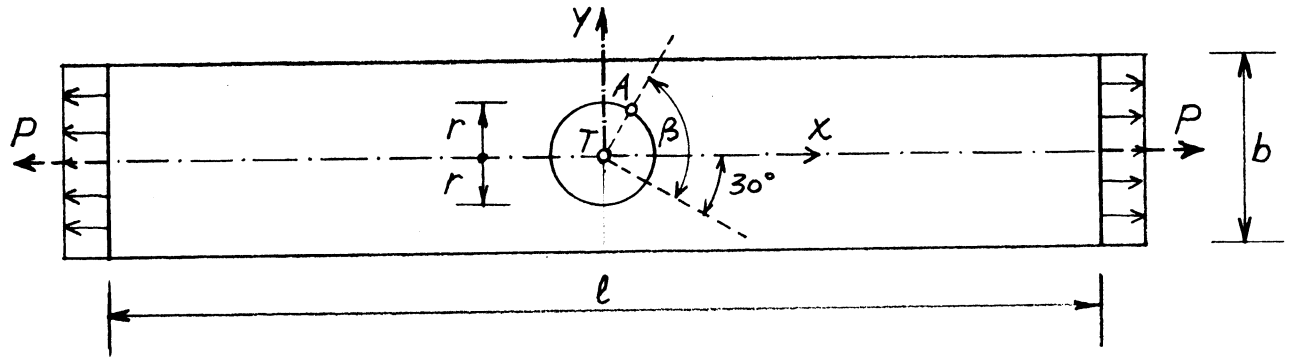
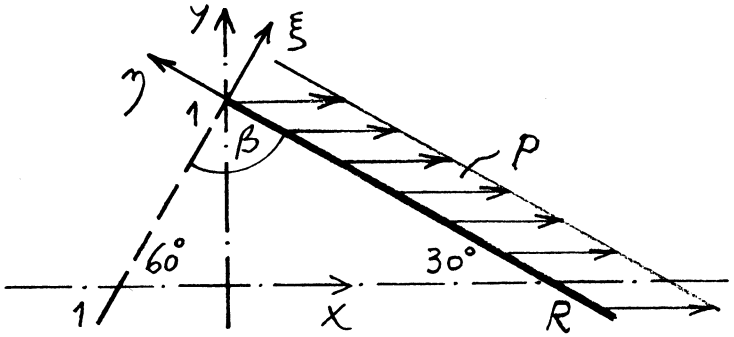
1. V sredini ravnega kovinskega traku z dolžino $l = 100\,cm$, širino $b = 10\,cm$ in debelino $d = 1\,mm$ narišemo krog s polmerom $r = 2\,cm$. Ožja konca traku enakomerno obtežimo s silo $P = 42\,kN$. Pri tem se dolžina traku poveča za $2\,mm$, širina pa se zmanjša za $0.06\,mm$. Narisani krog se spremeni v pravilno elipso. Določi:
   a. elastični modul E in koeficient prečne kontrakcije ν uporabljene kovine ter spremembo debeline traku,
   b. velikosti polosi dobljene elipse,
   c. velikosti in ravnine največjih normalnih in največjih strižnih napetosti (skica!),
   d. spremembo pravega kota β,
   e. novo dolžino polmera TA,
   f. specifično spremembo prostornine v točki T!

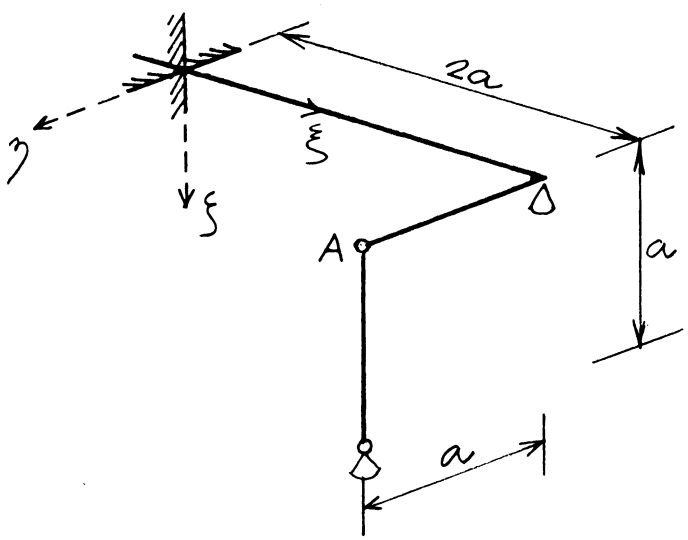


2. Na rob R enakomerno debele homogene in izotropne stene deluje enakomerna zvezna obtežba p, kot je prikazano na skici. V prerezu 1–1 ni normalnih napetosti. Določi ustrezno spremembo pravega kota β ter velikosti in smeri glavnih linearnih deformacij!



