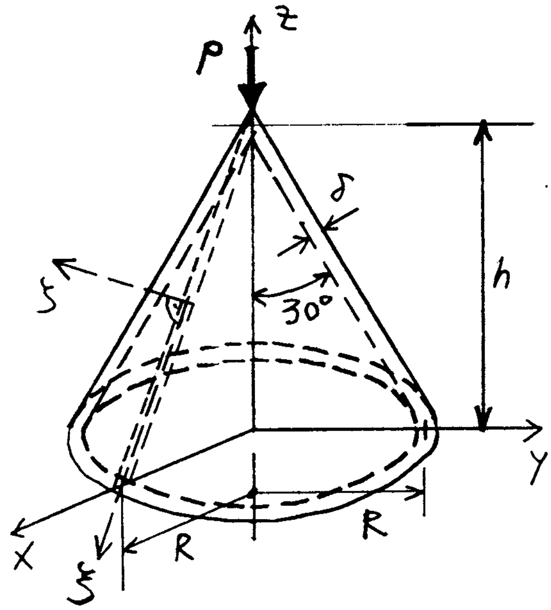
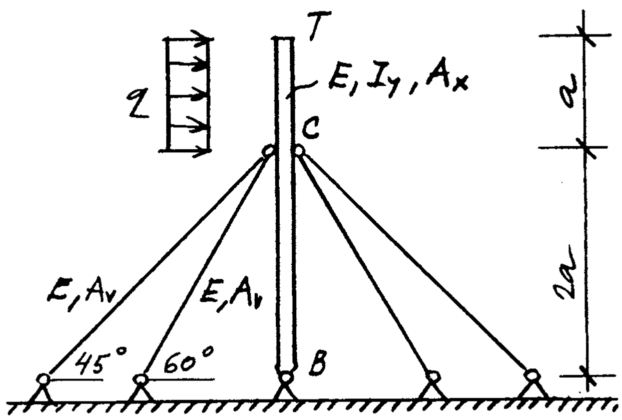
1. 

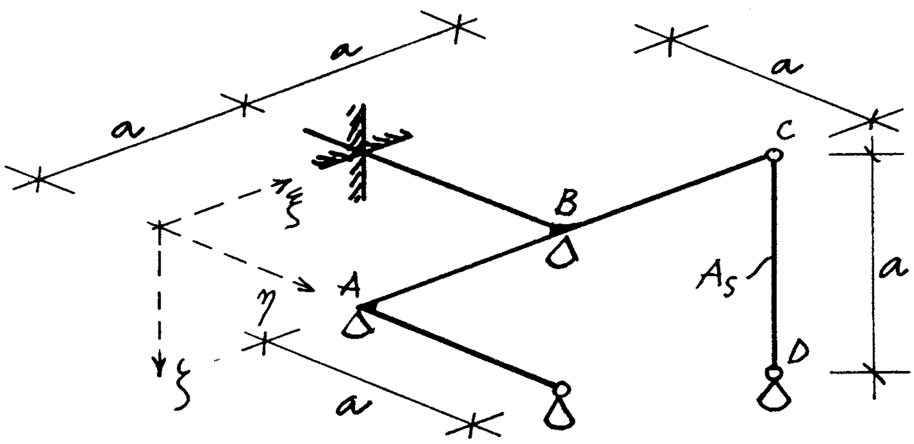
Tanka stožčasta lupina je prilepljena na vodoravno podlago. Srednji polmer osnovne ploskve je R, višina stožca pa je h. Vrh stožca je obtežen z navpično silo P. Določi normalno napetost $\sigma_{\xi\xi}$ v steni lupine v odvisnosti od koordinate z ter normalno napetost σ_{zz} in strižno napetost $\sigma_{zx} = \sigma_{zy}$ v onovni ploskvi ($z = 0$)! Ker gre za tanko lupino ($\delta << R$), lahko predpostaviš enakomeren potek napetosti po debelini. Lastne teže lupine ni treba upoštevati.

2. 

