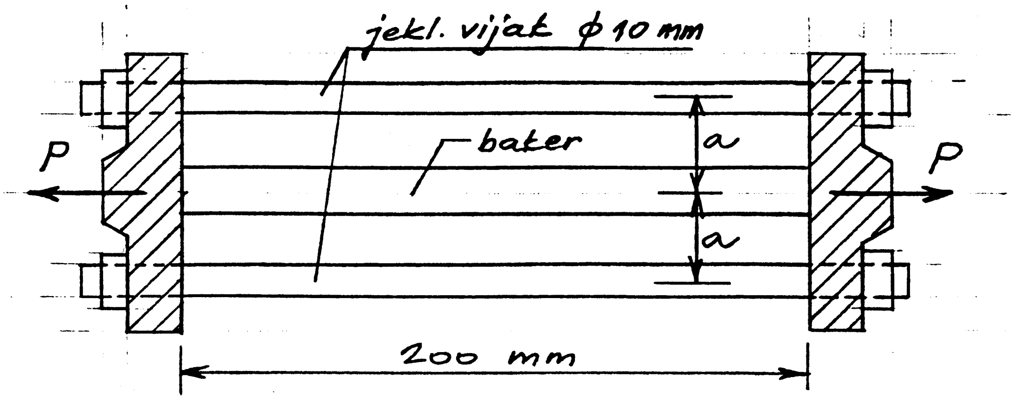
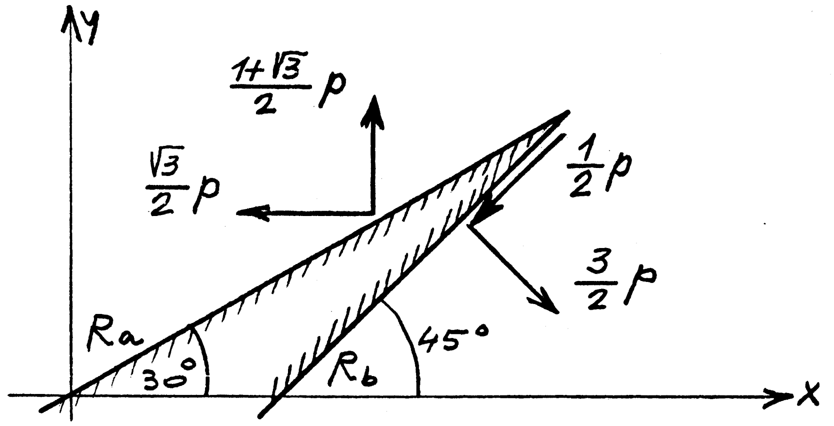
1. Med dva toga odlitka, ki sta spojena z dvema jeklenima vijakoma premera $10\,mm$ in dolžine $200\,mm$, moramo vstaviti bakreno palico s prečnim prerezom $A_b = 600\,mm^2$ in dolžino $200.2\,mm$.
   a. Določi silo P tako, da je mogoče vstaviti bakreno palico med odlitka!
   b. Določi razdaljo med odlitkoma, če po vstavitvi bakrene palice odstranimo sili P!
   c. Določi napetosti v jeklenih vijakih in bakrenem vložku po odstranitvi sile P!



$E_j = 2.1 \cdot 10^5\,MPa$

$E_b = 1.0 \cdot 10^5\,MPa$

2. Na skici so prikazane intenzitete enakomerne zvezne obtežbe na robovih R_a in R_b tanke stene, v kateri vlada homogeno ravninsko napetostno stanje ($\sigma_{xz} = \sigma_{yz} = \sigma_{zz} = 0$).
   a. Določi napetosti v koordinatnem sistemu (x, y)!
   b. Določi specifično spremembo dolžine robu R_a!
   c. Določi velikosti in smeri glavnih linearnih deformacij!

