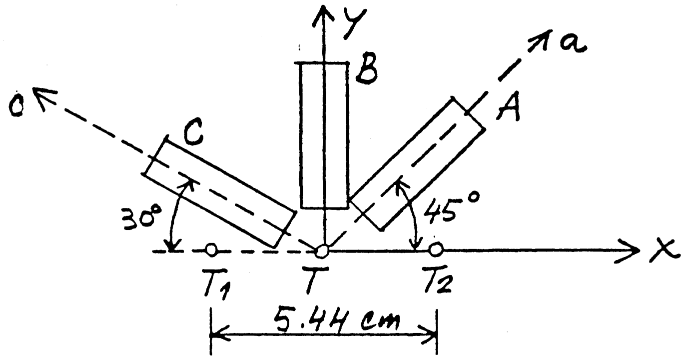
1. V točki T, ki leži na **neobteženi** površini stojine I-nosilca, nalepimo mersko baterijo, sestavljeno iz treh uporovnih merilnih lističev. Debelina stojine je 1 cm. Po obtežitvi nosilca izmerimo na lističu A linearno deformacijo 0.2 %, na lističu B −0.08 %, in na lističu C 0.12 %. Za kontrolo izmerimo z mehanskim ekstenzometrom spremembo razdalje med točkama T_1 in T_2. Ta sprememba znaša 0.016 cm.

   a. Določi komponente tenzorja majhnih deformacij in tenzorja napetosti v točki T. Ob upoštevanju meritve z ekstenzometrom preveri, ali je bila meritev z merilnimi lističi točna!

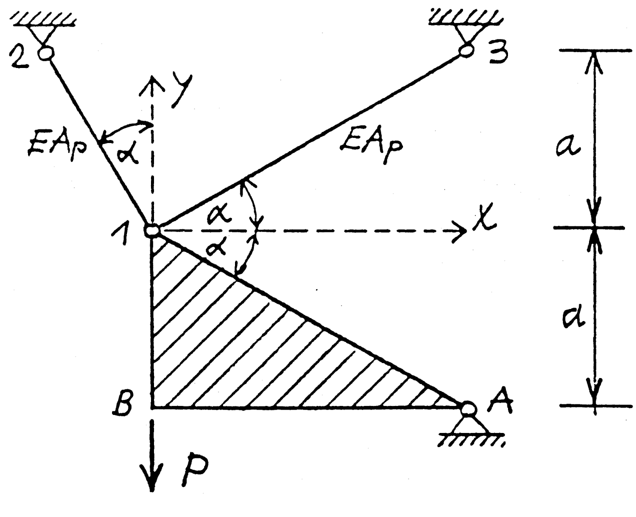
   b. Določi velikosti in smeri glavnih normalnih napetosti v točki T!



$E = 20\,000\, kN/cm^2$

$\nu = 0.25$

2. Toga šipa AB1 je podprta kot kaže skica. Določi osni sili v palicah $\overline{12}$ in $\overline{13}$, reakcije v podpori A ter pomike točke B!



