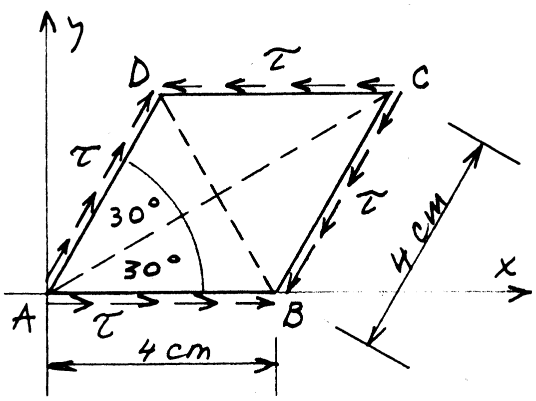
1. Na enakomerno debelo steno rombaste oblike deluje zunanja obtežba $\tau = 16\,\mathrm{kN/cm^2}$, kot kaže skica. Pri tem se diagonala $\overline{AC}$ skrajša za 0.0177 cm, diagonala $\overline{BD}$ pa se podaljša za 0.0060 cm. V steni, ki je narejena iz linearno elastične izotropne snovi, vlada homogeno ravninsko napetostno stanje ($\sigma_{xz} = \sigma_{yz} = \sigma_{zz} = 0$).



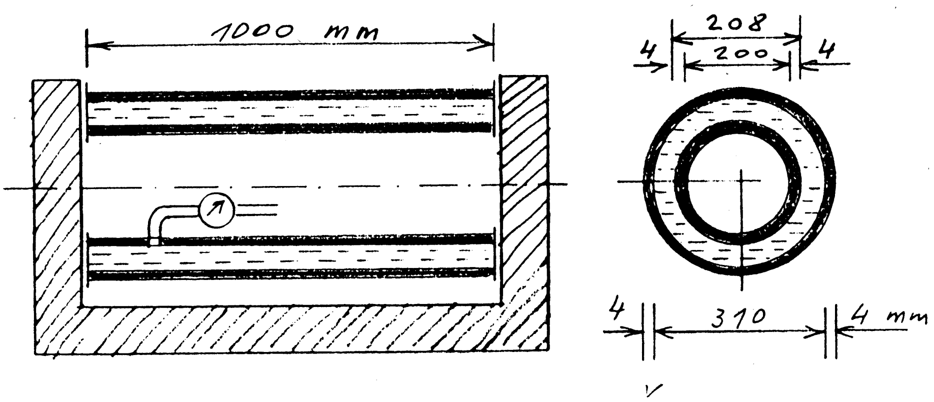
a. Določi napetosti glede na koordinatni sistem (x, y) ter dokaži, da v prerezih AC in BD ni strižnih napetosti. Določi tudi normalne napetosti v teh dveh prerezih!
b. Določi elastični modul E in koeficient prečne kontrakcije ν!
c. Določi spremembo pravega kota med diagonalama in specifično spremembo debeline stene!
c. Določi velikosti in smeri glavnih normalnih deformacij!

(45 točk)

2. V vmesni prostor med dvema bakrenima cevema z debelino stene $\delta = 4$ mm načrpamo nestisljivo hladilno tekočino. Koliko tekočine porabimo, da znaša hidrostatični tlak $p = 6$ MPa? Kolikšne so tedaj normalne napetosti v tangencialni smeri v obeh ceveh? (Tesnila ob nepodajnih priključnih ploščah omogočajo neovirano deformiranje cevi. Vzdolžne normalne napetosti so zanemarljive.)