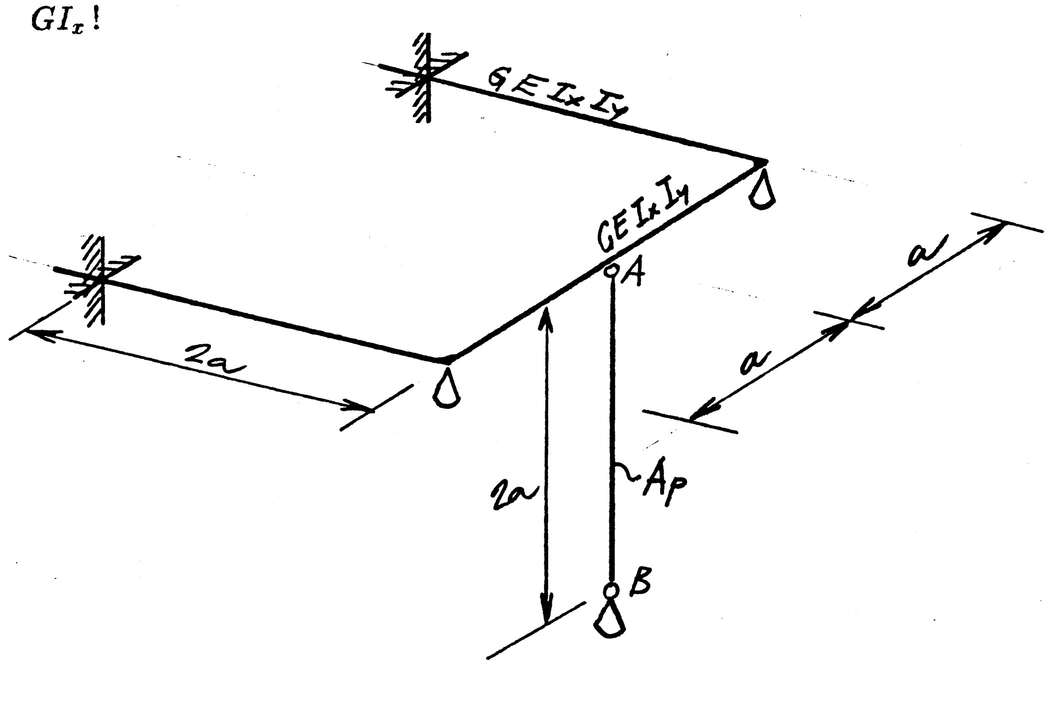


$E = 2.1 \cdot 10^5\,MPa$

$\nu = 0.3$

$p = 100\,MPa$

3. Določi osno silo v palici AB, če konstrukcijo segrejemo za ΔT! Pri tem upoštevaj, da je $EI_y = GI_x$!



MTT IZPIT 19. 12. 1994

Ad 1.)

$$\Delta l = l' - l_0 = 200{,}2 - 200$$
$$\Delta l = 0{,}2\ mm = 0{,}02\ cm$$

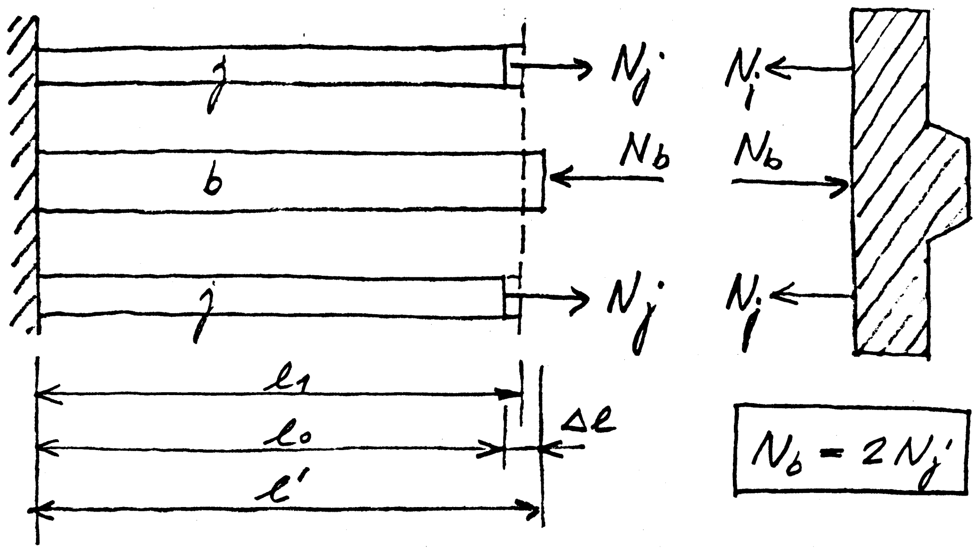
a)

