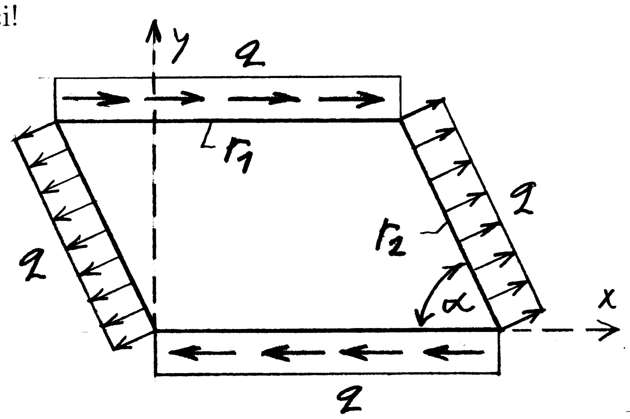
1. Na element enakomerno debele stene deluje zunanja obtežba $q$, kot kaže skica. Ob predpostavki, da vlada v elementu homogeno ravninsko napetostno stanje, določi kot $\alpha$, pri katerem je element v ravnotežju! Določi velikosti in smeri glavnih normalnih deformacij v tem primeru ter jih označi na skici!

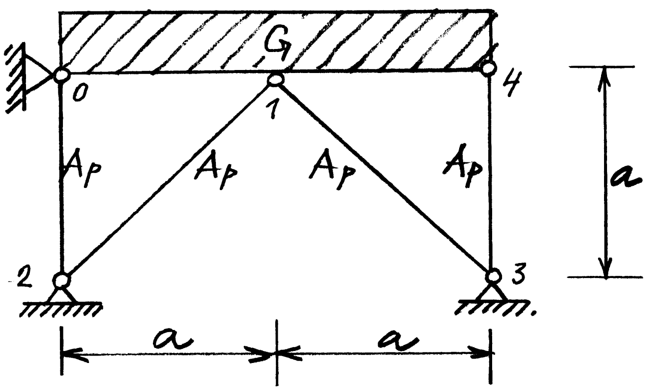



$E = 200\,000\,\mathrm{MPa}$

$\nu = 0.25$

NASVET: Zapiši ravnotežna pogoja na robovih $r_1$ in $r_2$, določi $\alpha$, nato pa še preostale napetosti!

2. Absolutno toga greda teže $G$ je podprta, kot kaže skica. Določi osne sile v podpornih palicah! Za koliko moramo spremeniti temperaturo palice $\overline{34}$, da v palicah $\overline{12}$ in $\overline{13}$ ne bo napetosti?




$E = 200\,000\,\mathrm{MPa}$

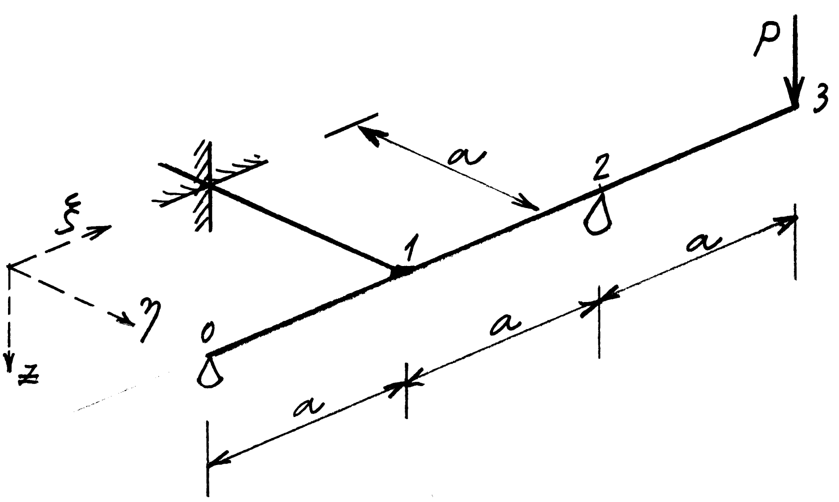
$\alpha_T = 1.25 \cdot 10^{-5}/\mathrm{K}$