Vrvi, s katerimi je stabiliziran steber BT, lahko prevzamejo natezne ne pa tudi tlačnih osnih sil. Določi in skiciraj potek notranjih sil vzdolž stebra ter vodoravni pomik točke T! Pri tem lahko zanemariš osno podajnost stebra v primerjavi z osno podajnostjo vrvi.

$$E = 210\,000\,\mathrm{MPa}$$
$$A_x = 114\,\mathrm{cm}^2$$
$$I_y = 5870\,\mathrm{cm}^4$$
$$A_v = 1.5\,\mathrm{cm}^2$$
$$a = 5\,\mathrm{m}$$
$$q = 9.6\,\mathrm{kN/m}$$

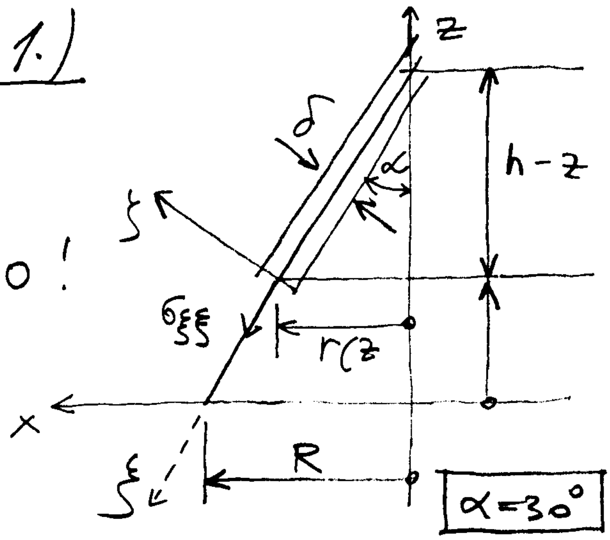
3. Določi in skiciraj notranje sile, ki nastopijo v prikazani konstrukciji, če steber CD segrejemo za ΔT! Temperaturni razteznostni koeficient materiala je α_T. Upoštevaj, da je $E = 2G$ in $I_x = 2I_y$!



MTT IZPIT 7.7.97

Ad 1.)

$\sigma_{\zeta\zeta} = 0$!



$r(z) = (h-z)\,\mathrm{tg}\,\alpha$

$$\boxed{r(z) = (h-z)\cdot\frac{1}{\sqrt{3}}}$$

$e_{\xi x} = \frac{1}{2}$

$e_{\xi z} = -\frac{\sqrt{3}}{2}$

$e_{\zeta x} = \frac{\sqrt{3}}{2}$

$e_{\zeta z} = \frac{1}{2}$

$$\sum P_z = 0 \quad \rightarrow \quad -P - 2\pi\, r(z)\cdot\delta\cdot\sigma_{\xi\xi}\cdot\cos\alpha = 0$$

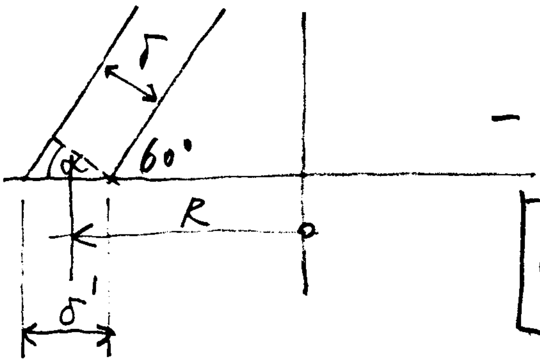
$$\boxed{\sigma_{\xi\xi} = -\frac{P}{\pi\delta(h-z)} = \sigma_{\xi\xi}(z)}$$

$$\sigma_{zz} = \sigma_{\xi\xi}\, e_{\xi z}^2 = \sigma_{\xi\xi}\cdot\frac{3}{4} \quad \rightarrow \quad \boxed{\sigma_{zz} = -\frac{3P}{4\pi\delta(h-z)}}$$

$$\sigma_{zx} = \sigma_{\xi\xi}\, e_{\xi z}\, e_{\xi x} = -\sigma_{\xi\xi}\cdot\frac{\sqrt{3}}{4} \rightarrow \boxed{\sigma_{zx} = \frac{P\sqrt{3}}{4\pi\delta(h-z)}}$$

$$z = 0 \quad \rightarrow \quad \boxed{\sigma_{zz} = -\frac{3P}{4\pi\delta h}} \quad \boxed{\sigma_{zx} = \frac{P\sqrt{3}}{4\pi\delta h}}$$

Kontrola :

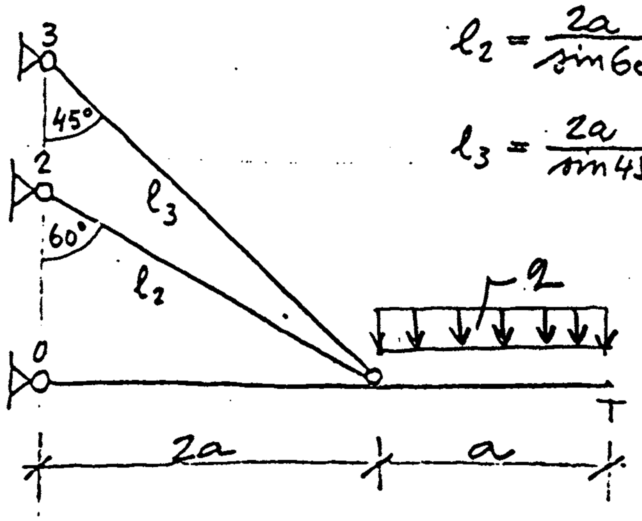


$$\delta' = \frac{\delta}{\cos\alpha} = \frac{2\delta}{\sqrt{3}}, \quad R = \frac{h}{\sqrt{3}}$$

