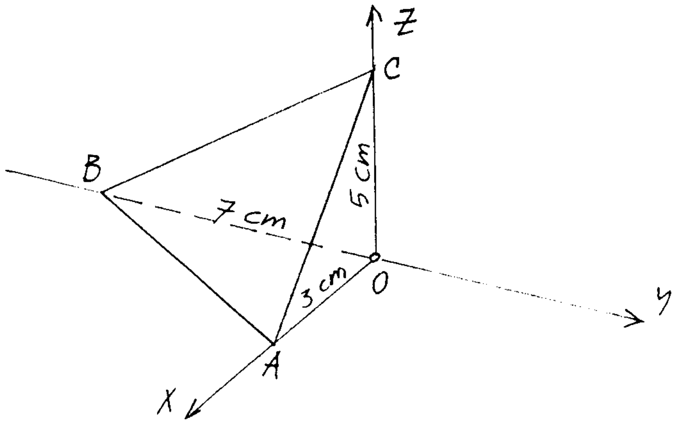
1. Na poševno mejno ploskev ABC prikazane elementarne piramide deluje enakomerna površinska obtežba $\vec{q} = 13.982\,\vec{e}_x + 6.439\,\vec{e}_y + 1.610\,\vec{e}_z$ [kN/cm^2]. Normalna napetost v mejni ploskvi $z = 0$ je tlačna in znaša $10\,\text{kN/cm}^2$. V mejni ploskvi $y = 0$ nastopa samo strižna napetost v smeri x.

   a. Določi vse komponente tenzorja napetosti v koordinatnem sistemu x, y, z!
   b. Razstavi tenzor napetosti na hidrostatični in deviatorični del ter ob upoštevanju Misesovega kriterija plastičnega tečenja preveri, ali je elementarna piramida v elastičnem območju!
   c. Določi dolžini stranic $\overline{AB}$ in $\overline{AO}$ po deformaciji piramide!



$E = 21\,000\,\text{kN/cm}^2$

$\sigma_Y = 36\,\text{kN/cm}^2$

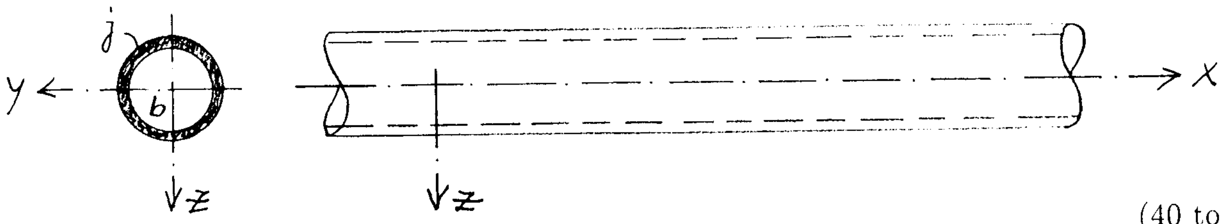
$\nu = 0.3$

(35 točk)

2. Na okroglo bakreno palico premera 2 cm želimo natakniti jekleno cev z notranjim premerom 1.996 cm in debelino stene 2 mm. Za koliko moramo segreti cev, da jo lahko nataknemo na jedro? Določi napetosti v cevi in jedru, ko se temperaturi cevi in jedra spet izenačita s temperaturo okolja! Kolikšen je tedaj premer bakrenega jedra? (Trenje med jedrom in cevjo lahko zanemarimo. V vzdolžni smeri deformacije niso ovirane.)