$$\Delta l = \frac{P}{2}\,\frac{l_0}{A_j E_j} \longrightarrow P = \frac{2\Delta l}{l_0 E_j A_j}$$

$$A_j = \frac{1}{4}\pi \cdot 10^2 \longrightarrow A_j = 78{,}5\ mm^2$$

$$\boxed{P = 32\,987\ N = 32{,}987\ kN}$$

b)



$$\boxed{N_b = 2N_j}$$

$$f_j = \frac{l_0}{E_j A_j} = \frac{200}{2{,}1 \cdot 10^5 \cdot 78{,}5} \rightarrow f_j = 12{,}126 \cdot 10^{-6}\ mm/N$$

$$f_b = \frac{l'}{E_b A_b} = \frac{200{,}2}{10^5 \cdot 600} \longrightarrow f_b = 3{,}337 \cdot 10^{-6}\ mm/N$$

$$l_0 + N_j f_j = l' - N_b f_b = l_0 + \Delta l - N_b f_b$$

$$N_j f_j + N_b f_b = \Delta l \longrightarrow N_j (f_j + 2 f_b) = \Delta l$$

$$N_j = \frac{\Delta l}{f_j + 2 f_b} \longrightarrow \left\{ \boxed{\begin{aligned} N_j &= 10\,639\ N \\ N_b &= 21\,277\ N \end{aligned}} \right.$$

$$l_1 = l_0 + N_j \cdot f_j = 200 + 10639 \cdot 12{,}126 \cdot 10^{-6}$$

$$\boxed{l_1 = 200{,}129 \text{ mm}}$$

Kontrola :

$$l_1 = l' - N_b f_b = 200{,}2 - 21277 \cdot 3{,}337 \cdot 10^{-6}$$

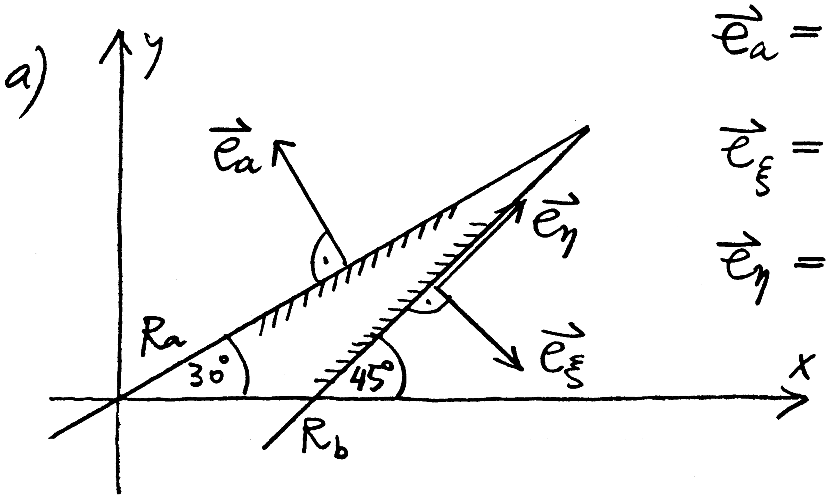
$$= 200{,}129 \text{ mm} \checkmark$$

c) $\sigma_j = \dfrac{N_j}{A_j} = \dfrac{10639}{78{,}5} \longrightarrow \boxed{\sigma_j = 135{,}46 \text{ MPa}}$

$\sigma_b = -\dfrac{N_b}{A_b} = -\dfrac{21277}{600} \longrightarrow \boxed{\sigma_b = -35{,}46 \text{ MPa}}$

Ad 2.)

a)



$$\vec{e}_a = -\frac{1}{2}\vec{e}_x + \frac{\sqrt{3}}{2}\vec{e}_y$$

$$\vec{e}_\xi = \frac{\sqrt{2}}{2}\vec{e}_x - \frac{\sqrt{2}}{2}\vec{e}_y$$

$$\vec{e}_\eta = \frac{\sqrt{2}}{2}\vec{e}_x + \frac{\sqrt{2}}{2}\vec{e}_y$$

$$\sigma_{\xi\xi} = \frac{3}{2}p$$

$$\sigma_{\xi\eta} = -\frac{1}{2}p$$

Ra : $\vec{p}_a = -\dfrac{\sqrt{3}}{2}p\,\vec{e}_x + \dfrac{1+\sqrt{3}}{2}p\,\vec{e}_y$

