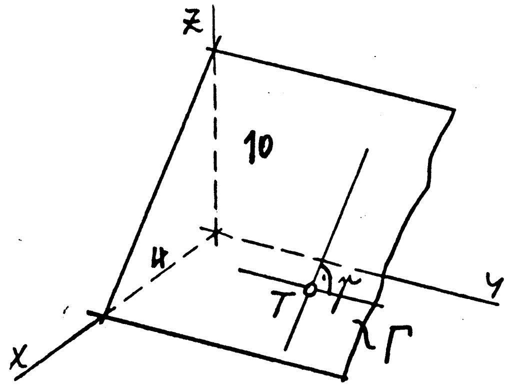
1. Z jeklenim merilnim trakom, ki je bil umerjen pri temperaturi 20°C, želimo pri temperaturi 36°C izmeriti razdaljo med točkama A in B. Trak nategnemo s silo 200 N in med točkama A in B odčitamo odmerek $L = 26,3290\,\text{m}$. Trak ima pravokoten prečni prerez dimenzij $16 \times 0.5\,\text{mm}$, modul elastičnosti pa je 250 000 MPa. Določi dejansko razdaljo L_0 med točkama A in B!

$\alpha_T = 1.25 \cdot 10^{-5}/K$

[10 točk]

2. Napetostno stanje telesa je opisano s komponentami σ_{ij}[MPa] tenzorja napetosti v koordinatnem sistemu (x, y, z). V točki $T(x, 4, 4)$, ki leži v ravnini Γ, določi:
   a. glavne normalne napetosti,
   b. specifično spremembo doložine normale na ravnino Γ,
   c. spremembo pravega kota γ!



$$[\sigma_{ij}] = \begin{bmatrix} 2(x^2+y) & xy^2 & 5z+2 \\ xy^2 & 3yz & 10(y^2-4z) \\ 5z+2 & 10(y^2-4z) & 2x-7y \end{bmatrix}$$

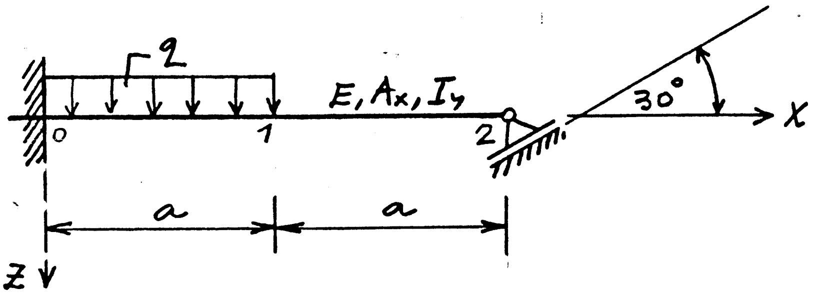
$T(3,4,4)$!

$E = 210\,000\,\text{MPa}$

$\nu = 0.3$

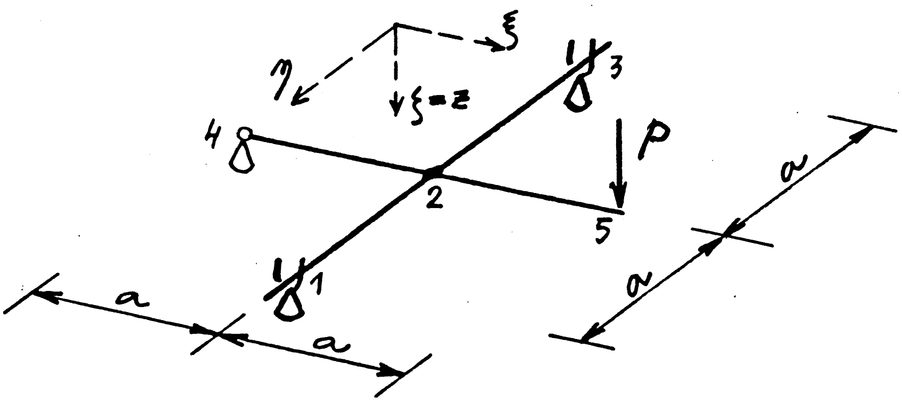
[30 točk]

3. Določi vektor pomika točke 2 glede na koordinatni sistem (x, z)! Določi in skiciraj diagrame notranjih sil!



[30 točk]

4. Določi in skiciraj notranje sile! Za koliko se spremeni navpični pomik točke 5, če nosilec $\overline{45}$ v točki 2 ni togo povezan z nosilcem $\overline{13}$, temveč je nanj le prosto položen? V podporah 1 in 3 sta preprečena navpična pomika in torzijska zasuka. $(EI_y = GI_x)$.