$a = 2\,\mathrm{m}$

$A_p = 40\,\mathrm{cm}^2$

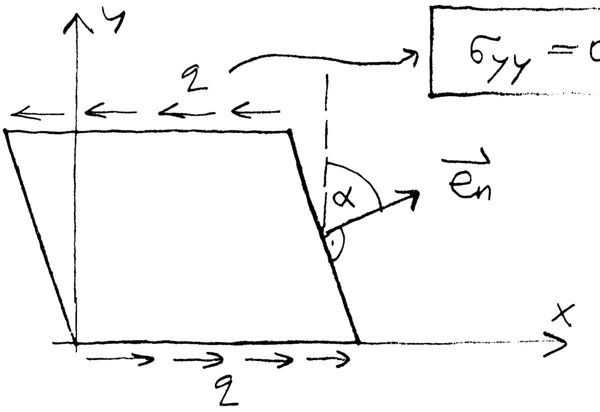
$G = 0.4\,\mathrm{MN}$

3. Določi navpični pomik točke 3! Za vse elemente konstrukcije velja $EI_y = GI_x$. Rezultat izrazi v odvisnosti od obtežbe $P$!



MTT IZPIT 10. 2. 2000

## Ad 1.)




$\boxed{\sigma_{yy} = 0, \quad \sigma_{yx} = \sigma_{xy} = q}$

$e_{nx} = \sin\alpha$

$e_{ny} = \cos\alpha$

$e_{nz} = 0$

$\vec{e}_n = \sin\alpha\, \vec{e}_x + \cos\alpha\, \vec{e}_y \qquad \vec{p}_n = q\, \vec{e}_n$

$\vec{p}_n = q \sin\alpha\, \vec{e}_x + q\cos\alpha\, \vec{e}_y = \vec{\sigma}_x e_{nx} + \vec{\sigma}_y e_{ny}$

$p_{nx} = q\sin\alpha = \sigma_{xx} e_{nx} + \sigma_{yx} e_{ny} = \sigma_{xx}\sin\alpha + q\cos\alpha$

$p_{ny} = q\cos\alpha = \sigma_{xy} e_{nx} + \sigma_{yy} e_{ny} = q \sin\alpha$

$\to \cos\alpha = \sin\alpha \longrightarrow \operatorname{tg}\alpha = 1 \longrightarrow \boxed{\alpha = 45^\circ}$

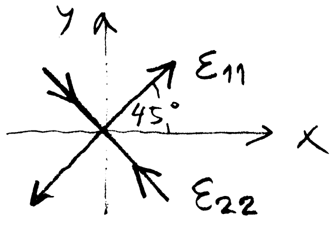
$\sin\alpha = \cos\alpha = \frac{\sqrt{2}}{2} \to q\frac{\sqrt{2}}{2} = \sigma_{xx}\frac{\sqrt{2}}{2} + q\frac{\sqrt{2}}{2}$

$\boxed{\sigma_{xx} = 0}$

$$[\sigma_{ij}] = \begin{bmatrix} 0 & q & 0 \\ q & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \to \sigma_{11,22} = \pm\sqrt{\sigma_{xy}^2} = \pm q$$

$\boxed{\sigma_{11} = q \quad , \quad \sigma_{22} = -q}$

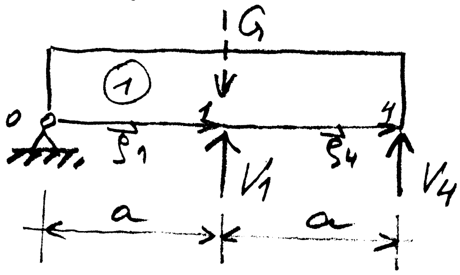
$$\operatorname{tg} 2\alpha_\sigma = \frac{2\sigma_{xy}}{\sigma_{xx} - \sigma_{yy}} = \infty \longrightarrow 2\alpha_\sigma = 90^\circ \to \boxed{\alpha_\sigma = 45^\circ}$$




$$\boxed{\begin{aligned} \varepsilon_{11} &= \frac{1+\nu}{E} q \\ \varepsilon_{22} &= -\frac{1+\nu}{E} q \\ \varepsilon_{33} &= 0 \end{aligned}}$$

