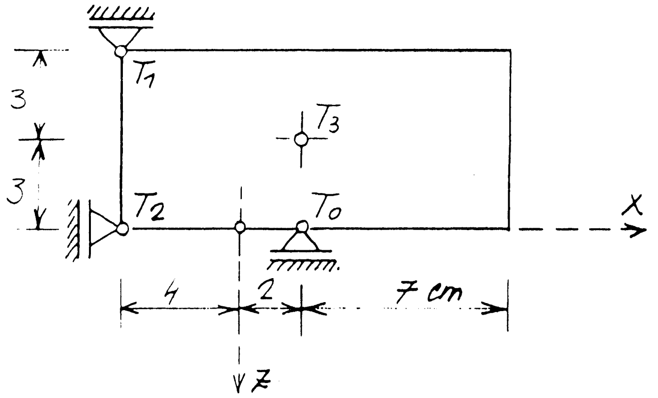
1. V prerezu telesa z ravnino $y = 0$ vlada ravninsko deformacijsko stanje $\varepsilon_{yx} = \varepsilon_{yy} = \varepsilon_{yz} = 0$; $u_y = 0$. Nenične komponente tenzorja majhnih deformacij ε_{ij} so podane v odvisnosti od obtežnega faktorja Ψ. Način podpiranja je simbolično prikazan na skici. Meja plastičnega tečenja uporabljenega materiala je σ_Y. V točki $T_3(2,0,-3)$ določi:
   a. pomike in zasuke,
   b. velikosti glavnih normalnih napetosti,
   c. kritično vrednost obtežnega faktorja Ψ_Y, pri katerem se v točki T pojavijo plastične deformacije (uporabi Misesov kriterij za začetek plstičnega tečenja).

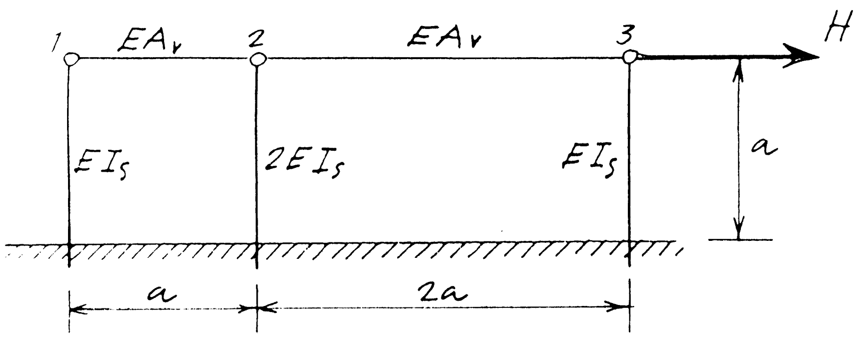


$$[\varepsilon_{ij}] = 10^{-4}\Psi \begin{bmatrix} 12xz & 0 & 3x^2 \\ 0 & 0 & 0 \\ 3x^2 & 0 & 2x \end{bmatrix}$$

$$2\mu = 16\,000 \text{ kN/cm}^2$$
$$\lambda = 8\,000 \text{ kN/cm}^2$$
$$\sigma_Y = 24 \text{ kN/cm}^2$$