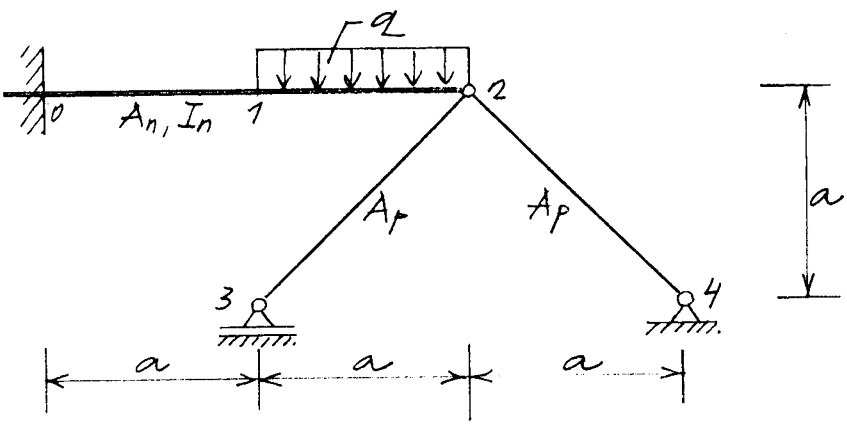
$E_j = 210\,000\,\text{MPa}$ $\quad \nu_j = 0.30$ $\quad \alpha_j = 1.2 \cdot 10^{-5}/K$

$E_b = 70\,000\,\text{MPa}$ $\quad \nu_b = 0.32$ $\quad \alpha_j = 2.4 \cdot 10^{-5}/K$



(40 točk)

3. Določi osni sili v palicah $\overline{23}$ in $\overline{24}$ v odvisnosti od obtežbe q!



$E = 200\,000\,\text{MPa}$

$a = 4\,\text{m}$

$A_p = 0.0008\,\text{m}^2$

$A_n = 0.01\,\text{m}^2$

$I_n = 0.0002\,\text{m}^4$

(35 točk)

Ad 1.) Ploha ABC:

$$\frac{x}{3} - \frac{y}{7} + \frac{z}{5} = 1 \longrightarrow \phi = 35x - 15y + 21z - 105 = 0$$

$$\vec{n} = 35\vec{e}_x - 15\vec{e}_y + 21z$$

$$|\vec{n}| = \sqrt{35^2 + 15^2 + 21^2} = 43{,}486$$

$$\vec{e}_n = \frac{\vec{n}}{n} \longrightarrow \boxed{\vec{e}_n = 0{,}805\,\vec{e}_x - 0{,}345\,\vec{e}_y + 0{,}483\,\vec{e}_z}$$

$$\boxed{\sigma_{zz} = -10 \;,\quad \sigma_{yy} = \sigma_{yz} = 0}$$

a)
$$\begin{Bmatrix} p_{nx} \\ p_{ny} \\ p_{nz} \end{Bmatrix} = \begin{Bmatrix} 13{,}982 \\ 6{,}439 \\ 1{,}610 \end{Bmatrix} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & 0 & 0 \\ \sigma_{zx} & 0 & -10 \end{bmatrix} \begin{Bmatrix} 0{,}805 \\ -0{,}345 \\ 0{,}483 \end{Bmatrix}$$

$$0{,}805\,\sigma_{xx} - 0{,}345\,\sigma_{xy} + 0{,}483\,\sigma_{xz} = 13{,}982$$

$$0{,}805\,\sigma_{xy} = 6{,}439 \longrightarrow \sigma_{xy} = 8\ kN/cm^2$$

$$0{,}805\,\sigma_{xz} - 0{,}483 \cdot 10 = 1{,}610 \longrightarrow \sigma_{xz} = 8\ kN/cm^2$$

$$\rightarrow 0{,}805\,\sigma_{xx} = 13{,}982 + 0{,}345 \cdot 8 - 0{,}483 \cdot 8$$

$$\sigma_{xx} = 16\ kN/cm^2$$

$$[\sigma_{ij}] = \begin{bmatrix} 16 & 8 & 8 \\ 8 & 0 & 0 \\ 8 & 0 & -10 \end{bmatrix} \qquad I_1^\sigma = 16 - 10 = 6$$