$$E = 10\,000\,kN/cm^2$$
$$\nu = 0.25\,cm^2$$
$$p = 6\,kN/cm^2$$
$$\sigma_{zx} = \sigma_{zy} = \sigma_{zz} = 0\,.$$

3. Določi pomike točke A, če se konstrukcija segreje za $60K$! Skiciraj tudi potek in značilne vrednosti notranjih sil!



$$E = 20\,000\,kN/cm^2$$
$$\nu = 0.25$$
$$\alpha_T = 1.2 \cdot 10^{-5}/K$$
$$A_x = 50\,cm^2$$
$$I_x = 2040\,cm^4$$
$$I_y = 1020\,cm^4$$
$$a = 2.5\,m$$

Ad 1.

a) $E = \frac{\sigma_{xx}}{\varepsilon_{xx}}$

$$\sigma_{xx} = \frac{P}{A_x} = \frac{42}{10 \cdot 0{,}1}$$

$$\boxed{\sigma_{xx} = 42\ kN/cm^2}$$

$$\varepsilon_{xx} = \frac{\Delta l}{l} = \frac{0{,}2}{100}$$

$$\boxed{\varepsilon_{xx} = 0{,}002}$$

$$E = \frac{42}{0{,}002} \rightarrow \boxed{E = 21000\ kN/cm^2}$$

$$\varepsilon_{yy} = \frac{\Delta b}{b} = -\frac{0{,}006}{10} \rightarrow \boxed{\varepsilon_{yy} = -0{,}0006}$$

$$\nu = -\frac{\varepsilon_{yy}}{\varepsilon_{xx}} = \frac{0{,}0006}{0{,}002} \rightarrow \boxed{\nu = 0{,}3}$$

$$\Delta d = d \cdot \varepsilon_{yy} = -0{,}0006 \cdot 0{,}1 \rightarrow \boxed{\Delta d = -6 \cdot 10^{-5}\ cm}$$

b) $a' = r(1 + \varepsilon_{xx}) = 2 \cdot 1{,}002 \rightarrow \boxed{a' = 2{,}004\ cm}$

$b' = r(1 + \varepsilon_{yy}) = 2 \cdot 0{,}9994 \rightarrow \boxed{b' = 1{,}9988\ cm}$

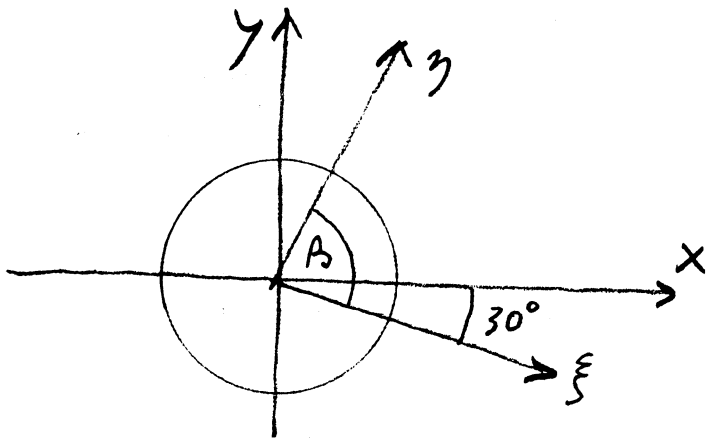
c) $\sigma_{xx} = 42\ kN/cm^2$, $\sigma_{yy} = \sigma_{xy} = 0$

$$\boxed{\sigma_{11} = 42\ kN/cm^2,\ \sigma_{22} = 0,\ \alpha_\sigma = 0}$$

$$\tau_{1,2} = \pm \frac{1}{2}(\sigma_{11} - \sigma_{22}) \qquad \boxed{\tau_{1,2} = \pm 21\ kN/cm^2}$$

$$\boxed{\alpha_\tau = \pm 45^\circ}$$

d)



$$e_{\xi x} = \frac{\sqrt{3}}{2}$$

$$e_{\xi y} = -\frac{1}{2}$$

$$e_{\eta x} = \frac{1}{2}$$

$$e_{\eta y} = \frac{\sqrt{3}}{2}$$

$$\varepsilon_{\xi\eta} = \varepsilon_{xx} e_{\xi x} e_{\eta x} + \varepsilon_{xy}(e_{\xi x} e_{\eta y} + e_{\xi y} e_{\eta x}) + \varepsilon_{yy} e_{\xi y} e_{\eta y}$$

$$\varepsilon_{\xi\eta} = 0{,}002 \cdot \frac{\sqrt{3}}{2} \cdot \frac{1}{2} - 0{,}0006 \cdot \left(-\frac{1}{2}\right) \cdot \frac{\sqrt{3}}{2}$$

$$\varepsilon_{\xi\eta} = 0{,}001126 \longrightarrow \boxed{\Delta_{\xi\eta} = 0{,}00225 = 0{,}13^\circ}$$

e)