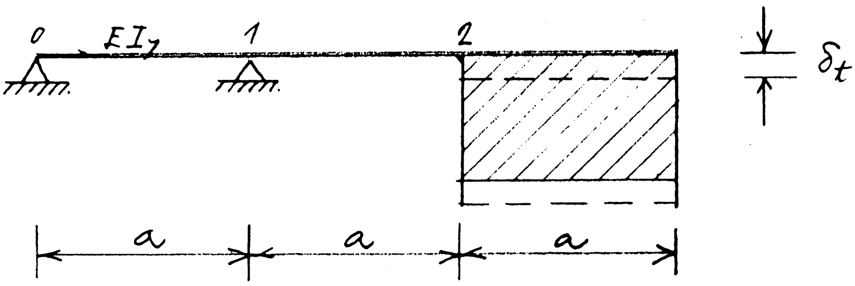
$E = 100\,000$ MPa

$\nu = 0.3$



(35 točk)

3. Elastičen nosilec je v točki 2 togo vpet v masiven temelj. Določi in reakcije, ki nastopijo v podporah 0 in 1, vce se masivni temelj enakomerno posede za δ_t! Rezultate izrazi v odvisnosti od posedka δ_t!

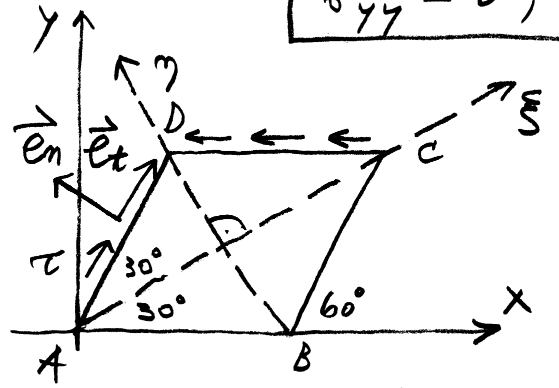


(35 točk)

Ad 1.)

$$\boxed{\begin{array}{l}\sigma_{xz}=\sigma_{yz}=\sigma_{zz}=0\\ \sigma_{yy}=0,\quad \sigma_{yx}=\sigma_{xy}=-\tau\end{array}}$$

a)



Rob $\overline{AD}$: $\sigma_{nn}=0$, $\sigma_{nt}=\tau$

$e_{nx}=-\frac{\sqrt{3}}{2}$ $\quad e_{ny}=\frac{1}{2}$

$e_{tx}=\frac{1}{2}$ $\quad e_{ty}=\frac{\sqrt{3}}{2}$

$$\sigma_{nn}=\sigma_{xx}e_{nx}^2+2\sigma_{xy}e_{nx}e_{ny}$$

$$=\frac{3}{4}\sigma_{xx}+2\tau\frac{\sqrt{3}}{4}=0 \rightarrow \boxed{\sigma_{xx}=-\frac{2\sqrt{3}}{3}\tau}$$

Kontrola: $\sigma_{nt}=\sigma_{xx}e_{nx}e_{tx}+\sigma_{xy}(e_{nx}e_{ty}+e_{ny}e_{tx})$

$$\sigma_{nt}=\frac{2\sqrt{3}}{3}\tau\cdot\frac{\sqrt{3}}{4}-\tau\left(-\frac{3}{4}+\frac{1}{4}\right)=\tau \quad \text{O.K.}$$

Koord. sist. (ξ,η): $\quad e_{\xi x}=\frac{\sqrt{3}}{2}$, $\quad e_{\xi y}=\frac{1}{2}$

$e_{\eta x}=-\frac{1}{2}$, $\quad e_{\eta y}=\frac{\sqrt{3}}{2}$

$$\sigma_{\xi\xi}=\sigma_{xx}e_{\xi x}^2+2\sigma_{xy}e_{\xi x}e_{\xi y}$$

$$=-\frac{2\sqrt{3}}{3}\tau\cdot\frac{3}{4}-2\tau\cdot\frac{\sqrt{3}}{4} \rightarrow \boxed{\sigma_{\xi\xi}=-\sqrt{3}\,\tau}$$

