fx



LC1: Load case 1: Shear Forces Fz



LC1: Load case 1: Displacements and Reactions



LC1: Load case 1: Rotations