[30 točk]

MTT IZPIT 26. 1. 1996

Ad 1.) $A = 16 \cdot 0{,}5 = 8 \; mm^2$

$$\varepsilon = \frac{N}{EA} + \alpha_T \Delta T = \frac{200}{2.5 \cdot 10^5 \cdot 8} + 1.25 \cdot 10^{-5} \cdot 16$$

$$\varepsilon = (10 + 20) \cdot 10^{-5} \longrightarrow \varepsilon = 30 \cdot 10^{-5}$$

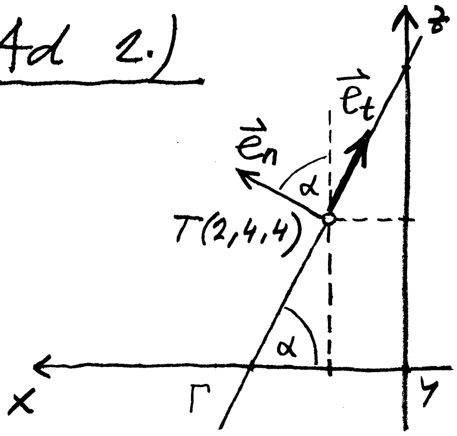
Dolžina deformirane enote : $e' = 1 + \varepsilon$

$$\boxed{e' = 1{,}0003}$$

Dejanska razdalja $\overline{AB}$: $L_0 = L\, e'$

$$L_0 = 26{,}3290 \cdot 1{,}0003 \longrightarrow \boxed{L_0 = 26{,}3369 \; m}$$

Ad 2.)



$z_T = 2 \; cm \longrightarrow T(2,4,4)$

$$[\sigma_{ij}]_T = \begin{bmatrix} 16 & 32 & 22 \\ 32 & 48 & 0 \\ 22 & 0 & -24 \end{bmatrix}$$

a) $I_1 = 16 + 48 - 24 = 40$

$$I_2 = \begin{vmatrix} 48 & 0 \\ 0 & -24 \end{vmatrix} + \begin{vmatrix} 16 & 22 \\ 22 & -24 \end{vmatrix} + \begin{vmatrix} 16 & 32 \\ 32 & 48 \end{vmatrix} = -2276$$

$$I_3 = 22(32 \cdot 0 - 48 \cdot 22) - 24(16 \cdot 48 - 32^2) = -17.088$$

$$\boxed{\sigma^3 - 40\sigma^2 - 2276\sigma + 17088 = 0}$$

$$\boxed{\begin{aligned} \sigma_{11} &= 69{,}289 \\ \sigma_{22} &= 6{,}828 \\ \sigma_{33} &= -36{,}117 \end{aligned}}$$

b) $d = \sqrt{4^2+8^2} = 8,944 \longrightarrow \sin\alpha = 0,8944 = e_{nx}$

$\cos\alpha = 0,4472 = e_{nz}$

$e_{ny} = 0$

$$\boxed{\vec{e}_n = 0,8944\, \vec{e}_x + 0,4472\, e_z}$$

$$\frac{1+\nu}{E} = \frac{1,3}{210\,000} = 0,619 \cdot 10^{-5}$$

$$\frac{\nu}{E} = \frac{0,3}{210\,000} = 0,143 \cdot 10^{-5}$$

$$\varepsilon_{ij} = \frac{1+\nu}{E}\sigma_{ij} - \frac{\nu}{E}\delta_{ij} I_1^\sigma$$

$$\varepsilon_{xx} = 10^{-5}(0,619 \cdot 16 - 0,143 \cdot 40) = 4,190 \cdot 10^{-5}$$

$$\varepsilon_{yy} = 10^{-5}(0,619 \cdot 48 - 0,143 \cdot 40) = 24,000 \cdot 10^{-5}$$

$$\varepsilon_{zz} = 10^{-5}(-0,619 \cdot 24 - 0,143 \cdot 40) = -20,571 \cdot 10^{-5}$$