Rb : $\vec{\sigma}_\xi = \dfrac{3}{2}p\,\vec{e}_\xi + \dfrac{1}{2}p\,\vec{e}_\eta$

Ra:

$$-\frac{\sqrt{3}}{2}p = \sigma_{xx} e_{ax} + \sigma_{xy} e_{ay} = -\frac{1}{2}\sigma_{xx} + \frac{\sqrt{3}}{2}\sigma_{xy}$$

$$\frac{1+\sqrt{3}}{2}p = \sigma_{xy} e_{ax} + \sigma_{yy} e_{ay} = -\frac{1}{2}\sigma_{xy} + \frac{\sqrt{3}}{2}\sigma_{yy}$$

R_b: $\sigma_{\xi\xi} = \sigma_{xx}\, e_{\xi x}^2 + 2\sigma_{xy}\, e_{\xi x}\, e_{\xi y} + \sigma_{yy}\, e_{\xi y}^2$

$$\sigma_{\xi\eta} = \sigma_{xx}\, e_{\xi x}\, e_{\eta x} + \sigma_{xy}(e_{\xi x}\, e_{\eta y} + e_{\xi y}\, e_{\eta x}) + \sigma_{yy}\, e_{\xi y}\, e_{\eta y}$$

R_a:

$$\sigma_{xx} - \sqrt{3}\,\sigma_{xy} = \sqrt{3}\,p \;\rightarrow\; \sigma_{xx} = \sqrt{3}\,p + \sqrt{3}\,\sigma_{xy}$$

$$\sqrt{3}\,\sigma_{yy} - \sigma_{xy} = (1+\sqrt{3})\,p \;\rightarrow\; \sigma_{yy} = \frac{1}{\sqrt{3}}(1+\sqrt{3})\,p + \frac{1}{\sqrt{3}}\sigma_{xy}$$

R_b:

$$\frac{3}{2}p = \sigma_{xx}\cdot\frac{1}{2} - 2\sigma_{xy}\cdot\frac{1}{2} + \sigma_{yy}\cdot\frac{1}{2} \quad (= \sigma_{\xi\xi})$$

$$\sigma_{xx} - 2\sigma_{xy} + \sigma_{yy} = 3p$$

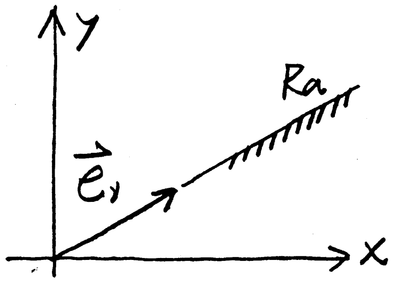
$$\sqrt{3}\,p + \sqrt{3}\,\sigma_{xy} - 2\sigma_{xy} + \frac{1}{\sqrt{3}}(1+\sqrt{3})\,p + \frac{1}{\sqrt{3}}\sigma_{xy} = 3p \;\Big|\cdot\sqrt{3}$$

$$\boxed{\sigma_{xy} = -p} \;\longrightarrow\; \boxed{\sigma_{xx} = 0}\;;\quad \boxed{\sigma_{yy} = p}$$

Kontrola: (R_b)

$$\sigma_{\xi\eta} = \sigma_{xx}\cdot\frac{1}{2} + \sigma_{xy}\left(\frac{1}{2} - \frac{1}{2}\right) + \sigma_{yy}\cdot\left(-\frac{1}{2}\right) = -\frac{p}{2} \;\checkmark$$

b)



$$\vec{e}_\nu = \frac{\sqrt{3}}{2}\vec{e}_x + \frac{1}{2}\vec{e}_y$$

$$I_1^\sigma = \sigma_{xx} + \sigma_{yy} \;\rightarrow\; \boxed{I_1^\sigma = p}$$

$$D_{\nu\nu} = \varepsilon_{\nu\nu} = \frac{1+\nu}{E}\sigma_{\nu\nu} - \frac{\nu}{E} I_1^\sigma$$