40%

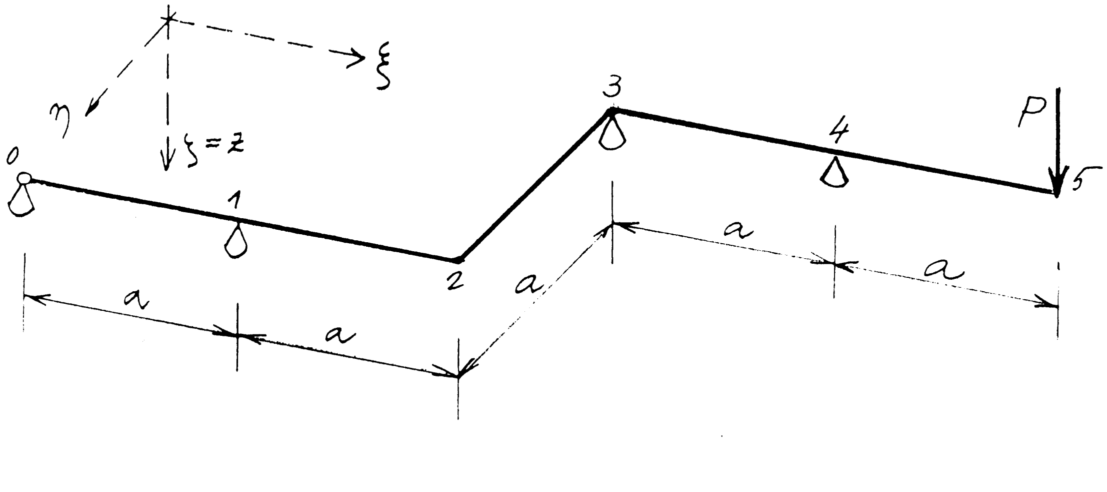
2. Dopustni vodoravni pomik glave srednjega stebra (vozlišče 2) je δ_d. Določi dopustno obtežbo H_d!



$a = 4$ m
$E = 200\,000$ MPa
$A_v = 2$ cm^2
$I_s = 4800$ cm^4
$\delta_d = 1{,}6$ cm

35%

3. Določi pomik točke 2! Zaradi lažjega računanja vzamemo $GI_x = EI_y$.



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MTT IZPIT

Ad 1.) $\boxed{\nu = 1}$ $\vec{\varepsilon}_y = \vec{0}$; $\omega_x = \omega_z = u_y = 0$

$T_0(2,0,0) \rightarrow u_z^0 = 0$

a)

$$\omega_y = \omega_y^0 + \int_2^x \vec{e}_y(\vec{\nabla}\times\vec{\varepsilon}_x)_{z=0}\,dx + \int_0^z \vec{e}_y(\vec{\nabla}\times\vec{\varepsilon}_z)\,dz$$

$$\vec{e}_y(\vec{\nabla}\times\vec{\varepsilon}_x) = \begin{vmatrix} 0 & 1 & 0 \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \varepsilon_{xx} & 0 & \varepsilon_{xz} \end{vmatrix} = \frac{\partial\varepsilon_{xx}}{\partial z} - \frac{\partial\varepsilon_{xz}}{\partial x} = 6x\cdot 10^{-4}$$

$$\vec{e}_y(\vec{\nabla}\times\vec{\varepsilon}_z) = \begin{vmatrix} 0 & 1 & 0 \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \varepsilon_{zx} & 0 & \varepsilon_{zz} \end{vmatrix} = \frac{\partial\varepsilon_{zx}}{\partial z} - \frac{\partial\varepsilon_{zz}}{\partial x} = -2\cdot 10^{-4}$$

$$\omega_y = \omega_y^0 + 10^{-4}\left[\int_2^x 6x\,dx - \int_0^z 2\,dz\right]$$

$$\boxed{\omega_y = \omega_y^0 + 10^{-4}(3x^2 - 2z - 12)} \qquad \boxed{\omega_y = \omega_{zx} = -\omega_{xz}}$$

$$u_x = u_x^0 + \int_2^x (\varepsilon_{xx} + \omega_{xx})_{z=0}\,dx + \int_0^z (\varepsilon_{zx} + \omega_{zx})\,dz$$

$$u_x = u_x^0 + 10^{-4}\left[\int_2^x (12xz)_{z=0}\,dx + \right.$$
$$\left. + \int_0^z (3x^2 + 10^4\omega_y^0 + 3x^2 - 2z - 12)\,dz\right]$$

$$\boxed{u_x = u_x^0 + z\,\omega_y^0 + 10^{-4}(6x^2z - z^2 - 12z)}$$

$$u_z = u_z^0 + \int_2^x (\varepsilon_{xz} + \omega_{xz})_{z=0}\,dx + \int_0^z (\varepsilon_{zz} + \omega_{zz})\,dz$$