$\alpha = 30^o$

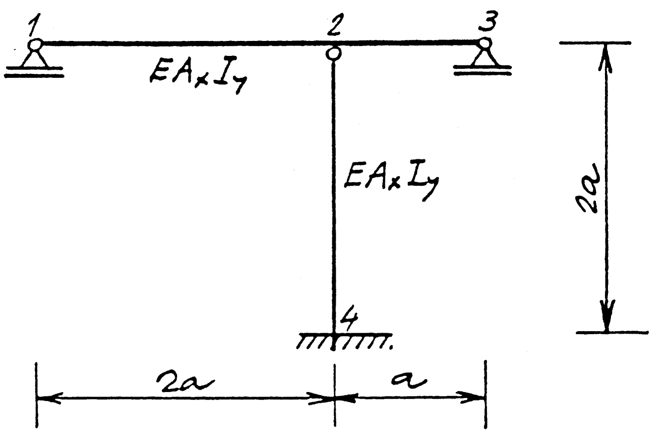
$a = 1\, m$

$E = 21\,000\, kN/cm^2$

$A_p = 1\, cm^2$

$P = 40\, kN$

3. Določi in skiciraj notranje sile, ki nastopijo v prikazani konstrukciji, če jo enakomerno segrejemo za 30 K!



$a = 2.5\, m$

$I_y = 6000\, cm^2$

$A_x = 88\, cm^2$

$E = 21\,000\, kN/cm^2$

$\alpha_T = 1.2 \cdot 10^{-5}/K$

MTT IZPIT 11. 2. 1994

Ad 1.) $\varepsilon_{yy} = -0{,}0008$

$\varepsilon_{aa} = 0{,}0020$; $e_{ax} = \frac{\sqrt{2}}{2}$, $e_{ay} = \frac{\sqrt{2}}{2}$, $e_{az} = 0$

$\varepsilon_{cc} = 0{,}0012$; $e_{cx} = -\frac{\sqrt{3}}{2}$, $e_{cy} = \frac{1}{2}$, $e_{cz} = 0$

$$2\mu = \frac{E}{1+\nu} = 16\,000 \ kN/cm^2$$

$$\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)} = 8\,000 \ kN/cm^2$$

a) $$\varepsilon_{aa} = \varepsilon_{xx} e_{ax}^2 + 2\varepsilon_{xy} e_{ax} e_{ay} + \varepsilon_{yy} e_{ay}^2$$
$$\varepsilon_{cc} = \varepsilon_{xx} e_{cx}^2 + 2\varepsilon_{xy} e_{cx} e_{cy} + \varepsilon_{yy} e_{cy}^2$$

$$0{,}0020 = \varepsilon_{xx} \cdot \frac{1}{2} + 2\varepsilon_{xy} \cdot \frac{1}{2} - 0{,}0008 \cdot \frac{1}{2}$$
$$0{,}0012 = \varepsilon_{xx} \cdot \frac{3}{4} - 2\varepsilon_{xy} \cdot \frac{\sqrt{3}}{4} - 0{,}0008 \cdot \frac{1}{4}$$

$$\left.\begin{aligned} \varepsilon_{xx} + 2\varepsilon_{xy} &= 0{,}0048 \\ 3\varepsilon_{xx} - 2\sqrt{3}\,\varepsilon_{xy} &= 0{,}0056 \end{aligned}\right\} \quad \boxed{\begin{aligned} \varepsilon_{xx} &= 0{,}00294 \\ \varepsilon_{xy} &= 0{,}00093 \end{aligned}}$$

Kontrola meritve :

$$\varepsilon_{xx} = \frac{\Delta l_x}{l_x} = \frac{0{,}016}{5{,}44} = 0{,}00294 \rightarrow \left\{ \begin{array}{l} \text{meritev} \\ \text{je OK} \end{array} \right.$$

$$\sigma_{zz} = 2\mu\,\varepsilon_{zz} + \lambda(\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}) = 0$$

$$\varepsilon_{zz} = -\frac{\lambda}{2\mu+\lambda}(\varepsilon_{xx} + \varepsilon_{yy}) \rightarrow \boxed{\varepsilon_{zz} = -0{,}0007}$$

$$I_1^{\varepsilon} = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz} = 0{,}00294 - 0{,}0008 - 0{,}0007$$

$$\boxed{I_1^{\varepsilon} = 0{,}00144}$$

$$\sigma_{xx} = 2\mu\,\varepsilon_{xx} + \lambda I_1^{\varepsilon} \longrightarrow \sigma_{xx} = 58{,}56\ kN/cm^2$$

$$\sigma_{yy} = 2\mu\,\varepsilon_{yy} + \lambda I_1^{\varepsilon} \longrightarrow \sigma_{yy} = -1{,}28\ kN/cm^2$$

$$\sigma_{xy} = 2\mu\,\varepsilon_{xy} \longrightarrow \sigma_{xy} = 14{,}88\ kN/cm^2$$

$$\sigma_{11,22} = \frac{\sigma_{xx}+\sigma_{yy}}{2} \pm \sqrt{\left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)^2 + \sigma_{xy}^2}$$

