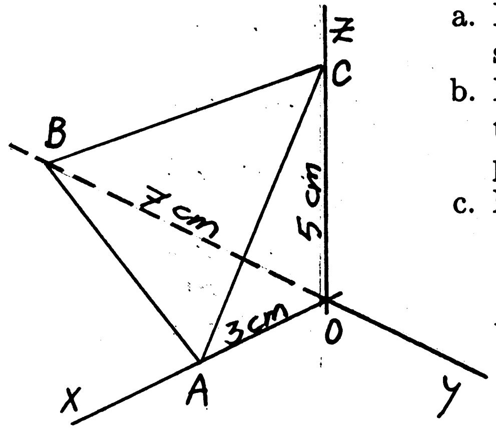
1. Na poševno mejno ploskev ABC prikazane elementarne piramide deluje enakomerna površinska obtežba $\vec{q} = 13.982\,\vec{e}_x + 6.439\,\vec{e}_y + 1.610\,\vec{e}_z$ [kN/cm^2]. Napetostna vektorja v koordinatnih ravninah $y = 0$ in $z = 0$ sta $\vec{\sigma}_y = \sigma_{yx}\,\vec{e}_x$ in $\vec{\sigma}_z = \sigma_{zx}\,\vec{e}_x - 10\,\vec{e}_z$.



a. Določi vse komponente tenzorja napetosti v koordinatnem sistemu x, y, z!

b. Razstavi tenzor napetosti na hidrostatični in deviatorični del ter ob upoštevanju Misesovega kriterija plastičnega tečenja preveri, ali je elementarna piramida v elastičnem območju!

c. Določi dolžino stranice $\overline{AO}$ po deformaciji piramide!

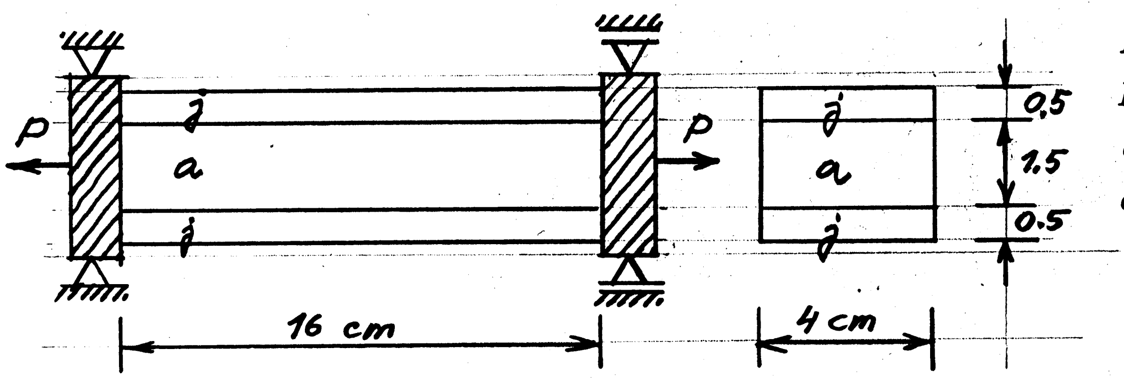
$E = 21\,000\,\mathrm{kN/cm^2}$ $\sigma_Y = 36\,\mathrm{kN/cm^2}$ $\nu = 0.3$

(40 točk)

2. Dve jekleni in ena aluminijska lamela so prosto sestavljene v prizmo in na obeh osnovnih ploskvah nepodajno priključene na absolutno togi plošči. Največja natezna normalna napetost, ki lahko nastopi v jekleni lameli, je $[\sigma_j] = 165$ MPa, v aluminijski pa $[\sigma_a] = 60$ MPa.

a. Določi dopustno natezno silo $[P]$ tako, da v nobeni od lamel ne bo presežena dovoljena vzdolžna normalna napetost.

b. Določi napetosti v lamelah, če je $P = 0$ in segrejemo prizmo za $\Delta T = 80$ K.