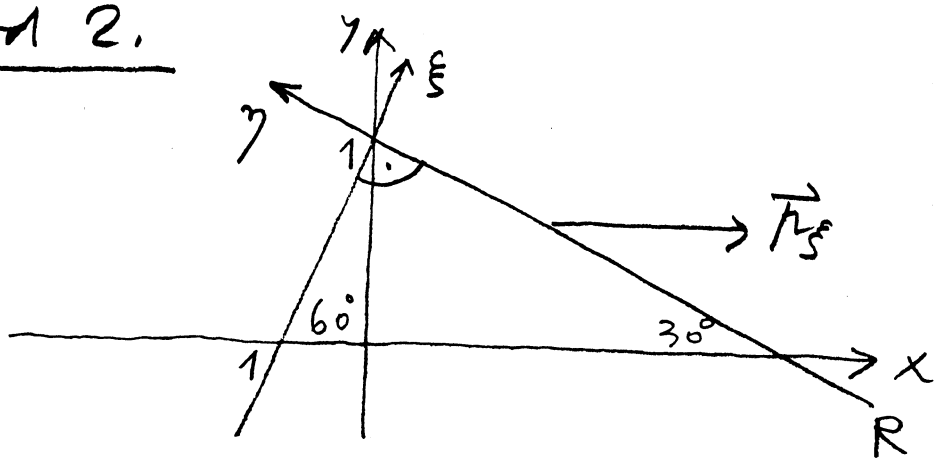
$$\varepsilon_{\eta\eta} = \varepsilon_{xx} e_{\eta x}^2 + \varepsilon_{yy} e_{\eta y}^2 = 0{,}002 \cdot \frac{1}{4} - 0{,}0006 \cdot \frac{3}{4}$$

$$\boxed{\varepsilon_{\eta\eta} = 5 \cdot 10^{-5}} \qquad \boxed{\overline{TA'} = 2(1 + \varepsilon_{\eta\eta}) = 2{,}0001 \text{ cm}}$$

f) $\varepsilon_v(T) = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz} = 0{,}002 - 2 \cdot 0{,}0006$

$$\boxed{\varepsilon_v(T) = 0{,}0008}$$

Am 2.



$$e_{\xi x} = \frac{1}{2}$$

$$e_{\xi y} = \frac{\sqrt{3}}{2}$$

$$e_{\eta x} = -\frac{\sqrt{3}}{2}$$

$$e_{\eta y} = \frac{1}{2}$$

Rot R: $\vec{p}_N = \vec{p}_\xi = p\vec{e}_x = \vec{\sigma}_x e_{\xi x} + \vec{\sigma}_y e_{\xi y}$

$p_{\xi x} = \sigma_{xx} e_{\xi x} + \sigma_{xy} e_{\xi y} = p \rightarrow \sigma_{xx} + \sqrt{3}\,\sigma_{xy} = 2p$

$p_{\xi y} = \sigma_{xy} e_{\xi x} + \sigma_{yy} e_{\xi y} = 0 \rightarrow \sigma_{xy} + \sqrt{3}\,\sigma_{yy} = 0$

$\sigma_{\eta\eta} = \sigma_{xx} e_{\eta x}^2 + 2\sigma_{xy} e_{\eta x} e_{\eta y} + \sigma_{yy} e_{\eta y}^2 = 0$

$\sigma_{xx} \cdot \frac{3}{4} - 2\sigma_{xy} \frac{\sqrt{3}}{4} + \sigma_{yy} \cdot \frac{1}{4} = 0$

$3\sigma_{xx} - 2\sqrt{3}\,\sigma_{xy} + \sigma_{yy} = 0$

1	$\sqrt{3}$	0	σ_{xx}	=	$2p$
0	1	$\sqrt{3}$	σ_{xy}		0
3	$-2\sqrt{3}$	1	σ_{yy}		0

$\rightarrow$

$\sigma_{xx} = \frac{7}{8} p$

$\sigma_{xy} = \frac{3\sqrt{3}}{8} p$

$\sigma_{yy} = -\frac{3}{8} p$

$\sigma_{xx} = 5{,}25 \ kN/cm^2$

$\sigma_{xy} = 3{,}90 \ kN/cm^2$

$\sigma_{yy} = -2{,}25 \ kN/cm^2$

$\sigma_{xz} = \sigma_{yz} = \sigma_{zz} = 0$

$\sigma_{\xi\eta} = \sigma_{xx} e_{\xi x} e_{\eta x} + \sigma_{xy}(e_{\xi x} e_{\eta y} + e_{\xi y} e_{\eta x}) + \sigma_{yy} e_{\xi y} e_{\eta y}$