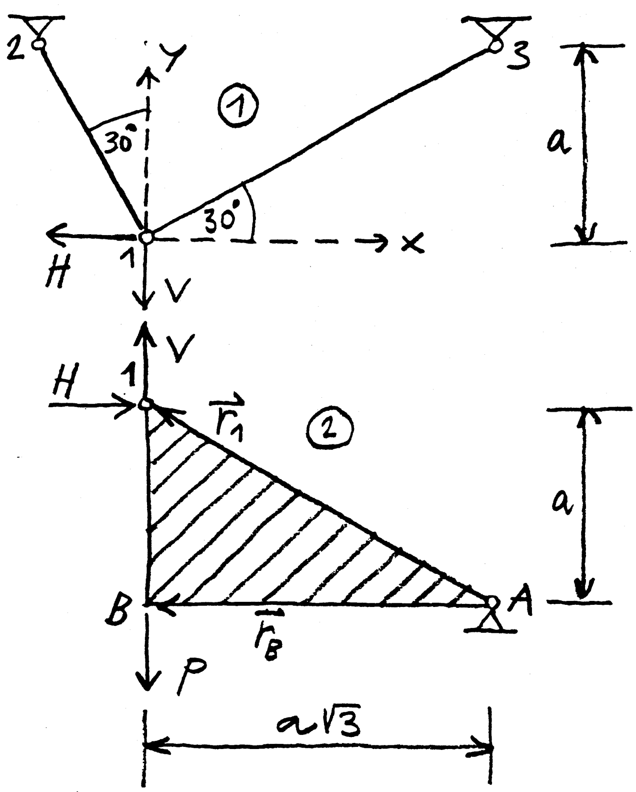
$$\sigma_{11,22} = \frac{58{,}56-1{,}28}{2} \pm \sqrt{\left(\frac{58{,}56+1{,}28}{2}\right)^2 + 14{,}88^2}$$

$\sigma_{11} = 62{,}06\ kN/cm^2$	$\sigma_{22} = -4{,}78\ kN/cm^2$	$\sigma_{33} = 0$

$$tg\,2\alpha_\sigma = \frac{2\cdot 14{,}88}{58{,}56+1{,}28} = 0{,}497 \longrightarrow 2\alpha_\sigma = 26{,}44^\circ$$

$$\boxed{\alpha_\sigma = 13{,}22^\circ}$$

Ad 2.)



1

$$l_{12} = \frac{2a}{\sqrt{3}}$$

$$l_{13} = 2a$$

$$k_{12} = \frac{EA_p}{l_{12}} = \frac{EA_p\sqrt{3}}{2a}$$

$$k_{12} = 182\ kN/cm$$

$$k_{13} = \frac{EA_p}{l_{13}} = \frac{EA_p}{2a}$$

$$k_{13} = 105\ kN/cm$$

$$[K_{12}] = 182 \quad \begin{array}{c|cc} & -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ \hline -\frac{1}{2} & \frac{1}{4} & -\frac{\sqrt{3}}{4} \\ \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{4} & \frac{3}{4} \end{array} \longrightarrow [K_{12}] = \begin{bmatrix} 45{,}47 & -78{,}75 \\ -78{,}75 & 136{,}40 \end{bmatrix}$$

$$[K_{13}] = 105 \quad \begin{array}{c|cc} & \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \hline \frac{\sqrt{3}}{2} & \frac{3}{4} & \frac{\sqrt{3}}{4} \\ \frac{1}{2} & \frac{\sqrt{3}}{4} & \frac{1}{4} \end{array} \longrightarrow [K_{13}] = \begin{bmatrix} 78{,}75 & 45{,}47 \\ 45{,}47 & 26{,}25 \end{bmatrix}$$