$\boxed{I_1^\sigma = 0}$

# Ad 2.)




$$\Sigma M^0 = 0 \rightarrow V_1 a + V_4 \cdot 2a - Ga = 0$$

$$\boxed{V_1 + 2V_4 = G}$$

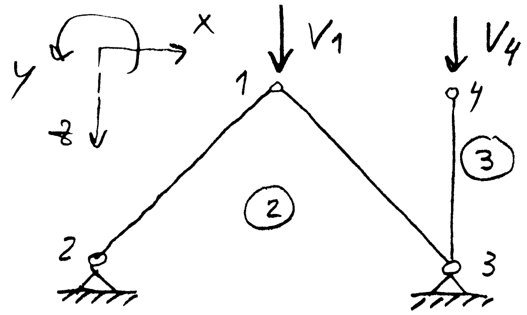
$$\vec{u}_0 = \vec{0} \quad , \quad \vec{\omega}_0 = \omega_y \vec{e}_y$$

$$\vec{\rho}_1 = a\vec{e}_x \quad , \quad \vec{\rho}_4 = 2a\,\vec{e}_x$$

$$\vec{u}_1 = \vec{u}_0 + \vec{\omega}_0 \times \vec{\rho}_1 = \omega \vec{e}_y \times a\vec{e}_x \rightarrow \vec{u}_1 = -a\omega_y \vec{e}_z$$

$$\vec{u}_4 = -2a\omega_y \vec{e}_z$$

$$\boxed{w_1^{(1)} = -a\omega_y} \quad \boxed{w_4^{(1)} = -2a\omega_y}$$




$$\boxed{w_4^{(3)} = V_4 \frac{a}{EA_P}}$$

$$w_4^{(3)} = \frac{V_4}{4000} = 0{,}00025\, V_4$$

$$② : \ l_{12} = l_{13} = a\sqrt{2}$$

$$k_{12} = k_{13} = \frac{E A_p}{a\sqrt{2}} = \frac{20000 \cdot 40}{200\sqrt{2}} = 2828{,}4 \ kN/cm$$

$$[\check{K}_{12}] = 2828{,}4 \begin{array}{c|cc} & -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ \hline -\frac{\sqrt{2}}{2} & \frac{1}{2} & -\frac{1}{2} \\ \frac{\sqrt{2}}{2} & -\frac{1}{2} & \frac{1}{2} \end{array} \rightarrow [K_{12}] = \begin{bmatrix} 1414.2 & -1414.2 \\ -1414.2 & 1414.2 \end{bmatrix}$$

$$[K_{11}] = \begin{bmatrix} -2828{,}4 & 0 \\ 0 & -2828{,}4 \end{bmatrix} \qquad [K_{13}] = \begin{bmatrix} 1414.2 & 1414.2 \\ 1414.2 & 1414.2 \end{bmatrix}$$

$$[K_{11}] \begin{Bmatrix} u_1 \\ w_1 \end{Bmatrix} + \begin{Bmatrix} 0 \\ V_1 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$\begin{bmatrix} -2828{,}4 & 0 \\ 0 & -2828{,}4 \end{bmatrix} \begin{Bmatrix} u_1 \\ w_1 \end{Bmatrix} = \begin{Bmatrix} 0 \\ -V_1 \end{Bmatrix} \rightarrow \begin{array}{l} u_1^{②} = 0 \\ \boxed{w_1^{②} = \frac{V_1}{2828{,}4}} \end{array}$$

$$w_1^{①} = w_1^{②} \rightarrow -200 \ \omega_y = \frac{V_1}{2828{,}4} = 0{,}0003536 \ V_1$$

$$w_4^{①} = w_4^{③} \rightarrow -400 \ \omega_y = 0{,}00025 \ V_4$$

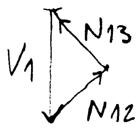
$$\omega_y = -\frac{V_1}{200} \cdot 0{,}0003536 = -\frac{V_4}{400} \cdot 0{,}00025$$

$$1{.}7678 \ V_1 = 0{,}6250 \ V_4$$

$$V_1 = 0{,}3536 \ V_4 = G - 2V_4 \rightarrow 2{,}3536 \ V_4 = G$$

$$\boxed{V_4 = 0{,}4249 \ G} \rightarrow \boxed{V_4 = 0{,}1700 \ MN}$$

$$V_1 = G - 2V_4 \rightarrow \boxed{V_1 = 0{,}1502 \ G} \rightarrow \boxed{V_1 = 0{,}0600 \ MN}$$




$$N_{12} = N_{13} = -\frac{V_1}{\sqrt{2}} = \underline{\underline{-0{,}0424 \ MN}}$$