$\sigma_{\xi\eta} = -5{,}20 \ kN/cm^2$

$\varepsilon_{\xi\eta} = \sigma_{\xi\eta} \cdot \frac{1+\nu}{E} = -5{,}20 \cdot \frac{1{,}25}{10000} = -6{,}5 \cdot 10^{-4}$

$D_{\xi\eta} = 2\varepsilon_{\xi\eta} = -0{,}0013 = -0{,}0745°$

$$\sigma_{11,22} = \frac{5{,}25 - 2{,}25}{2} \pm \sqrt{\left(\frac{5{,}25 + 2{,}25}{2}\right)^2 + 3{,}90^2}$$

$$\sigma_{11,22} = 1{,}5 \pm 5{,}41 \longrightarrow$$

$$\boxed{\begin{aligned} \sigma_{11} &= 6{,}91 \text{ kN/cm}^2 \\ \sigma_{22} &= -3{,}91 \text{ kN/cm}^2 \\ \sigma_{33} &= 0 \end{aligned}}$$

$$\frac{1+\nu}{E} = \frac{1{,}25}{10000} = 1{,}25 \cdot 10^{-4}$$

$$\frac{\nu}{E} = \frac{0{,}25}{10000} = 0{,}25 \cdot 10^{-4}$$

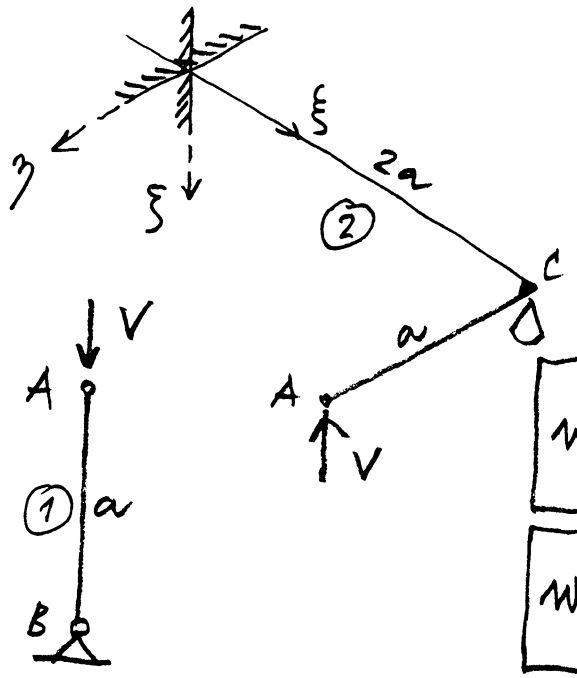
$$I_1^\sigma = 5{,}25 - 2{,}25 = 3 \text{ kN/cm}^2$$

$$\varepsilon_{11} = 10^{-4}(6{,}91 \cdot 1{,}25 - 0{,}25 \cdot 3) \longrightarrow \boxed{\varepsilon_{11} = 7{,}89 \cdot 10^{-4}}$$

$$\varepsilon_{22} = 10^{-4}(-3{,}91 \cdot 1{,}25 - 0{,}25 \cdot 3) \longrightarrow \boxed{\varepsilon_{22} = -4{,}14 \cdot 10^{-4}}$$

$$\varepsilon_{33} = 10^{-4}(-0{,}25 \cdot 3) \longrightarrow \boxed{\varepsilon_{33} = -0{,}75 \cdot 10^{-4}}$$

Ad 3.



$$\omega_\xi^{②}(C) = -Va \cdot \frac{2a}{GI_x}$$

$$u_\xi^{②}(A) = w_A^{②} =$$

$$= -V\frac{a^3}{3EI_y} - \omega_\xi^{②}(C) \cdot a$$

$$\boxed{w_A^{②} = -\frac{Va^3}{3EI_y} - \frac{2Va^3}{GI_x}}$$

$$\boxed{w_A^{①} = V\frac{a}{EA_x} - a\alpha_T \Delta T}$$

$$-V\frac{a^3}{3EI_y} - V\frac{2a^3}{GI_x} = V\frac{a}{EA_x} - a\alpha_T \Delta T$$

$$V\left(\frac{a^3}{3EI_y} + \frac{2a^3}{GI_x} + \frac{a}{EA_x}\right) = a\alpha_T \Delta T$$

$$2{,}1704\, V = 0{,}18 \longrightarrow \boxed{V = 0{,}083\ kN}$$

$$w_A = 0{,}083 \cdot \frac{250}{20000 \cdot 50} - 0{,}18$$

$$\boxed{w_A = -0{,}18\ cm}$$