$$[K_{11}] = -\left([K_{12}] + [K_{13}]\right) \longrightarrow [K_{11}] = \begin{bmatrix} -124{,}22 & 33{,}28 \\ 33{,}28 & -166{,}65 \end{bmatrix}$$

$$[K_{11}] \begin{Bmatrix} u_1 \\ v_1 \end{Bmatrix} + \begin{Bmatrix} -H \\ -V \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$\begin{bmatrix} -124{,}22 & 33{,}28 \\ 33{,}28 & -166{,}65 \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \end{Bmatrix} = \begin{Bmatrix} H \\ V \end{Bmatrix}$$

$$\boxed{\begin{aligned} u_1^{①} &= 10^{-3}\left(-8{,}518\,H + 1{,}743\,V\right) \\ v_1^{①} &= 10^{-3}\left(1{,}743\,H - 6{,}505\,V\right) \end{aligned}}$$

② $\sum M^A = 0 \rightarrow Pa\sqrt{3} - Ha - Va\sqrt{3} = 0$

$$\boxed{V\sqrt{3} + H = P\sqrt{3}}$$

$$\vec{u}_A = \vec{0}\,, \quad \vec{\omega}_A = \varphi \vec{e}_z\,, \quad \vec{r}_1 = -a\sqrt{3}\,\vec{e}_x + a\,\vec{e}_y$$

$$\vec{u}_1 = \vec{u}_A + \vec{\omega}_A \times \vec{r}_1 = \vec{0} + \varphi \vec{e}_z \times \left(-a\sqrt{3}\,\vec{e}_x + a\,\vec{e}_y\right)$$

$$\boxed{\vec{u}_1 = -\varphi a\,\vec{e}_x - \varphi a\sqrt{3}\,\vec{e}_y} \qquad \boxed{\begin{aligned} u_1^{②} &= -\varphi a \\ v_1^{②} &= -\varphi a\sqrt{3} \end{aligned}}$$

$$\boxed{\begin{matrix} u_1^{①} = u_1^{②} \\ v_1^{①} = v_1^{②} \end{matrix}} \begin{matrix} \to -8{,}518\,H - 1{,}743\,V = -\varphi a \cdot 10^{-3} \quad |\cdot\sqrt{3} \\ \to -1{,}743\,H - 6{,}505\,V = -\varphi a\sqrt{3}\cdot 10^{-3} \end{matrix} \Bigg\} \ominus$$

$$H(8{,}518\cdot\sqrt{3} - 1{,}743) = V(6{,}505 - 1{,}743\cdot\sqrt{3})$$

$$\boxed{H = 0{,}268\,V} \longrightarrow \quad V\cdot\sqrt{3} + 0{,}268\,V = P\sqrt{3}$$

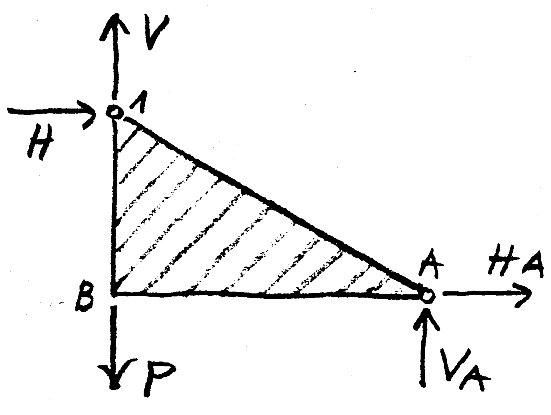
$$\boxed{\begin{matrix} V = 0{,}866\,P = 34{,}641\ kN \\ H = 0{,}232\,P = 9{,}282\ kN \end{matrix}}$$

$$u_1 = -10^{-3}(8{,}518\cdot 9{,}282 + 1{,}743\cdot 34{,}641) \to \boxed{u_1 = -0{,}139\ cm}$$
$$v_1 = -10^{-3}(1{,}743\cdot 9{,}282 + 6{,}505\cdot 34{,}641) \to \boxed{v_1 = -0{,}242\ cm}$$