$$\sigma_{\nu\nu} = \sigma_{xx}\, e_{\nu x}^2 + 2\sigma_{xy}\, e_{\nu x}\, e_{\nu y} + \sigma_{yy}\, e_{\nu y}^2$$

$$\sigma_{\nu\nu} = -2p\,\frac{\sqrt{3}}{4} + \frac{p}{4} \;\longrightarrow\; \sigma_{\nu\nu} = \frac{p}{4}(1 - 2\sqrt{3})$$

$$\boxed{\sigma_{\nu\nu} = -61{,}6 \text{ MPa}}$$

$$D_{\nu\nu} = \frac{1}{2{,}1\cdot 10^5}\left[1{,}3\cdot(-61{,}6) - 0{,}3\cdot 100\right]$$

$$\boxed{D_{\nu\nu} = -52{,}42\cdot 10^{-5}}$$

c)

$$\sigma_{11,22} = \frac{p}{2} \pm \sqrt{\frac{p^2}{4} + p^2} \quad \boxed{\begin{aligned}\sigma_{11} &= \frac{p}{2}\left(1+\sqrt{5}\right)\\ \sigma_{22} &= \frac{p}{2}\left(1-\sqrt{5}\right)\end{aligned}}$$

$$\boxed{\begin{aligned}\sigma_{11} &= 161{,}80 \text{ MPa}\\ \sigma_{22} &= -61{,}80 \text{ MPa}\end{aligned}}$$

$$\operatorname{tg} 2\alpha_\sigma = \frac{2\sigma_{xy}}{\sigma_{xx}-\sigma_{yy}} = 2 \longrightarrow \boxed{\alpha_\sigma = 31{,}72^\circ}$$

$$\varepsilon_{11} = \frac{1+\nu}{E}\sigma_{11} - \frac{\nu}{E} I_1^\sigma = \frac{1}{2{,}1\cdot 10^5}\left[1{,}3\cdot 161{,}80 - 0{,}3\cdot 100\right]$$

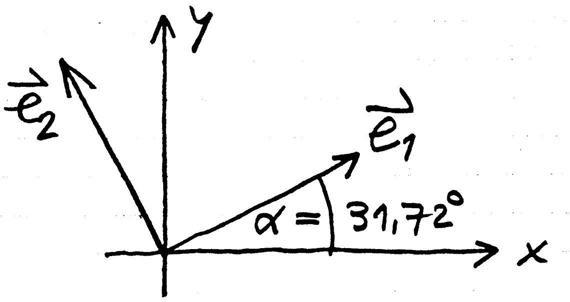
$$\boxed{\varepsilon_{11} = 85{,}88\cdot 10^{-5}}$$

$$\varepsilon_{22} = \frac{1+\nu}{E}\sigma_{22} - \frac{\nu}{E} I_1^\sigma = \frac{1}{2{,}1\cdot 10^5}\left[-1{,}3\cdot 61{,}80 - 0{,}3\cdot 100\right]$$