$$u_z = 10^{-4}\left[\int_2^x (3x^2 - 10^4\omega_y^0 - 3x^2 + 2z + 12)_{z=0}\,dx + \int_0^z 2x\,dz\right]$$

$$\boxed{u_z = (2-x)\,\omega_y^0 + 10^{-4}(2xz + 12x - 24)}$$

$T_1(-4,0,-6) \rightarrow u_z = 0$

$u_z(T_1) = 6\omega_y^0 + 10^{-4}(48-48-24) = 0 \rightarrow \boxed{\omega_y^0 = 4 \cdot 10^{-4}}$

$$\boxed{\omega_y = 10^{-4}(3x^2 - 2z - 8)}$$

$u_x = u_x^0 + (6x^2 z - z^2 - 8z) \cdot 10^{-4}$

$T_2(-4,0,0) \rightarrow u_x = 0 \rightarrow \boxed{u_x^0 = 0}$

$$\boxed{u_x = 10^{-4}(6x^2 z - z^2 - 8z)}$$

$$\boxed{u_z = 10^{-4}(2x^3 + 8x - 16)}$$

Kontrola: $\varepsilon_{xx} = \frac{\partial u_x}{\partial x} = 12xz \cdot 10^{-4}$

$\varepsilon_{zz} = \frac{\partial u_z}{\partial z} = 2x \cdot 10^{-4}$

$\varepsilon_{zx} = \frac{1}{2}\left(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x}\right) = 3x^2 \cdot 10^{-4}$

$\omega_y = \frac{1}{2}\left(\frac{\partial u_x}{\partial z} - \frac{\partial u_z}{\partial x}\right) = (3x^2 - 2z - 8) \cdot 10^{-4}$

Točka $T_3(2,0,-3)$: $\boxed{\begin{aligned} u_x &= -57 \cdot 10^{-4} \text{ cm} \\ u_z &= -12 \cdot 10^{-4} \text{ cm} \\ \omega_y &= 10 \cdot 10^{-4} \end{aligned}}$

b)

$$[\varepsilon_{ij}]_{T_3} = 10^{-4}\begin{bmatrix} -72 & 0 & 12 \\ 0 & 0 & 0 \\ 12 & 0 & 4 \end{bmatrix}$$

$$\varepsilon_{11,22} = 10^{-4}\left[\frac{\varepsilon_{xx}+\varepsilon_{zz}}{2} \pm \sqrt{\left(\frac{\varepsilon_{xx}-\varepsilon_{zz}}{2}\right)^2 + \varepsilon_{xz}^2}\right]$$

$$\varepsilon_{11,22} = 10^{-4}\left[\frac{-72+4}{2} \pm \sqrt{\left(\frac{-72-4}{2}\right)^2 + 12^2}\right]$$

$$\boxed{\varepsilon_{11} = 5,85 \cdot 10^{-4}} \quad \boxed{\varepsilon_{22} = -73,85 \cdot 10^{-4}} \quad \boxed{\varepsilon_{33} = 0}$$

$$\boxed{I_1^{\varepsilon} = -68 \cdot 10^{-4}} \qquad \boxed{\gamma = 1} \; ;$$

$$\sigma_{11} = 2\mu\,\varepsilon_{11} + \lambda I_1^{\varepsilon} \rightarrow \sigma_{11} = -45,04 \text{ kN/cm}^2$$
$$\sigma_{22} = 2\mu\,\varepsilon_{22} + \lambda I_1^{\varepsilon} \rightarrow \sigma_{22} = -172,56 \text{ kN/cm}^2$$
$$\sigma_{33} = 2\mu\,\varepsilon_{33} + \lambda I_1^{\varepsilon} \rightarrow \sigma_{33} = -54,40 \text{ kN/cm}^2$$