$$-2\pi R\delta'\,\sigma_{zz}(z=0) - P = 0$$

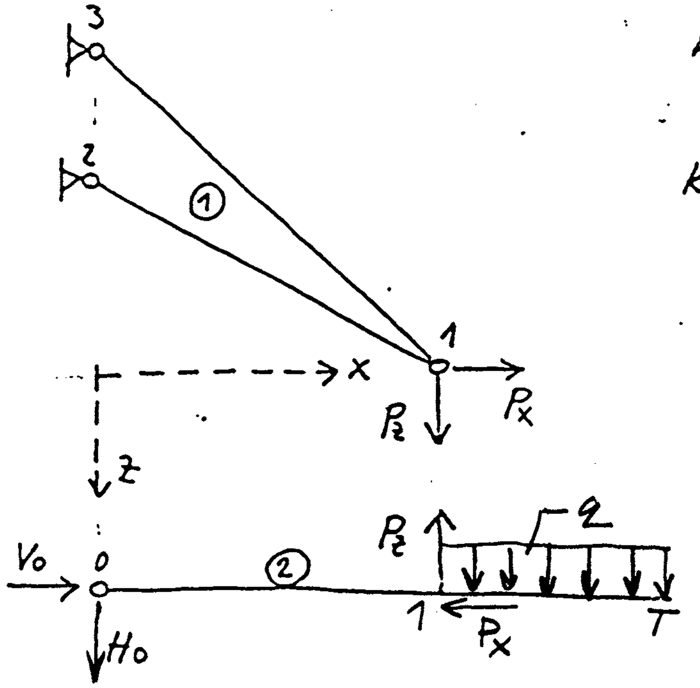
$$\boxed{\sigma_{zz}(z=0) = -\frac{3P}{4\pi\delta h}} \quad ✓$$

Ad 2.



$$l_2 = \frac{2a}{\sin 60°} \longrightarrow l_2 = 11{,}55\ m$$

$$l_3 = \frac{2a}{\sin 45°} \longrightarrow l_3 = 14{,}14\ m$$



$$k_{12} = \frac{EA_v}{l_2} \longrightarrow k_{12} = 27{,}28\ \frac{kN}{cm}$$

$$k_{13} = \frac{EA_v}{l_3} \longrightarrow k_{13} = 22{,}27\ \frac{kN}{cm}$$

1

$$[K_{12}] = \begin{array}{c|cc} & -\sqrt{3}/2 & -1/2 \\ \hline -\sqrt{3}/2 & 3/4 & \sqrt{3}/4 \\ -1/2 & \sqrt{3}/4 & 1/4 \end{array} \cdot k_{12} \longrightarrow [K_{12}] = \begin{bmatrix} 20{,}46 & 11{,}81 \\ 11{,}81 & 6{,}82 \end{bmatrix}$$

$$[K_{13}] = \begin{array}{c|cc} & -\sqrt{2}/2 & -\sqrt{2}/2 \\ \hline -\sqrt{2}/2 & 1/2 & 1/2 \\ -\sqrt{2}/2 & 1/2 & 1/2 \end{array} \cdot k_{13} \longrightarrow [K_{13}] = \begin{bmatrix} 11{,}14 & 11{,}14 \\ 11{,}14 & 11{,}14 \end{bmatrix}$$

$$[K_{11}] = -\left([K_{12}] + [K_{13}]\right) \longrightarrow [K_{11}] = -\begin{bmatrix} 31{,}60 & 22{,}95 \\ 22{,}95 & 17{,}96 \end{bmatrix}$$

$$[K_{11}]\begin{Bmatrix}u_1\\w_1\end{Bmatrix} = -\begin{Bmatrix}P_x\\P_z\end{Bmatrix} \longrightarrow \begin{bmatrix}31,60 & 22,95\\22,95 & 17,96\end{bmatrix}\begin{Bmatrix}u_1\\w_1\end{Bmatrix} = \begin{Bmatrix}P_x\\P_z\end{Bmatrix}$$