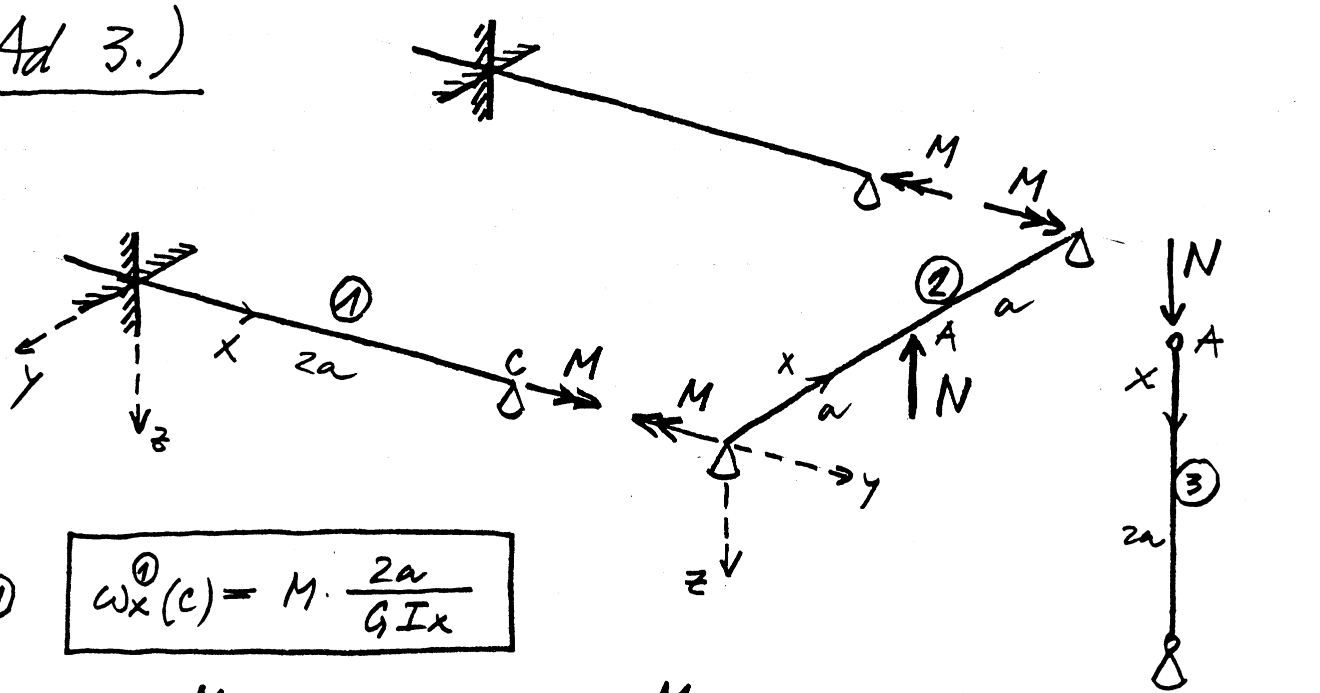
$$\boxed{\varepsilon_{22} = -52{,}54\cdot 10^{-5}}$$

$$\varepsilon_{33} = -\frac{\nu}{E} I_1^\sigma = -\frac{0{,}3}{2{,}1\cdot 10^5}\cdot 100$$

$$\boxed{\varepsilon_{33} = \varepsilon_{zz} = -14{,}29\cdot 10^{-5}} \qquad \boxed{\alpha_\varepsilon = \alpha_\sigma = 31{,}72^\circ}$$

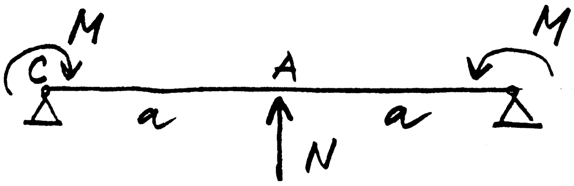


Ad 3.)



① $\boxed{\omega_x^{①}(c) = M \cdot \dfrac{2a}{GI_x}}$

②



$$w_A^{②} = M\frac{(2a)^2}{8EI_y} - N\frac{(2a)^3}{48EI_y} \longrightarrow \boxed{w_A^{②} = M\frac{a^2}{2EI_y} - N\frac{a^3}{6EI_y}}$$

$$\omega_y^{②}(c) = -M\frac{2a}{2EI_y} + N\frac{(2a)^2}{16EI_y} \longrightarrow$$

$$\boxed{\omega_y^{②}(c) = -M\frac{a}{EI_y} + N\frac{a^2}{4EI_y}}$$

③ $\boxed{u_A^{③} = N\dfrac{2a}{EA_p} - 2a\,\alpha_T\,\Delta T}$

$$\omega_x^{①}(c) = \omega_y^{②}(c) \longrightarrow M\cdot\frac{2a}{GI_x} = -M\frac{a}{EI_y} + N\frac{a^2}{4EI_y}$$

$$\boxed{M = N\frac{a}{12}}$$

$$w_A^{②} = N\frac{a}{12}\cdot\frac{a^2}{2EI_y} - N\frac{a^3}{6EI_y} \longrightarrow \boxed{w_A^{②} = -N\frac{a^3}{8EI_y}}$$

$$w_A^{②} = u_A^{③} \longrightarrow -N\frac{a^3}{8EI_y} = N\frac{2a}{EA_p} - 2a\,\alpha_T\,\Delta T$$

$$\boxed{N = \frac{16\,\alpha_T\,EA_p I_y}{a^2A_p + 16I_y}\,\Delta T} = 0{,}025 \text{ MN}$$