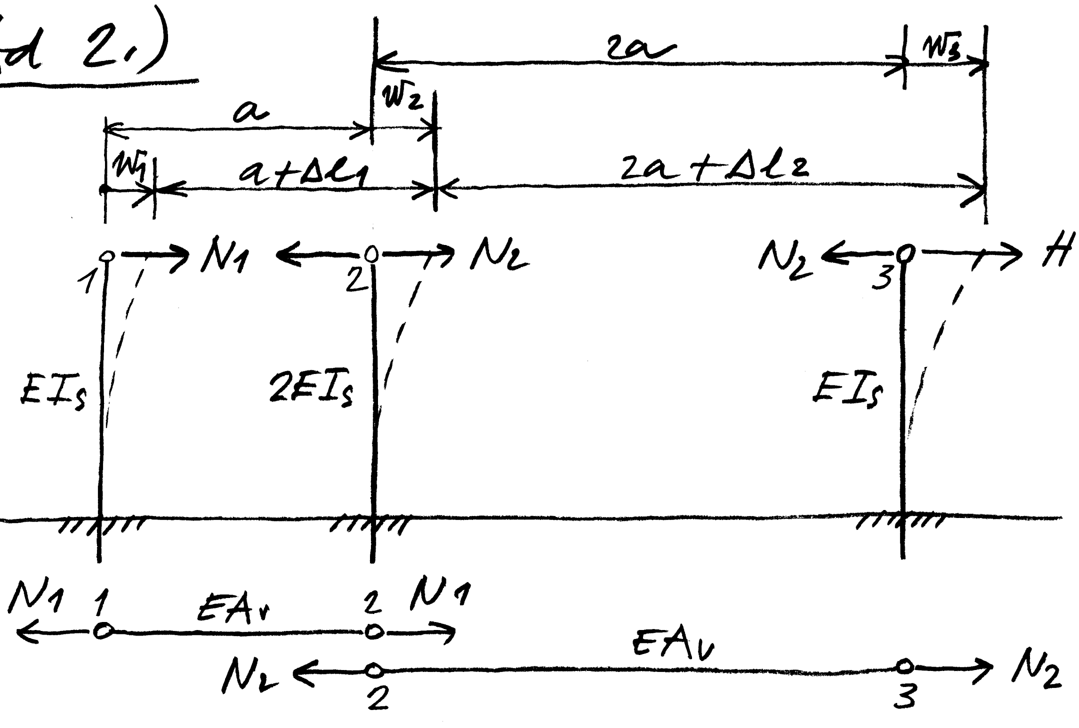
2) Mises: $(\sigma_{11}-\sigma_{22})^2 + (\sigma_{22}-\sigma_{33})^2 + (\sigma_{33}-\sigma_{11})^2 = 2\sigma_Y^2$

$$\gamma_Y^2\left[(-45,04+172,56)^2 + (-172,56+54,40)^2 + (-54,40+45,04)^2\right] = 2\sigma_Y^2$$

$$30310,40\,\gamma_Y^2 = 2\sigma_Y^2 \rightarrow \gamma_Y = \sigma_Y\sqrt{\frac{2}{30310,4}}$$

$$\gamma_Y = 0,00812\,\sigma_Y \longrightarrow \boxed{\gamma_Y = 0,195}$$

Ad 2.)



$\overline{12}$: $w_1 + a + \Delta l_1 = a + w_2 \rightarrow$ $\boxed{w_2 = w_1 + \Delta l_1}$

$\overline{23}$: $w_2 + 2a + \Delta l_2 = 2a + w_3 \rightarrow$ $\boxed{w_3 = w_2 + \Delta l_2}$

$$w_1 = N_1 \frac{a^3}{3EI_s} \qquad \Delta l_1 = N_1 \frac{a}{EA_v}$$

$$w_2 = (N_2 - N_1)\frac{a^3}{3 \cdot 2EI_s} \qquad \Delta l_2 = N_2 \frac{2a}{EA_v}$$

$$w_3 = (H - N_2)\frac{a^3}{3EI_s}$$

$$(N_2 - N_1)\frac{a^3}{6EI_s} = N_1 \frac{a^3}{3EI_s} + N_1 \frac{a}{EA_v}$$

$$(H - N_2)\frac{a^3}{3EI_s} = (N_2 - N_1)\frac{a^3}{6EI_s} + N_2 \frac{2a}{EA_v}$$