$$\sigma_{\eta\eta}=\sigma_{xx}e_{\eta x}^2+2\sigma_{xy}e_{\eta x}e_{\eta y}$$

$$=-\frac{2\sqrt{3}}{3}\tau\cdot\frac{1}{4}-2\tau\cdot\left(-\frac{\sqrt{3}}{4}\right) \rightarrow \boxed{\sigma_{\eta\eta}=\frac{\sqrt{3}}{3}\tau}$$

$$\sigma_{\xi\eta}=\sigma_{xx}e_{\xi x}e_{\eta x}+\sigma_{xy}(e_{\xi x}e_{\eta y}+e_{\xi y}e_{\eta x})$$

$$=-\frac{2\sqrt{3}}{3}\tau\left(-\frac{\sqrt{3}}{4}\right)-\tau\left(\frac{3}{4}-\frac{1}{4}\right) \rightarrow \boxed{\sigma_{\xi\eta}=0}$$

V prerezu AC ni strižnih nap. Zaradi simetrije tudi v prerezu BD ne.

$$(x,y,z): \quad [\sigma_{ij}] = \begin{bmatrix} -18{,}475 & -16 & 0 \\ -16 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$(\xi,\eta,\zeta): \quad [\sigma_{\alpha\beta}] = \begin{bmatrix} -27{,}713 & 0 & 0 \\ 0 & 9{,}238 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \boxed{\begin{aligned} \sigma_{\xi\xi} &= \sigma_{11} \\ \sigma_{\eta\eta} &= \sigma_{22} \\ \sigma_{\zeta\zeta} &= \sigma_{33} \\ &= 0 \end{aligned}}$$

b) $I_1^{\sigma} = \sigma_{xx} + \sigma_{yy} = -18{,}475 \ kN/cm^2$

$\overline{AC} = 2 \cdot 4 \cdot \cos 30^\circ \longrightarrow \overline{AC} = 6{,}92820 \ cm$

$\overline{BD} = 2 \cdot 4 \cdot \sin 30^\circ \longrightarrow \overline{BD} = 4 \ cm$

$$\varepsilon_{\xi\xi} = \frac{\Delta \overline{AC}}{\overline{AC}} = \frac{-0{,}0177}{6{,}92820} \longrightarrow \varepsilon_{\xi\xi} = -0{,}002555$$

$$\varepsilon_{\eta\eta} = \frac{\Delta \overline{BD}}{\overline{BD}} = \frac{0{,}0060}{4} \longrightarrow \varepsilon_{\eta\eta} = 0{,}001500$$

$$\varepsilon_{\xi\xi} = \frac{1}{E}(\sigma_{\xi\xi} - \nu\sigma_{\eta\eta}) \rightarrow E\varepsilon_{\xi\xi} = \sigma_{\xi\xi} - \nu\sigma_{\eta\eta}$$

$$\varepsilon_{\eta\eta} = \frac{1}{E}(\sigma_{\eta\eta} - \nu\sigma_{\xi\xi}) \rightarrow E\varepsilon_{\eta\eta} = \sigma_{\eta\eta} - \nu\sigma_{\xi\xi}$$

$$\begin{bmatrix} \varepsilon_{\xi\xi} & \sigma_{\eta\eta} \\ \varepsilon_{\eta\eta} & \sigma_{\xi\xi} \end{bmatrix} \cdot \begin{bmatrix} E \\ \nu \end{bmatrix} = \begin{bmatrix} \sigma_{\xi\xi} \\ \sigma_{\eta\eta} \end{bmatrix} \rightarrow \begin{bmatrix} 0{,}002555 & 9{,}238 \\ 0{,}001500 & -27{,}713 \end{bmatrix} \begin{bmatrix} E \\ \nu \end{bmatrix} = \begin{bmatrix} -27{,}713 \\ 9{,}238 \end{bmatrix}$$