$$\sigma^H = \frac{1}{3} I_1^\sigma \rightarrow \sigma^H = 2$$

b.)
$$[\sigma_{ij}^H] = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \qquad [s_{ij}^H] = \begin{bmatrix} 14 & 8 & 8 \\ 8 & -2 & 0 \\ 8 & 0 & -12 \end{bmatrix}$$

$$I_2^s = \begin{vmatrix} -2 & 0 \\ 0 & -12 \end{vmatrix} + \begin{vmatrix} 14 & 8 \\ 8 & -12 \end{vmatrix} + \begin{vmatrix} 14 & 8 \\ 8 & -2 \end{vmatrix} \rightarrow \boxed{I_2^s = -300}$$

$$f(s) = |I_2^s| - k_M^2 \quad \ldots \quad k_M = \frac{\sigma_Y}{\sqrt{3}}$$

$$f(s) = 300 - \frac{36^2}{3} = -132 \longrightarrow \boxed{EL.ST. \quad !}$$

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c) $\frac{1+\nu}{E} = 6,19 \cdot 10^{-5}$; $\frac{\nu}{E} = 1,43 \cdot 10^{-5}$

$$\varepsilon_{ij} = \frac{1+\nu}{E}\sigma_{ij} - \frac{\nu}{E} I_1^\sigma \delta_{ij}$$

$$\varepsilon_{xx} = 10^{-5}(6,19 \cdot 16 - 1,43 \cdot 6) \rightarrow \varepsilon_{xx} = 90,476 \cdot 10^{-5}$$

$$\varepsilon_{yy} = 10^{-5}(6,19 \cdot 0 - 1,43 \cdot 6) \rightarrow \varepsilon_{yy} = -8,571 \cdot 10^{-5}$$

$$\varepsilon_{zz} = 10^{-5}(6,19 \cdot -10 - 1,43 \cdot 6) \rightarrow \varepsilon_{zz} = -70,476 \cdot 10^{-5}$$

$$\varepsilon_{xy} = \varepsilon_{xz} = 10^{-5} \cdot 6,19 \cdot 8 \rightarrow \varepsilon_{xy} = \varepsilon_{xz} = 49,523 \cdot 10^{-5}$$

$$\varepsilon_{yz} = 0$$

$$[\varepsilon_{ij}] = 10^{-5} \begin{bmatrix} 90,476 & 49,523 & 49,523 \\ 49,523 & -8,571 & 0 \\ 49,523 & 0 & -70,476 \end{bmatrix}$$

$\overline{AO}$: $\overline{AO}' = (1+\varepsilon_{xx}) \cdot \overline{AO} \longrightarrow \boxed{\overline{AO}' = 3,0027 \text{ cm}}$

$\overline{AB}$: $\vec{r} = -3\vec{e}_x - 7\vec{e}_y \rightarrow |\vec{r}| = \sqrt{3^2+7^2} = \sqrt{58}$

$$\vec{e}_s = \frac{\vec{r}}{r} = -0,394\,\vec{e}_x - 0,919\,\vec{e}_y \quad ; \quad e_{sz} = 0 \; !$$

$$\varepsilon_{ss} = \varepsilon_{xx} e_{sx}^2 + 2\,\varepsilon_{xy} e_{sx} e_{sy} + \varepsilon_{yy} e_{sy}^2$$

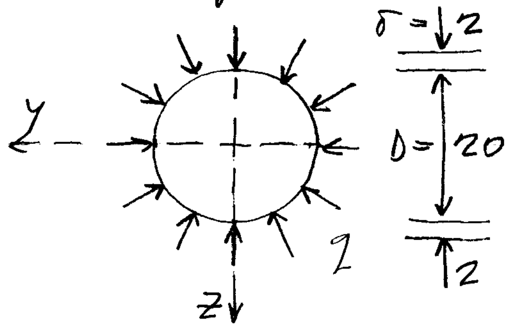
$$= 10^{-5}(90,476 \cdot 0,394^2 + 2 \cdot 49,523 \cdot 0,394 \cdot 0,919 - 8,571 \cdot 0,919^2) \longrightarrow \boxed{\varepsilon_{ss} = 42,670 \cdot 10^{-5}}$$