$$\boxed{u_1 = 0} \quad \dots \quad 2295\; w_1 = P_x \quad \rightarrow \quad \boxed{P_x = 1,278\, P_z}$$

$$1796\; w_1 = P_z \quad \rightarrow \quad \boxed{w_1 = \frac{P_z}{17,96}}$$

$$②:\; P_z \cdot 2a - qa \cdot \frac{5a}{2} = 0 \longrightarrow \boxed{P_z = q\,\frac{5a}{4} = 60\ kN}$$

$$w_1 = \frac{60}{17,96} \rightarrow \boxed{w_1 = 3,34\ cm}$$

$$P_x = 1,278 \cdot 60 \rightarrow \boxed{P_x = 76,68\ kN}$$

$$\boxed{V_0 = P_x = 76,68\ kN}$$

$$H_0 = P_z - qa \longrightarrow \boxed{H_0 = 12\ kN}$$

$$M_y = -H_0 x + P_z \langle x-2a\rangle - \frac{q}{2}\langle x-2a\rangle^2$$

$$M_y = P_z\left(\langle x-2a\rangle - x\right) + \frac{q}{2}\left(2ax - \langle x-2a\rangle^2\right)$$

$$EI_y w'' = P_z\left(x - \langle x-2a\rangle\right) + \frac{q}{2}\left(\langle x-2a\rangle^2 - 2ax\right)$$

$$EI_y w' = \frac{P_z}{2}\left(x^2 - \langle x-2a\rangle^2\right) + \frac{q}{2}\left(\frac{1}{3}\langle x-2a\rangle^3 - ax^2\right) + C_1$$

$$EI_y w = \frac{P_z}{6}\left(x^3 - \langle x-2a\rangle^3\right) + \frac{q}{2}\left(\frac{1}{12}\langle x-2a\rangle^4 - \frac{a}{3}x^3\right) + C_1 x + C_2$$

$$x=0 \;\dots\; w = 0 \longrightarrow \boxed{C_2 = 0}$$

$$x = 2a \;\dots\; w = w_1$$

$$EI_y w_1 = \frac{P_z}{6}\cdot 8a^3 - \frac{q}{2}\cdot\frac{a}{3}\,8a^3 + 2a\,C_1$$

$$\boxed{C_1 = w_1\,\frac{EI_y}{2a} - \frac{qa^3}{6}}$$

$$x = 3a \;\dots\; EI_y w_T = \frac{P_z}{6}\left(27a^3 - a^3\right) + \frac{q}{2}\left(\frac{a^4}{12} - \frac{27a^4}{3}\right) +$$

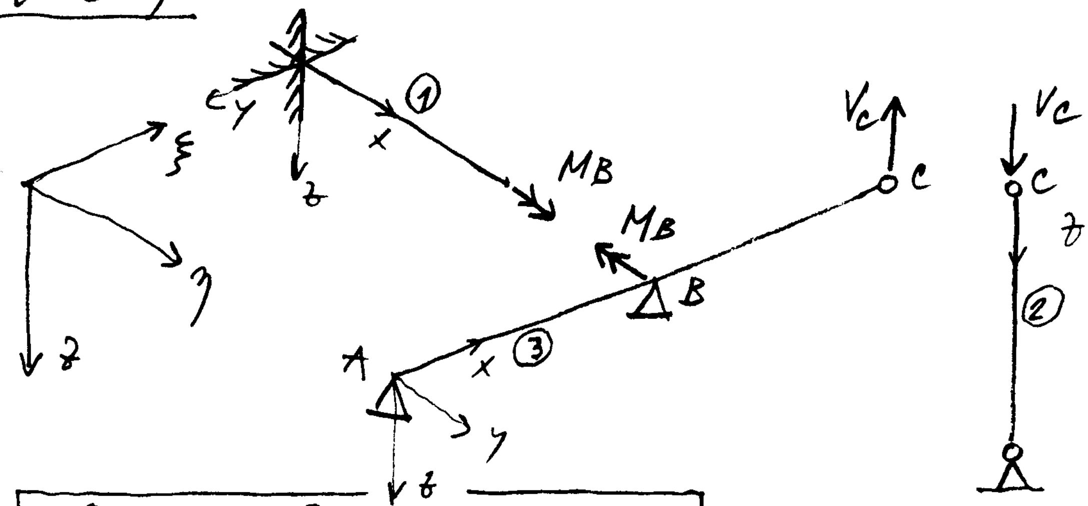
$$+\, 3a\left(w_1\,\frac{EI_y}{2a} - \frac{qa^3}{6}\right)$$