$$N_1\left(\frac{a^2}{2I_s} + \frac{1}{A_v}\right) = N_2 \frac{a^2}{6I_s}$$

$$\boxed{N_2 = N_1 \frac{3(a^2A_v + 2I_s)}{a^2A_v}} \rightarrow \boxed{N_2 = 3{,}09\ N_1}$$

$$N_2\left(\frac{a^2}{2I_s} + \frac{2}{A_v}\right) - N_1\frac{a^2}{6I_s} = H\frac{a^2}{3I_s}$$

$$N_2 \frac{a^2A_v + 4I_s}{2A_v} - N_1 \frac{a^2}{6} = H\frac{a^2}{3}$$

$$\boxed{84\,800\ N_2 - 26\,667\ N_1 = 53\,333\ H}$$

$$N_1(3{,}09 \cdot 84\,800 - 26\,667) = 53\,333\ H$$

$$\boxed{N_1 = 0{,}2266\ H} \longrightarrow \boxed{N_2 = 0{,}7002\ H}$$

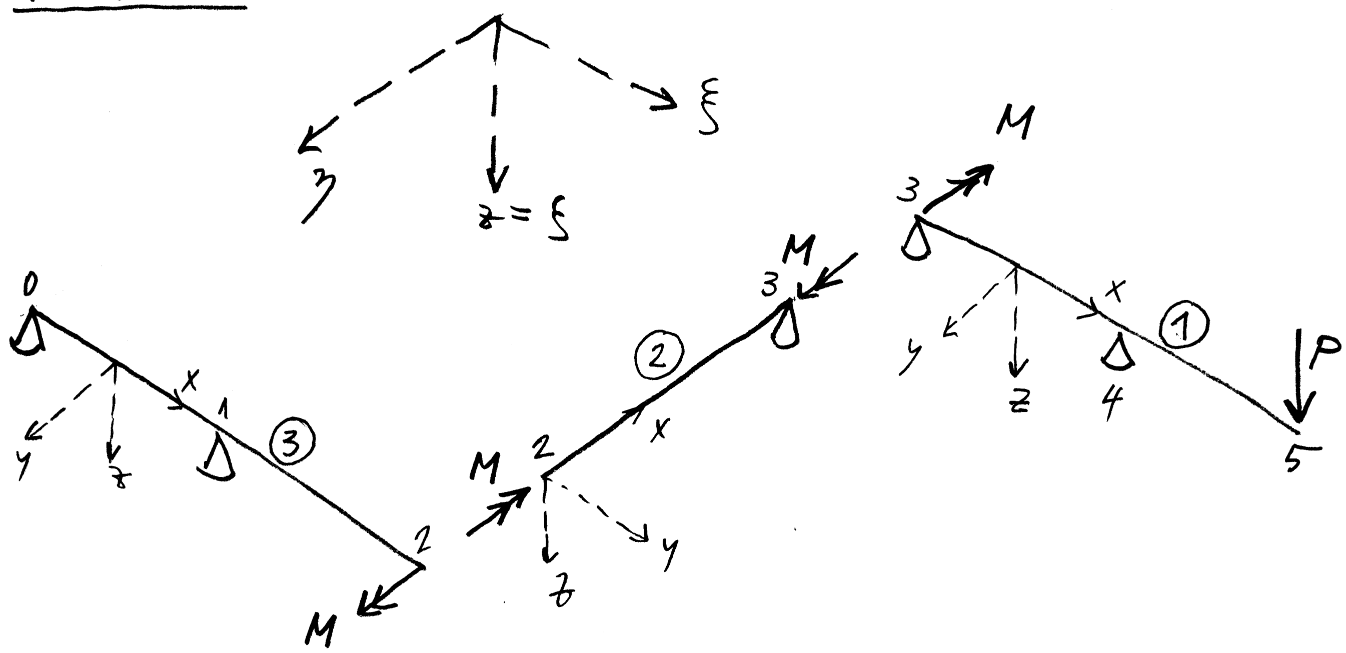
$= 6{,}459$ kN $\qquad = 19{,}960$ kN

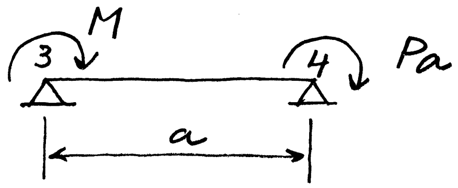
$$w_2 = (0{,}7002 - 0{,}2266)H\ \frac{a^3}{6EI_y}$$