$$\varepsilon_{xy} = 10^{-5} \cdot 0,619 \cdot 32 = 19,810 \cdot 10^{-5}$$

$$\varepsilon_{yz} = 0$$

$$\varepsilon_{zx} = 10^{-5} \cdot 0,619 \cdot 22 = 13,619 \cdot 10^{-5}$$

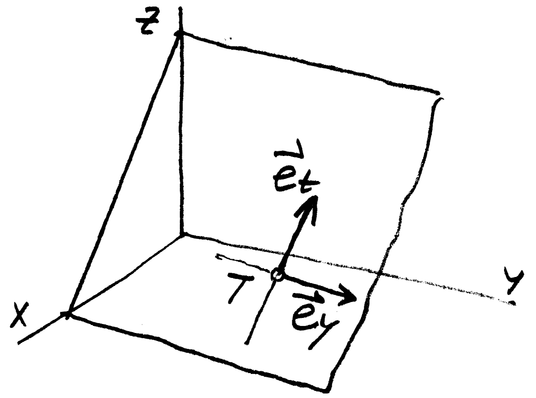
$$\begin{Bmatrix} \varepsilon_{nx} \\ \varepsilon_{ny} \\ \varepsilon_{nz} \end{Bmatrix} = 10^{-5} \begin{bmatrix} 4,190 & 19,810 & 13,619 \\ 19,810 & 24,000 & 0 \\ 13,619 & 0 & -20,571 \end{bmatrix} \begin{Bmatrix} 0,8944 \\ 0 \\ 0,4472 \end{Bmatrix}$$

$$= 10^{-5} \begin{Bmatrix} 9,838 \\ 17,719 \\ 2,982 \end{Bmatrix}$$

$$\vec{\varepsilon}_n = 10^{-5}(9,838\, \vec{e}_x + 17,719\, \vec{e}_y + 2,982 \cdot \vec{e}_z)$$

$$\varepsilon_{nn} = \vec{\varepsilon}_n \cdot \vec{e}_n = 10,133 \cdot 10^{-5} \longrightarrow \boxed{D_{nn} \doteq 10,133 \cdot 10^{-5}}$$

$$\boxed{\vec{e}_t = -0{,}4472\,\vec{e}_x + 0{,}8944\,\vec{e}_z}$$



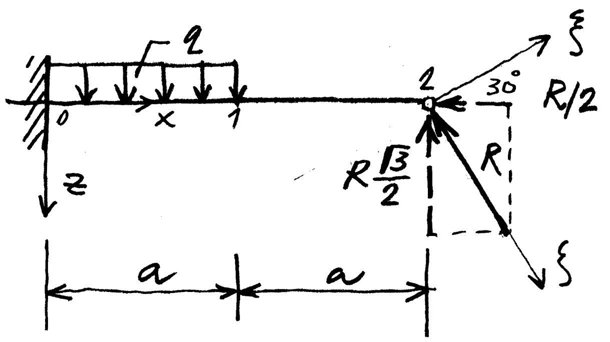
$$\varepsilon_{ty} = \varepsilon_{yt} = \vec{\varepsilon}_y \cdot \vec{e}_t$$

$$\vec{\varepsilon}_y = 10^{-5}(19{,}810\,\vec{e}_x + 24\,\vec{e}_y)$$

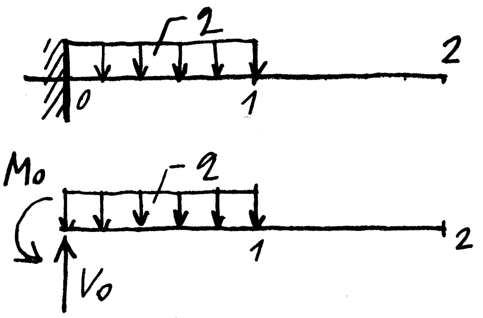
$$\varepsilon_{ty} = 10^{-5}\,19{,}810 \cdot (-0{,}4472) = -8{,}859 \cdot 10^{-5}$$

$$\boxed{D_{ty} \doteq 2\,\varepsilon_{ty} = -17{,}719 \cdot 10^{-5}}$$

Ad 2.)