$$\boxed{w_T = \frac{3 w_1}{2} + q \, \frac{11 a^4}{24 E I_y}}$$

$$w_T = \frac{3}{2} \cdot 3{,}34 + 0{,}096 \, \frac{11 \cdot 500^4}{24 \cdot 21000 \cdot 5870}$$

$$\boxed{w_T = 5{,}01 + 22{,}31 = 27{,}32 \text{ cm}}$$

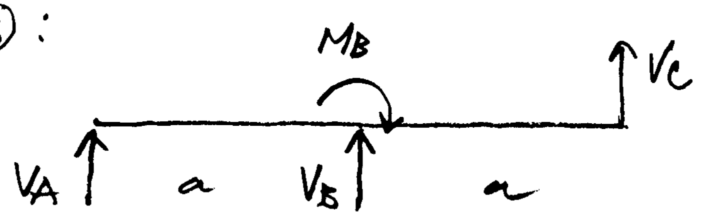
Ad 3.)



$$① : \quad \boxed{\omega_\eta^{①}(B) = \omega_x^{①}(B) = M_B \, \frac{a}{G I_x}}$$

$$② : \quad \boxed{u_z^{②}(C) = V_C \, \frac{a}{E A_S} - a \, \alpha_T \, \Delta T}$$

③ :



$$V_A = V_C - \frac{M_B}{a}$$

$$V_B = -2 V_C + \frac{M_B}{a}$$

$$M_y = V_A x + V_B \langle x-a \rangle + M_B \langle x-a \rangle^0$$

$$M_y = \left(V_C - \frac{M_B}{a} \right) x + \left(-2 V_C + \frac{M_B}{a} \right) \langle x-a \rangle + M_B \langle x-a \rangle^0$$

$$E I_y w'' = V_C \big(2 \langle x-a \rangle - x \big) + \frac{M_B}{a} \big(x - \langle x-a \rangle - a \langle x-a \rangle^0 \big)$$

$$E I_y w' = V_c\left(\langle x-a\rangle^2 - \frac{x^2}{2}\right) + \frac{M_B}{a}\left(\frac{x^2}{2} - \frac{1}{2}\langle x-a\rangle^2 - a\langle x-a\rangle\right) + C_1$$

$$E I_y w = V_c\left(\frac{1}{3}\langle x-a\rangle^3 - \frac{x^3}{6}\right) + \frac{M_B}{a}\left(\frac{x^3}{6} - \frac{1}{6}\langle x-a\rangle^3 - \frac{a}{2}\langle x-a\rangle^2\right) + C_1 x + C_2$$

$x = 0 \ \ldots \ w = 0 \rightarrow C_2 = 0$

$x = a \ \ldots \ w = 0 \rightarrow$

$$-V_c\frac{a^3}{6} + \frac{M_B}{a}\frac{a^3}{6} + C_1 a = 0 \rightarrow \boxed{C_1 = V_c\frac{a^2}{6} - M_B\frac{a}{6}}$$

$$x = a \ \ldots \ E I_y w' = -V_c\frac{a^2}{2} + M_B\frac{a}{2} + V_c\frac{a^2}{6} - M_B\frac{a}{6}$$

$$\boxed{\omega_y(B) = \frac{1}{E I_y}\left(V_c\frac{a^2}{3} - M_B\frac{a}{3}\right)}$$

$$x = 2a \ \ldots \ E I_y w = V_c a^3\left(\frac{1}{3} - \frac{8}{6}\right) + M_B a^2\left(\frac{8}{6} - \frac{1}{6} - \frac{1}{2}\right) + 2a\left(V_c\frac{a^2}{6} - M_B\frac{a}{6}\right)$$

$$\boxed{w(C) = -V_c\frac{2a^3}{3 E I_y} + M_B\frac{a^2}{3 E I_y}} = w^{(3)}(C)$$

$$\boxed{\omega_y^{(1)}(B) = \omega_y^{(3)}(B)} \rightarrow M_B a = V_c\frac{a^2}{3} - M_B\frac{a}{3}$$

$$\boxed{M_B = V_c\frac{a}{4}}$$

$$\boxed{u_z^{(2)}(C) = w^{(3)}(C)} \rightarrow V_c\frac{a}{E A_s} - a\alpha_T\Delta T = -V_c\frac{2a^3}{3 E I_y} + M_B\frac{a^2}{3 E I_y}$$

$$\boxed{V_c = \frac{12 E A_s I_y \alpha_T}{12 I_y + 7a^2 A_s}\Delta T}$$

$$\boxed{M_B = \frac{3 E a A_s I_y \alpha_T}{12 I_y + 7a^2 A_s}\Delta T}$$