$E = 11\,988 \ kN/cm^2$	$\nu = 0{,}316$

c) $\boxed{D_{\xi\eta} = 0}$ $\quad \sigma_{zz} = 2\mu\,\varepsilon_{zz} + \lambda(\varepsilon_{\xi\xi} + \varepsilon_{\eta\eta} + \varepsilon_{zz}) = 0$

$$\varepsilon_{zz} = -\frac{\lambda}{2\mu+\lambda}(\varepsilon_{\xi\xi} + \varepsilon_{\eta\eta})$$

$$\varepsilon_{zz} = -\frac{\nu}{1-\nu}(-0{,}002555 + 0{,}001500) \longrightarrow \boxed{\varepsilon_{zz} = 0{,}000487}$$

d) $\varepsilon_{11} = \varepsilon_{\xi\xi} \ldots \vec{e}_1 = \vec{e}_\xi$, $\varepsilon_{22} = \varepsilon_{\eta\eta} \ldots \vec{e}_2 = \vec{e}_\eta$, $\varepsilon_{33} = \varepsilon_{zz} \ldots \vec{e}_3 = \vec{e}_z$

~~Ad~~ 2) $D_1 = 208$ mm , $D_2 = 310$ mm

$V = \frac{\pi l}{4}(D_2^2 - D_1^2)$ ... $V' = \frac{\pi l}{4}(D_2'^2 - D_1'^2)$

Notranja cev: $\sigma_{ss}^1 = -\frac{p D_1}{2\delta}$, $\sigma_{rr}^1 = -p$

$\varepsilon_{ss}^1 = \frac{1}{E}(\sigma_{ss}^1 - \nu \sigma_{rr}^1) = -\frac{p}{E}\left(\frac{D_1}{2\delta} - \nu\right)$

$\varepsilon_{ss}^1 = -\frac{p}{E}\left(\frac{D_1}{2\delta} - \nu\right) \rightarrow \boxed{\varepsilon_{ss}^1 = -0{,}00154}$

Zunanja cev: $\sigma_{ss}^2 = \frac{p D_2}{2\delta}$, $\sigma_{rr}^2 = -p$

$\varepsilon_{ss}^2 = \frac{1}{E}(\sigma_{ss}^2 - \nu \sigma_{rr}^2) = \frac{p}{E}\left(\frac{D_2}{2\delta} - \nu\right)$

$\boxed{\varepsilon_{ss}^2 = 0{,}00231}$

$D_1' = D_1(1 + \varepsilon_{ss}^1) = 208(1 - 0{,}00154)$

$\underline{D_1' = 207{,}679 \text{ mm}}$

$D_2' = D_2(1 + \varepsilon_{ss}^2) = 310 \cdot (1 + 0{,}0023)$

$\underline{D_2' = 310{,}715 \text{ mm}}$

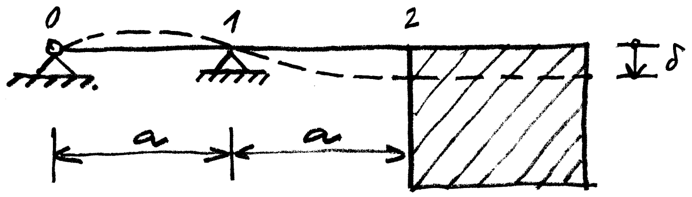
$V' = \frac{\pi \cdot 1000}{4}(310{,}715^2 - 207{,}679^2) = 41.950.661 \text{ mm}$

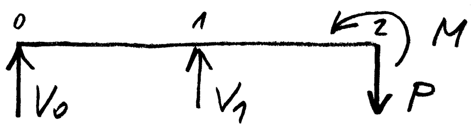
$\boxed{V' = 41{,}951 \text{ litrov}}$

$\sigma_{ss}^n = -\frac{6 \cdot 208}{2 \cdot 4} \rightarrow \sigma_{ss}^n = -156$ MPa