Vpliv q :



$$V_0 = qa \;,\quad M_0 = \frac{qa^2}{2}$$

$$M_y = V_0 x - M_0 - \frac{q}{2}\left(x^2 - \langle x-a\rangle^2\right)$$

$$M_y = \frac{q}{2}\left(2ax - a^2 - x^2 + \langle x-a\rangle^2\right) = -EI_y\, w''$$

$$EI_y\, w' = \frac{q}{2}\left(\frac{x^3}{3} - ax^2 + a^2x - \frac{1}{3}\langle x-a\rangle^3\right) + C_1$$

$$EI_y\, w = \frac{q}{2}\left(\frac{x^4}{12} - a\frac{x^3}{3} + a^2\frac{x^2}{2} - \frac{1}{12}\langle x-a\rangle^4\right) + C_1 x + C_2$$

$$x=0 \longrightarrow w=0,\; w'=0 \longrightarrow C_1 = C_2 = 0$$

$$x=2a \longrightarrow w_2(q) = q\,\frac{7a^4}{24EI_y}$$

Tč. 2 :

$$\boxed{\begin{aligned} u_x &= -R\,\frac{a}{EA_x} \\ u_z &= q\,\frac{7a^4}{24EI_y} - R\,\frac{4\sqrt{3}\,a^3}{3EI_y} \end{aligned}}$$

$$u_\xi = u_x e_{\xi x} + u_z e_{\xi z} = 0 \qquad e_{\xi x} = \frac{1}{2} \qquad e_{\xi z} = \frac{\sqrt{3}}{2}$$

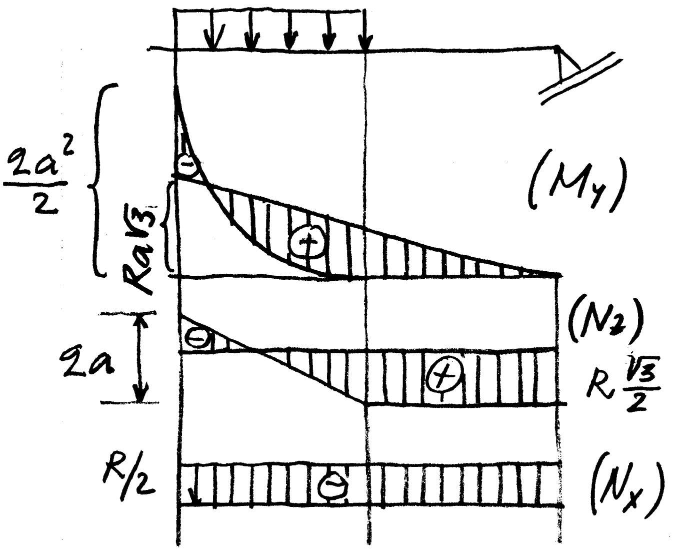
$$-R\frac{a}{EA_x}\cdot\frac{1}{2} + \left(q\frac{7a^4}{24EI_y} - R\frac{4\sqrt{3}a^3}{3EI_y}\right)\cdot\frac{\sqrt{3}}{2} = 0$$

$$R\left(\frac{1}{A_x} + \frac{4a^2}{I_y}\right) = q\frac{7a^3}{24I_y} \rightarrow \boxed{R = q\frac{7A_x a^3}{24(I_y + 4a^2A_x)}}$$

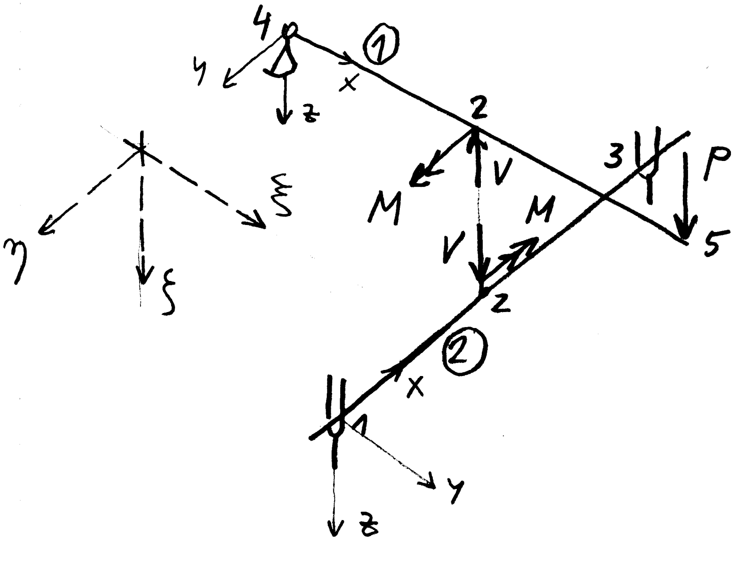
$$e_{\xi x} = \frac{\sqrt{3}}{2} \qquad e_{\xi z} = -\frac{1}{2}$$