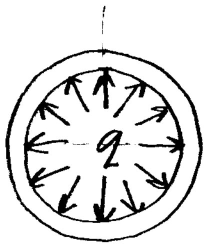
$$\overline{AB}' = (1+\varepsilon_{ss}) \cdot \overline{AB} \longrightarrow \boxed{\overline{AB}' = 7,6190 \text{ cm}}$$

Ad 2.)

$$\varepsilon_{ss} = \frac{\theta'}{\theta} - 1 = \frac{D'}{D} - 1 = \alpha_j \, \Delta T = \frac{D' - D}{D}$$

$$\Delta T = \frac{D' - D}{\alpha_j} = \frac{2{,}0 - 1{,}996}{1{,}996 \cdot 1{,}2 \cdot 10^{-5}} \longrightarrow \boxed{\Delta T = 167\,^\circ C}$$





Valj: $\sigma_{yy}^{b} = \sigma_{zz}^{b} = -q$

Cev: $\sigma_{rr}^{j} = -q$

$$\sigma_{ss}^{j} = q \, \frac{D}{2\delta}$$

$$\sigma_{xx}^{j} = \sigma_{xx}^{b} = 0$$

$$\boxed{\varepsilon_{yy}^{b} = \varepsilon_{ss}^{j}} \qquad \boxed{\Delta T_j = -333\,^\circ C}$$

$$\varepsilon_{yy}^{b} = \frac{1}{E_b}\left(\sigma_{yy}^{b} - \nu_b \, \sigma_{zz}^{b}\right) \longrightarrow \boxed{\varepsilon_{yy}^{b} = -\frac{q}{E_b}(1 - \nu_b)}$$

$$\varepsilon_{ss}^{j} = \frac{1}{E_j}\left(\sigma_{ss}^{j} - \nu_j \cdot \sigma_{rr}^{j}\right) + \alpha_j \cdot \Delta T$$

$$\boxed{\varepsilon_{ss}^{j} = \frac{q}{E_j}\left(\frac{D}{2\delta} + \nu_j\right) + \alpha_j \cdot \Delta T}$$

$$\varepsilon_{yy}^{b} = \varepsilon_{ss}^{j} \longrightarrow -\frac{q}{E_b}(1 - \nu_b) = \frac{q}{E_j}\left(\frac{D}{2\delta} + \nu_j\right) + \alpha_j \Delta T_j$$

$$\boxed{q = -\frac{E_j E_b \, \alpha_j \, \Delta T_j}{E_b\left(\frac{D}{2\delta} + \nu_j\right) + E_j (1 - \nu_b)}}$$

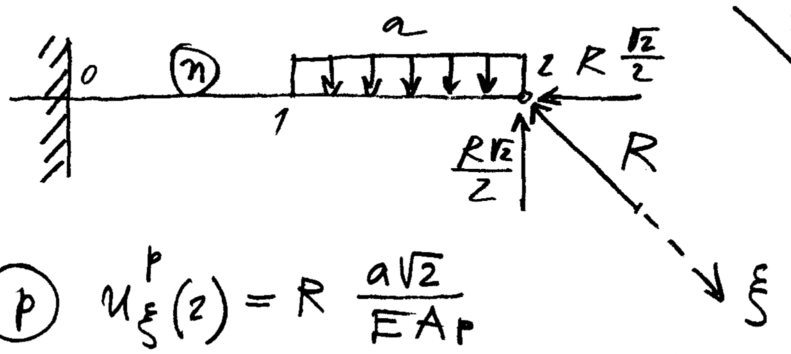
$$q = \frac{21000 \cdot 7000 \cdot 1{,}2 \cdot 10^{-5} \cdot 167}{7000\left(\frac{2}{2 \cdot 0{,}2} + 0{,}3\right) + 21000\,(1 - 0{,}32)}$$