$E_j = 2.1 \cdot 10^5$ MPa

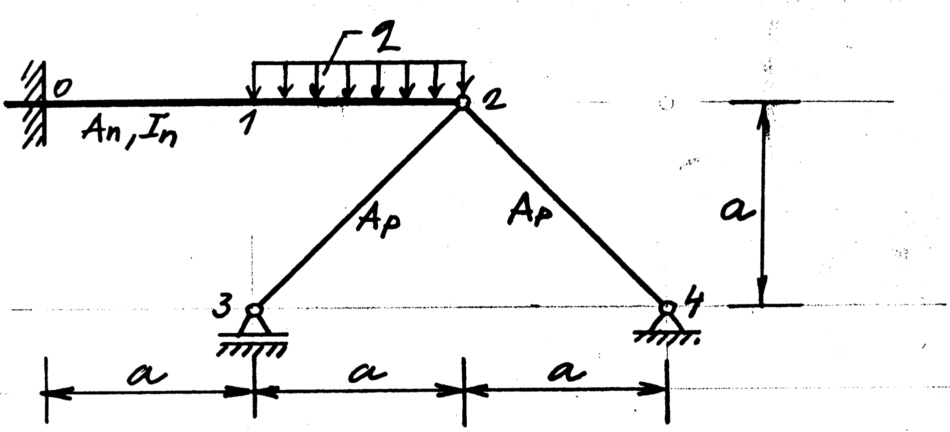
$E_a = 0.7 \cdot 10^5$ MPa

$\alpha_j = 1.25 \cdot 10^{-5}$/ K

$\alpha_a = 2.00 \cdot 10^{-5}$/ K

(30 točk)

2. Določi osni sili v palicah $\overline{23}$ in $\overline{24}$ v odvisnosti od obtežbe q!



$E = 200\,000$ MPa

$A_p = 0.0008\ \mathrm{m}^2$

$A_n = 0.0100\ \mathrm{m}^2$

$I_n = 0.0002\ \mathrm{m}^4$

$a = 4$ m

(40 točk)

Ad 1.) Ploshur ABC :

$$\frac{x}{3} - \frac{y}{7} + \frac{z}{5} = 1 \longrightarrow \phi = 35x - 15y + 21z - 105 = 0$$

$$\vec{n} = 35\vec{e}_x - 15\vec{e}_y + 21z$$

$$|\vec{n}| = \sqrt{35^2 + 15^2 + 21^2} = 43,486$$

$$\vec{e}_n = \frac{\vec{n}}{n} \longrightarrow \boxed{\vec{e}_n = 0,805\,\vec{e}_x - 0,345\,\vec{e}_y + 0,483\,\vec{e}_z}$$

$$\boxed{\sigma_{zz} = -10 \quad , \quad \sigma_{yy} = \sigma_{yz} = 0}$$

a)
$$\begin{Bmatrix} p_{nx} \\ p_{ny} \\ p_{nz} \end{Bmatrix} = \begin{Bmatrix} 13,982 \\ 6,439 \\ 1,610 \end{Bmatrix} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & 0 & 0 \\ \sigma_{zx} & 0 & -10 \end{bmatrix} \begin{Bmatrix} 0,805 \\ -0,345 \\ 0,483 \end{Bmatrix}$$

$$0,805\,\sigma_{xx} - 0,345\,\sigma_{xy} + 0,483\,\sigma_{xz} = 13,982$$

$$0,805\,\sigma_{xy} = 6,439 \longrightarrow \sigma_{xy} = 8\ kN/cm^2$$

$$0,805\,\sigma_{xz} - 0,483 \cdot 10 = 1,610 \longrightarrow \sigma_{xz} = 8\ kN/cm^2$$

$$\rightarrow 0,805\,\sigma_{xx} = 13,982 + 0,345 \cdot 8 - 0,483 \cdot 8$$

$$\sigma_{xx} = 16\ kN/cm^2$$

$$[\sigma_{ij}] = \begin{bmatrix} 16 & 8 & 8 \\ 8 & 0 & 0 \\ 8 & 0 & -10 \end{bmatrix} \qquad I_1^{\sigma} = 16 - 10 = 6$$

$$\sigma^H = \frac{1}{3} I_1^{\sigma} \rightarrow \sigma^H = 2$$

b.)
$$[\sigma_{ij}^H] = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \qquad [s_{ij}^H] = \begin{bmatrix} 14 & 8 & 8 \\ 8 & -2 & 0 \\ 8 & 0 & -12 \end{bmatrix}$$

$$I_2^s = \begin{vmatrix} -2 & 0 \\ 0 & -12 \end{vmatrix} + \begin{vmatrix} 14 & 8 \\ 8 & -12 \end{vmatrix} + \begin{vmatrix} 14 & 8 \\ 8 & -2 \end{vmatrix} \rightarrow \boxed{I_2^s = -300}$$