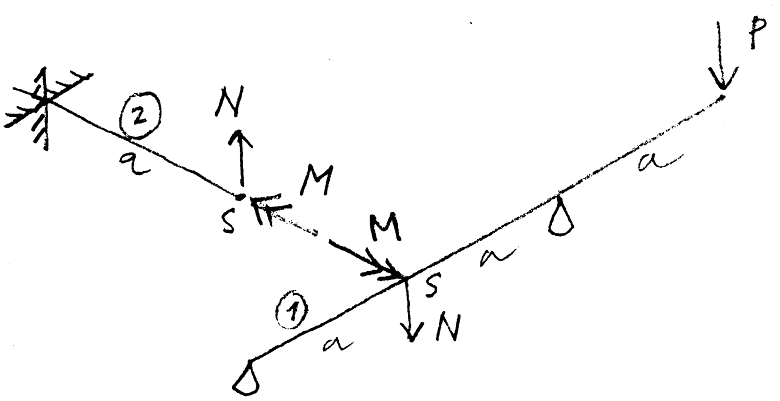
$$\boxed{\Delta l_{34} = 0} : \qquad \Delta l_{34} = -\tilde{V}_4 \frac{a}{EA_P} + \alpha_T \Delta T a$$

$$\tilde{V}_4 = \frac{G}{2} \quad \rightarrow \quad G \frac{a}{2EA_P} = \alpha_T \Delta T a$$

$$\boxed{\Delta T = \frac{G}{2EA_P \alpha_T}} \quad \rightarrow \quad \Delta T = \frac{400 \cdot 10^5}{2 \cdot 20000 \cdot 40 \cdot 1{,}25}$$

$$\boxed{\Delta T = 20 \ K}$$

## Ad 3)

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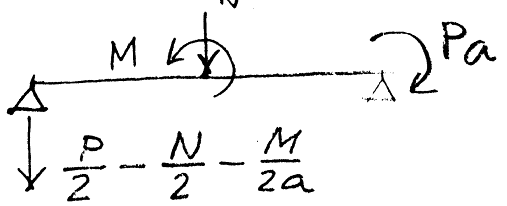



$$M_y = -\left(\frac{P}{2} - \frac{N}{2} - \frac{M}{2a}\right)x - N\langle x-a\rangle - M\langle x-a\rangle^0 = -EI_y w''$$

$$EI_y w'' = \frac{P}{2}x - \frac{N}{2}\left(x - 2\langle x-a\rangle\right) - \frac{M}{2a}\left(x - 2a\langle x-a\rangle^0\right)$$

$$EI_y w' = P\frac{x^2}{4} - \frac{N}{2}\left(\frac{x^2}{2} - \langle x-a\rangle^2\right) - \frac{M}{2a}\left(\frac{x^2}{2} - 2a\langle x-a\rangle\right) + C_1$$

$$EI_y w = P\frac{x^3}{12} - \frac{N}{2}\left(\frac{x^3}{6} - \frac{1}{3}\langle x-a\rangle^3\right) -$$

$$- \frac{M}{2a}\left(\frac{x^3}{6} - a\langle x-a\rangle^2\right) + C_1 x + C_2$$