$$\vec{u}_B = \vec{u}_A + \vec{\omega}_A \times \vec{r}_B = \varphi\vec{e}_z \times (-a\sqrt{3}\,\vec{e}_x) = -a\varphi\sqrt{3}\,\vec{e}_y$$

$$\boxed{u_B = 0 \qquad v_B = v_1 = -0{,}242\ cm}$$

$$N_{12} = 182\left(-0{,}139\cdot\frac{1}{2} + 0{,}242\cdot\frac{\sqrt{3}}{2}\right) \to \boxed{N_{12} = 25{,}359\ kN}$$
$$N_{13} = 105\left(0{,}139\cdot\frac{\sqrt{3}}{2} + 0{,}242\cdot\frac{1}{2}\right) \to \boxed{N_{13} = 25{,}359\ kN}$$



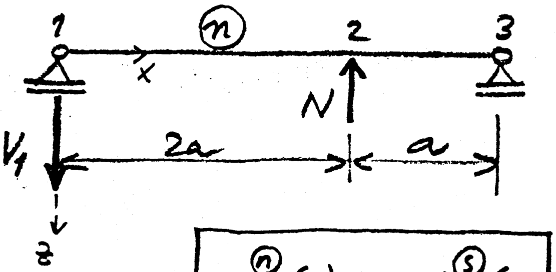
$$V_A = P - V$$

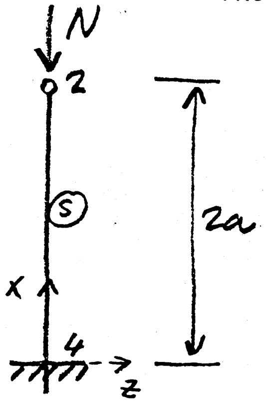
$$\boxed{V_A = 5{,}359\ kN}$$

$$H_A = -H$$

$$\boxed{H_A = -9{,}282\ kN}$$

Ad 3.)





$$\boxed{u_z^{(n)}(2) = -u_x^{(s)}(2)}$$

(n): $V_1 = \frac{N}{3}$

$$M_y = -\frac{N}{3}x + N\langle x-2a\rangle^1 = -EI_y\, w''$$

$$EI_y\, w'' = \frac{N}{3}\left(x - 3\langle x-2a\rangle^1\right)$$

$$EI_y\, w' = \frac{N}{6}\left(x^2 - 3\langle x-2a\rangle^2\right) + C_1$$

$$EI_y\, w = \frac{N}{18}\left(x^3 - 3\langle x-2a\rangle^3\right) + C_1 x + C_2$$

$$x=0 \;\ldots\; w=0 \longrightarrow C_2 = 0$$

$$x=3a \;\ldots\; w=0 \longrightarrow \frac{Na^3}{18}(27-3) + 3a\,C_1 = 0$$

$$C_1 = -\frac{4Na^2}{9}$$

$$EI_y\, w(2) = \frac{N}{18}\cdot 8a^3 - \frac{4Na^2}{9}\cdot 2a$$

$$\boxed{w^{(n)}(2) = -N\,\frac{4a^3}{9EI_y}}$$

(s): $$\boxed{u_x^{(s)}(2) = -2a\left(-\frac{N}{EA_x} + \alpha_T\Delta T\right)}$$

$$-N\,\frac{4a^3}{9EI_y} = N\,\frac{2a}{EA_x} - 2a\,\alpha_T\Delta T$$

$$N\left(\frac{4a^3}{9EI_y} + \frac{2a}{EA_x}\right) = 2a\,\alpha_T\Delta T \rightarrow \boxed{N = \frac{18\,\alpha_T\,\Delta T\,E A_x I_y}{4a^2 A_x + 18\,I_y}}$$

$$\boxed{N = 3{,}25\ N}$$