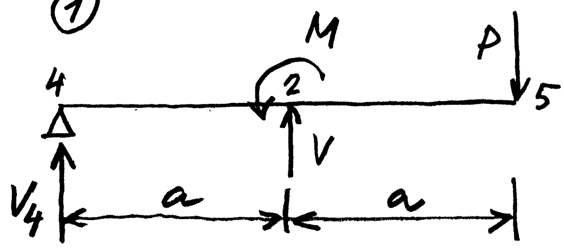
$$u_\xi = u_x e_{\xi x} + u_z e_{\xi z}$$

$$\boxed{u_\xi = -R\frac{a\sqrt{3}}{2EA_x} + R\frac{2\sqrt{3}a^3}{3EI_y} - q\frac{7a^4}{48EI_y}}$$

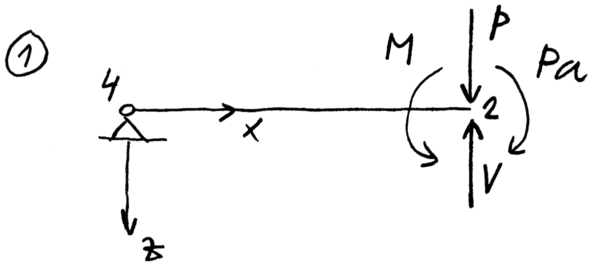


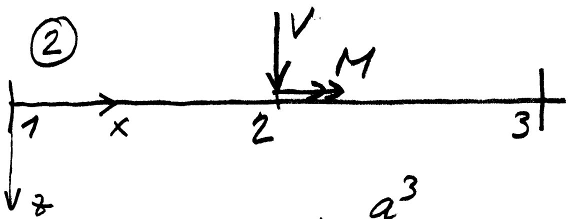
Ad 3.)

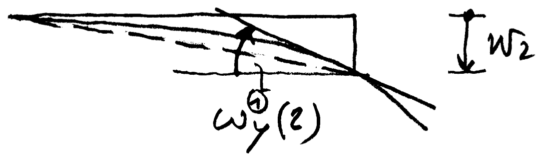




$$\boxed{\begin{aligned} V_4 &= P - V \\ M &= a(2P - V) \end{aligned}}$$







$$w_2 = V\frac{a^3}{6EI_y}$$

$$\omega_x^{(1)}(2) = M\frac{a}{2GI_x}$$

$$\boxed{EI_y = GI_x = 1}$$

$$\omega_y^{(1)}(2) = -\frac{w_2}{a} + (M - Pa)\frac{a}{3EI_y}$$

$$\omega_y^{(1)}(2) = -V\frac{a^2}{6EI_y} + M\frac{a}{3EI_y} - P\frac{a^2}{3EI_y}$$

$$\boxed{\omega_y^{(1)}(2) = -\omega_x^{(2)}(2)}$$

$$-V\frac{a}{6} + M\cdot\frac{1}{3} - P\frac{a}{3} = -M\cdot\frac{1}{2} \longrightarrow M\cdot\frac{5}{6} - V\frac{a}{6} = P\frac{a}{3}$$

$$\left.\begin{aligned} 5M - Va &= 2Pa \\ \text{R.P.:}\quad M + Va &= 2Pa \end{aligned}\right\} \quad 6M = 4Pa \longrightarrow \boxed{M = P\frac{2a}{3}}$$

$$\boxed{V = P\cdot\frac{4}{3}}$$

$$w_5 = w_2 - a\,\omega_y^{(1)}(2) + P\frac{a^3}{3EI_y}$$

$$w_5 = P\cdot\frac{4}{3}\cdot\frac{a^3}{6EI_y} + P\cdot\frac{2a}{3}\cdot\frac{a}{2GI_x}\cdot a + P\frac{a^3}{3EI_y}$$

$$w_5 = \frac{Pa^3}{EI_y}\left(\frac{4}{18} + \frac{2}{6} + \frac{1}{3}\right) \longrightarrow \boxed{w_5 = P\frac{8a^3}{9EI_y}}$$

Proth Aik : $\boxed{M = 0} \longrightarrow \boxed{V = 2P}$

$$\omega_y^{(1)}(2) = -2P\frac{a^2}{6EI_y} - P\frac{a^2}{3EI_y} = -P\frac{2a^2}{3EI_y}$$

$$w_5 = 2P\frac{a^3}{6EI_y} + P\frac{2a^3}{3EI_y} + P\frac{a^3}{3EI_y} \longrightarrow \boxed{w_5 = P\frac{4a^3}{3EI_y}}$$