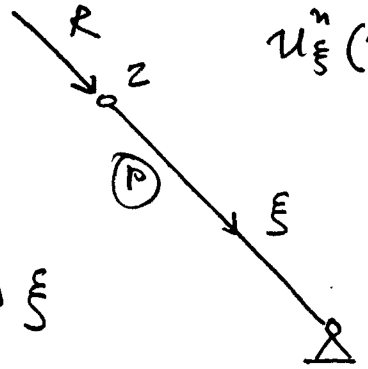
$$\boxed{q = 5{,}734 \ kN/cm^2}$$

$$\varepsilon_{yy}^{b} = -\frac{5{,}734}{7000}(1 - 0{,}32) = -0{,}000557$$

$$D_b' = 2\,(1 + \varepsilon_{yy}^{b}) \longrightarrow \boxed{D_b' = 1{,}9989 \ cm}$$

~~A1~~ 3.)　　$N_{23} = 0$!





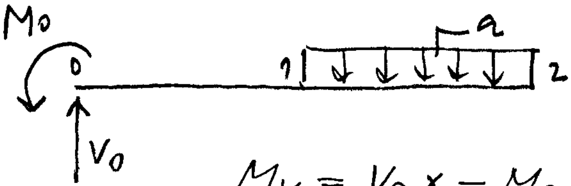
$u^n_\xi(2) = u^p_\xi(2)$!

(p)　$u^p_\xi(2) = R\,\dfrac{a\sqrt{2}}{EA_p}$

$$\boxed{u^p_\xi(2) = 0{,}03536\ R}$$

(n)　$u^{(n)}_x(2) = -R\,\dfrac{\sqrt{2}}{2}\cdot\dfrac{2a}{EA_n} \longrightarrow \boxed{u^{(n)}_x(2) = -0{,}00283\ R}$

$$u^n_z(2) = w_R + w_q = -R\,\frac{\sqrt{2}}{2}\cdot\frac{(2a)^3}{3EI_n} + w_q$$



$M_0 = \dfrac{3qa^2}{2}$,　$V_0 = qa$

$$M_y = V_0 x - M_0 - \frac{q}{2}\langle x-a\rangle^2 = qax - \frac{3qa^2}{2} - \frac{q}{2}\langle x-a\rangle^2$$

$$EI_y w'' = -M_y = -\frac{q}{2}\left(2ax - 3a^2 - \langle x-a\rangle^2\right)$$
$$EI_y w' = -\frac{q}{2}\left(ax^2 - 3a^2x - \frac{1}{3}\langle x-a\rangle^3\right) + C_1$$
$$EI_y w = -\frac{q}{2}\left(a\frac{x^3}{3} - \frac{3}{2}a^2x^2 - \frac{1}{12}\langle x-a\rangle^4\right) + C_1x + C_2$$

$x=0 \ \dots\ w = 0\ ,\ w' = 0 \longrightarrow C_1 = C_2 = 0$

$x = 2a \ \dots\ \ w = -\dfrac{q}{2}\left(a\cdot\dfrac{8a^3}{3} - \dfrac{3}{2}\,a^2\cdot 4a^2 - \dfrac{1}{12}\,a^4\right)$

$$w_q = q\,\frac{41\,a^4}{24EI_n}$$

$$u^n_z(2) = q\,\frac{41\,a^4}{24EI_n} - R\,\frac{4\sqrt{2}\,a^3}{3EI_n}$$

$$\boxed{u^n_z(2) = 10{,}93333\ q - 3{,}01699\ R}$$

tč. 2 :　　$u^p_\xi = u^n_x\,e_{\xi x} + u^n_z\,e_{\xi z}$

$$0{,}03536\,R = \frac{\sqrt{2}}{2}\left(-0{,}00283\,R + 10{,}93333\,g - 3{,}01699\,R\right)$$

$$\boxed{R = 3{,}562\ g}$$