$$\boxed{w_2 = 0{,}05262\ H}$$

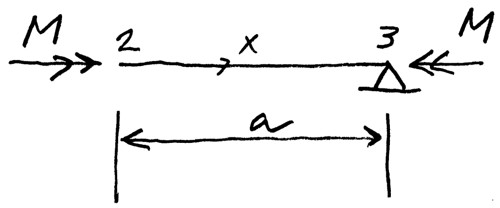
$$w_2 = \delta_d = 1{,}5 \text{ cm} \longrightarrow \boxed{H_d = 28{,}506\ \text{kN}}$$

Ad 3.



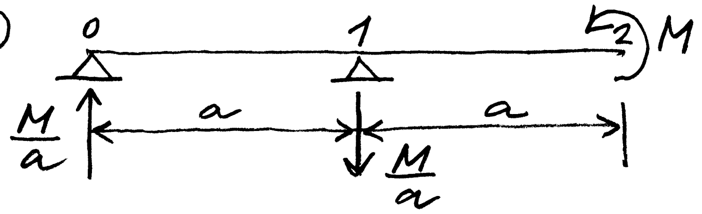
① 

$$\boxed{\omega_y^{(1)}(3) = P\frac{a^2}{6EI_y} - M\frac{a}{3EI_y}}$$

② 

$$M_x = -M - GI_x\frac{d\omega_x}{dx}$$

$$\boxed{\omega_x^{(2)}(3) = \omega_x^{(2)}(2) - \frac{M}{GI_x}a}$$

③ 

$$M_y = \frac{M}{a}x - \frac{M}{a}\langle x-a\rangle$$

$$M_y = \frac{M}{a}\left(x - \langle x-a\rangle\right)$$

$$EI_y w'' = -\frac{M}{a}\left(x - \langle x-a\rangle\right)$$

$$EI_y w' = -\frac{M}{2a}\left(x^2 - \langle x-a\rangle^2\right) + C_1$$

$$EI_y w = -\frac{M}{6a}\left(x^3 - \langle x-a\rangle^3\right) + C_1 x + C_2$$

$$x = 0 \rightarrow w = 0 \rightarrow C_2 = 0$$

$$x = a \rightarrow w = 0 \rightarrow -\frac{Ma^2}{6} + aC_1 = 0$$

$$C_1 = M\frac{a}{6}$$

$$\omega_y = -w' = \frac{M}{2EI_y a}\left(x^2 - \langle x-a\rangle^2\right) - \frac{Ma}{6EI_y}$$

$$x = 2a \rightarrow \omega_y^{(3)}(2) = \frac{M}{2EI_y a}\left(4a^2 - a^2\right) - \frac{Ma}{6EI_y}$$

$$\boxed{\omega_y^{(3)}(2) = M\frac{4a}{3EI_y}}$$

$$\omega_x^{(2)}(2) = -\omega_y^{(3)}(2) = -M\frac{4a}{3EI_y}$$

$$\omega_x^{(2)}(3) = -M\frac{4a}{3EI_y} - M\frac{a}{GI_x} = -M\frac{7a}{3EI_y}$$

$$\omega_y^{(1)}(3) = -\omega_x^{(2)}(3) = M\frac{7a}{3EI_y} = P\frac{a^2}{6EI_y} - M\frac{a}{3EI_y}$$

$$\boxed{M = P\frac{a}{16}}$$

$$w_2 = -\frac{M}{6aEI_y}\left(8a^3 - a^3\right) + \frac{Ma}{6EI_y}\cdot 2a = M\frac{5a^2}{6EI_y}$$

$$\boxed{w_2 = -P\frac{5a^3}{96EI_y}}$$