$$f(s) = |I_2^s| - k_M^2 \quad \dots \quad k_M = \frac{\sigma_Y}{\sqrt{3}}$$

$$f(s) = 300 - \frac{36^2}{3} = -132 \longrightarrow \boxed{EL.\,ST. \quad !}$$

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c) $\frac{1+\nu}{E} = 6{,}19 \cdot 10^{-5}$; $\frac{\nu}{E} = 1{,}43 \cdot 10^{-5}$

$$\varepsilon_{ij} = \frac{1+\nu}{E}\sigma_{ij} - \frac{\nu}{E} I_1^\sigma \delta_{ij}$$

$$\varepsilon_{xx} = 10^{-5}\left(6{,}19 \cdot 16 - 1{,}43 \cdot 6\right) \rightarrow \varepsilon_{xx} = 90{,}476 \cdot 10^{-5}$$

$\overline{AO}$: $\overline{AO}' = (1+\varepsilon_{xx}) \cdot \overline{AO} \longrightarrow \boxed{\overline{AO}' = 3{,}0027 \text{ cm}}$

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Ad 2.)

$$\boxed{\varepsilon_j = \varepsilon_a}$$

$$\boxed{\begin{aligned} A_j &= 2\ cm^2 \\ A_a &= 6\ cm^2 \end{aligned}}$$

a) $\varepsilon_j = \dfrac{\sigma_j}{E_j}$, $\varepsilon_a = \dfrac{\sigma_a}{E_a}$

$$\varepsilon_j = \varepsilon_a \rightarrow \frac{\sigma_j}{E_j} = \frac{\sigma_a}{E_a} \longrightarrow \sigma_a = \sigma_j \frac{E_a}{E_j}$$

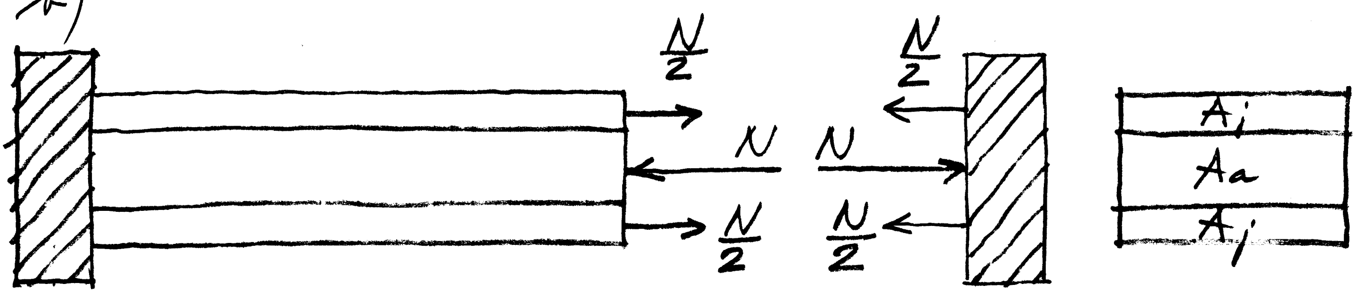
$$P = 2\sigma_j A_j + \sigma_a A_a = \sigma_j \left(2A_j + \frac{E_a}{E_j} A_a\right)$$

$$[P_j] = [\sigma_j]\left(2A_j + \frac{E_a}{E_j} A_a\right) = 0{,}099\ MN$$

$$[P_a] = [\sigma_a]\left(A_a + 2A_j \frac{E_j}{E_a}\right) = 0{,}108\ MN$$

$$\boxed{[P] = [P_j] = 0{,}099\ MN}$$

b)



$$\varepsilon_j = \frac{N}{2E_j A_j} + \alpha_j \Delta T \quad ; \quad \varepsilon_a = \frac{-N}{E_a A_a} + \alpha_a \Delta T$$

$$\varepsilon_j = \varepsilon_a \longrightarrow \frac{N}{2E_j A_j} + \alpha_j \Delta T = -\frac{N}{E_a A_a} + \alpha_a \Delta T$$

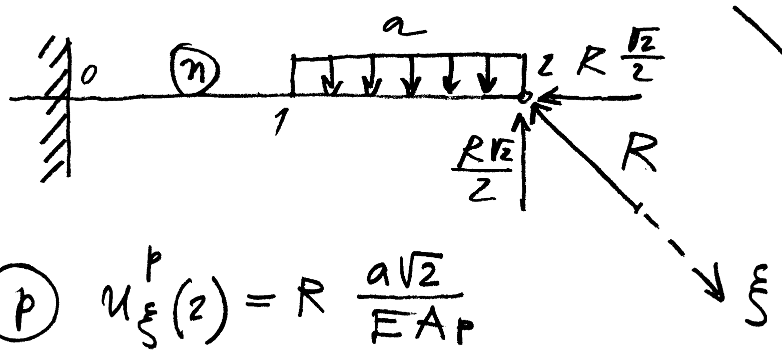
$$N \frac{2E_j A_j + E_a A_a}{2E_j E_a A_j A_a} = (\alpha_a - \alpha_j)\Delta T$$