$x = 0 \quad \ldots \quad w = 0 \quad \rightarrow \quad C_2 = 0$

$x = 2a \quad \ldots \quad w = 0$

$$P\cdot\frac{8a^3}{12} - \frac{N}{2}\left(\frac{8a^3}{6} - \frac{a^3}{3}\right) -$$

$$- \frac{M}{2a}\left(\frac{8a^3}{6} - a^3\right) + 2a\, C_1 = 0$$

$$\boxed{C_1 = -\frac{Pa^2}{3} + \frac{Na^2}{4} + \frac{Ma}{12}}$$

$$\boxed{w_s^{①} = -\frac{Pa^3}{4EI_y} + \frac{Na^3}{6EI_y}}$$

$$\boxed{\omega_s^{①} = \frac{Pa^2}{12EI_y} + \frac{Ma}{6EI_y}}$$

② $\boxed{w_s^{②} = -\frac{Na^3}{3EI_y}}$ $\qquad$ $\boxed{\omega_s^{②} = -\frac{Ma}{GI_x}}$

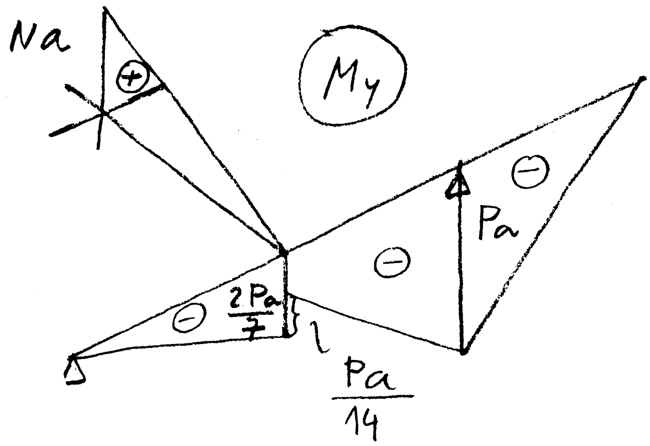
$$w_s^{①} = w_s^{②} \quad \ldots \quad \frac{a^3}{EI_y}\left(-\frac{P}{4} + \frac{N}{6}\right) = -\frac{Na^3}{3EI_y}$$

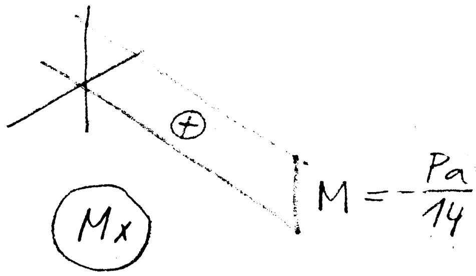
$$\boxed{N = \frac{P}{2}}$$

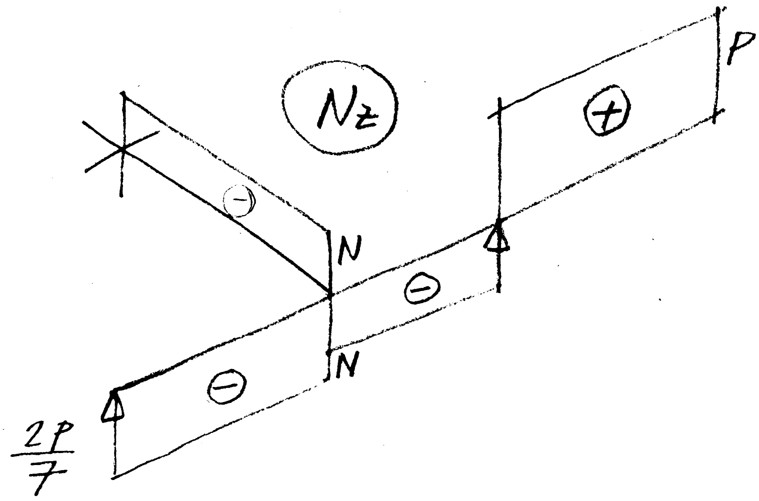
$$\omega_s^{①} = \omega_2^{②} \quad \ldots \quad \frac{a}{EI_y}\left(\frac{Pa}{12} + \frac{M}{6}\right) = -\frac{Ma}{EI_y}$$

$$\boxed{M = -\frac{Pa}{14}}$$

$$V_0 = \frac{P}{2} - \frac{P}{4} + \frac{P}{28} \quad \longrightarrow \quad \boxed{V_0 = \frac{2P}{7}}$$


Na
+
My
−
Pa
−
2Pa
7
−
Pa
14


+
Mx
M = − Pa/14


Nz
P
+
−
N
−
N
−
2P
7