$\sigma_{ss}^z = \frac{6 \cdot 310}{2 \cdot 4} \rightarrow \sigma_{ss}^z = 232{,}5$ MPa

Ad 3.





$$V_0 = \frac{M}{a} - P$$
$$V_1 = 2P - \frac{M}{a}$$

$$M_y = V_0 x + V_1 \langle x-a\rangle = \left(\frac{M}{a} - P\right)x + \left(2P - \frac{M}{a}\right)\langle x-a\rangle$$
$$M_y = -P\left(x - 2\langle x-a\rangle\right) + \frac{M}{a}\left(x - \langle x-a\rangle\right) = -EI_y w''$$

$$EI_y w'' = P\left(x - 2\langle x-a\rangle\right) - \frac{M}{a}\left(x - \langle x-a\rangle\right)$$
$$EI_y w' = \frac{P}{2}\left(x^2 - 2\langle x-a\rangle^2\right) - \frac{M}{2a}\left(x^2 - \langle x-a\rangle^2\right) + C_1$$
$$EI_y w = \frac{P}{6}\left(x^3 - 2\langle x-a\rangle^3\right) - \frac{M}{6a}\left(x^3 - \langle x-a\rangle^3\right) + C_1 x + C_2$$

$$x=0 \ \ldots \ w=0 \rightarrow \boxed{C_2 = 0}$$
$$x=a \ \ldots \ w=0 \rightarrow \frac{P}{6}a^3 - \frac{M}{6a}a^3 + C_1 a = 0$$
$$\boxed{C_1 = -\frac{Pa^2}{6} + \frac{Ma}{6}}$$

$$\omega_y = -\frac{1}{EI_y}\left[\frac{P}{2}\left(x^2 - 2\langle x-a\rangle^2\right) - \frac{M}{2a}\left(x^2 - \langle x-a\rangle^2\right) - \frac{Pa^2}{6} + \frac{Ma}{6}\right]$$

$$\boxed{\omega_y = \frac{P}{6EI_y}\left(a^2 - 3x^2 + 6\langle x-a\rangle^2\right) - \frac{M}{6aEI_y}\left(a^2 - 3x^2 + 3\langle x-a\rangle^2\right)}$$

$$\boxed{w = \frac{P}{6EI_y}\left(x^3 - 2\langle x-a\rangle^3 - a^2 x\right) - \frac{M}{6aEI_y}\left(x^3 - \langle x-a\rangle^3 - a^2 x\right)}$$

$x = 2a \; \ldots \; w_y = 0$

$$w_y(2a) = \frac{P}{6EI_y}\left(a^2 - 12a^2 + 6a^2\right) - \frac{M}{6aEI_y}\left(a^2 - 12a^2 + 3a^2\right)$$

$$w_y(2a) = -\frac{5Pa^2}{6EI_y} + \frac{4Ma}{3EI_y} = 0 \;\rightarrow\; \boxed{M = P \cdot \frac{5a}{8}}$$

$$w(2a) = \frac{P}{6EI_y}\left(8a^3 - 2a^3 - 2a^3\right) - \frac{M}{6aEI_y}\left(8a^3 - a^3 - 2a^3\right)$$

$$\boxed{w(2a) = P\frac{2a^3}{3EI_y} - M\frac{5a^2}{6EI_y}}$$

$$w(2a) = P\frac{2a^3}{3EI_y} - P\frac{5a}{8} \cdot \frac{5a^2}{6EI_y}$$

$$\boxed{w(2a) = P\frac{7a^3}{48EI_y}} \;\rightarrow\; \boxed{w(2a) = \delta}$$

$$\boxed{P = \delta\frac{48EI_y}{7a^3}} \;\longrightarrow\; \boxed{M = \delta\frac{30EI_y}{7a^2}}$$

$$\boxed{V_0 = -\delta\frac{18EI_y}{7a^3}} \qquad \boxed{V_1 = \delta\frac{66EI_y}{7a^3}}$$