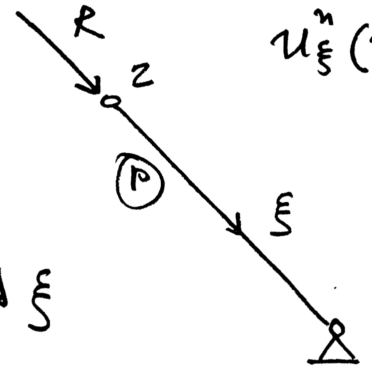
$$N = \frac{2E_j A_j E_a A_a}{2E_j A_j + E_a A_a}(\alpha_a - \alpha_j)\Delta T \rightarrow \boxed{N = 0{,}0168\ MN}$$

$$\sigma_a = \frac{-0{,}0168}{6} \cdot 10^4 \longrightarrow \boxed{\sigma_a = -28\ MPa}$$

$$\sigma_j = \frac{0{,}0168}{2 \cdot 2} \cdot 10^4 \longrightarrow \boxed{\sigma_j = 42\ MPa}$$

~~Ad~~ 3.) $N_{23} = 0$!





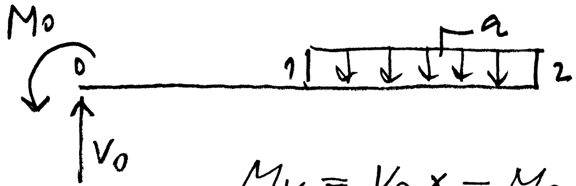
$$u_\xi^n(2) = u_\xi^p(2) \ !$$

(p) $u_\xi^p(2) = R\,\frac{a\sqrt{2}}{EA_p}$

$$\boxed{u_\xi^p(2) = 0{,}03536\ R}$$

(n) $u_x^{(n)}(2) = -R\frac{\sqrt{2}}{2}\cdot\frac{2a}{EA_n} \longrightarrow \boxed{u_x^{(n)}(2) = -0{,}00283\ R}$

$$u_z^n(2) = w_R + w_q = -R\frac{\sqrt{2}}{2}\cdot\frac{(2a)^3}{3EI_n} + w_q$$



$$M_0 = \frac{3qa^2}{2}\ , \quad V_0 = qa$$

$$M_y = V_0 x - M_0 - \frac{q}{2}\langle x-a\rangle^2 = qax - \frac{3qa^2}{2} - \frac{q}{2}\langle x-a\rangle^2$$

$$EI_y w'' = -M_y = -\frac{q}{2}\left(2ax - 3a^2 - \langle x-a\rangle^2\right)$$

$$EI_y w' = -\frac{q}{2}\left(ax^2 - 3a^2x - \frac{1}{3}\langle x-a\rangle^3\right) + C_1$$

$$EI_y w = -\frac{q}{2}\left(a\frac{x^3}{3} - \frac{3}{2}a^2x^2 - \frac{1}{12}\langle x-a\rangle^4\right) + C_1x + C_2$$

$x=0 \ \dots \ w=0\ ,\ w'=0 \longrightarrow C_1 = C_2 = 0$

$x = 2a \ \dots \ w = -\frac{q}{2}\left(a\cdot\frac{8a^3}{3} - \frac{3}{2}a^2\cdot 4a^2 - \frac{1}{12}a^4\right)$

$$w_q = q\,\frac{41a^4}{24EI_n}$$

$$u_z^n(2) = q\,\frac{41a^4}{24EI_n} - R\,\frac{4\sqrt{2}a^3}{3EI_n}$$

$$\boxed{u_z^n(2) = 10{,}93333\ q - 3{,}01699\ R}$$

tč. 2: $u_\xi^p = u_x^n e_{\xi x} + u_z^n e_{\xi z}$

$$0{,}03536\,R = \frac{\sqrt{2}}{2}\left(-0{,}00283\,R + 10{,}93333\,q - 3{,}01699\,R\right)$$

$$\boxed{R = 3{,}562\ q}$$