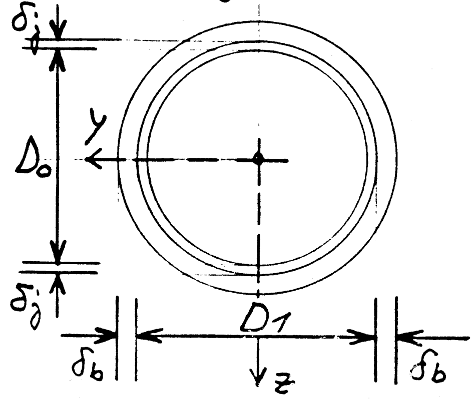
MTT IZPIT 30. 6. 1993

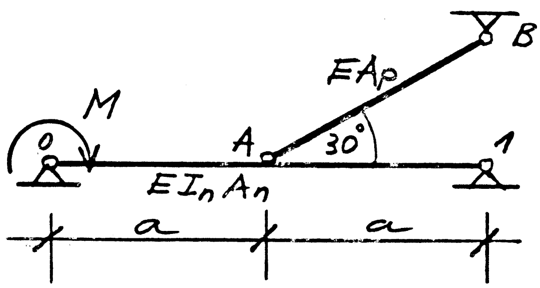
1. Podvodni cevovod je sestavljen iz jeklene notranje in varovalne bakrene zunanje cevi, ki se med seboj tesno, vendar brez napetosti prilegata. Določi normalne napetosti v tangencialni smeri v obeh ceveh med obratovanjem cevovoda v globini 50 m pri notranjem pritisku $p = 0{,}15\ kN/cm^2$



$D_0 = 50\ cm$, $D_1 = 52\ cm$
$\delta_j = 1\ cm$, $\delta_b = 1{,}5\ cm$
$E_j = 2 E_b$
$\nu_j = 0{,}3$ $\nu_b = 0{,}34$

Vzdolžne napetosti v ceveh lahko zanemariš.

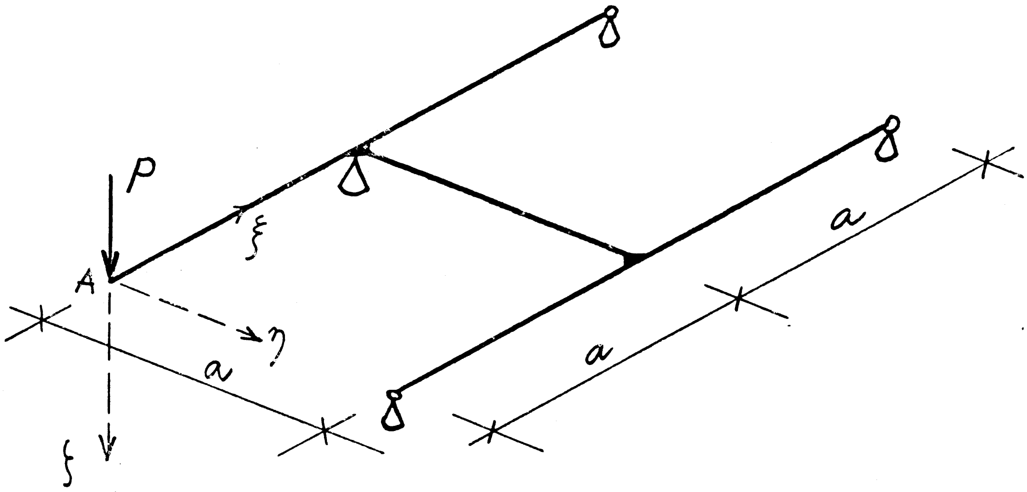
2. Določi potrebni prečni prerez vrvi $\overline{AB}$, če je dopustni navpični pomik točke A 1 cm! Določi in skiciraj notranje sile v tem primeru!



$E = 2 \cdot 10^5\ MPa$
$A_n = 40\ cm^2$
$I_n = 4246\ cm^4$
$a = 5\ m$
$M = 80\ kNm$

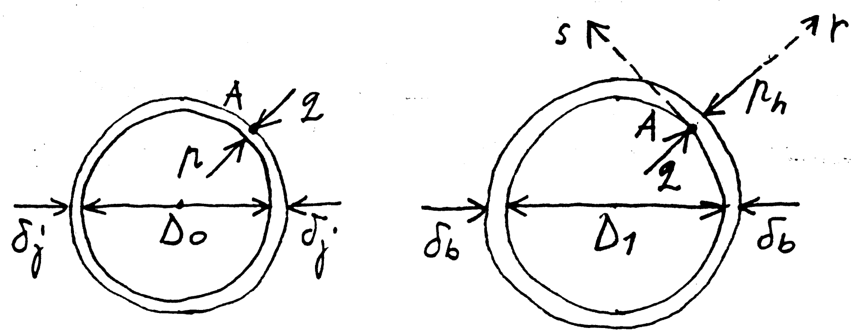
3. Določi poves točke A ter skiciraj notranje sile!

$EI_y = GI_x$



Ad 1.) $\gamma_v = 10\ kN/m^3$

$p_h = \gamma \cdot h = 10 \cdot 50 = 500\ kN/m^2 = 0{,}05\ kN/cm^2$



$\boxed{\sigma_{xx} = 0}$

v točki A :

$$\sigma_{ss}^{j} = (p-q)\frac{D_0}{2\delta_j} \qquad \sigma_{rr}^{j} = -q$$

$$\sigma_{ss}^{b} = (q-p_h)\frac{D_1}{2\delta_b} \qquad \sigma_{rr}^{b} = -q$$

$$\varepsilon_{ss}^{j} = \frac{1}{E_j}\left(\sigma_{ss}^{j} - \nu_j\,\sigma_{rr}^{j}\right) = \frac{1}{E_j}\left[(p-q)\frac{D_0}{2\delta_j} + \nu_j\, q\right]$$

$$\varepsilon_{ss}^{b} = \frac{1}{E_b}\left(\sigma_{ss}^{b} - \nu_b\,\sigma_{rr}^{b}\right) = \frac{1}{E_b}\left[(q-p_h)\frac{D_1}{2\delta_b} + \nu_b\, q\right]$$

$$\boxed{\varepsilon_{ss}^{b} = \varepsilon_{ss}^{j}}$$

$$(p-q)\frac{D_0}{2\delta_j} + \nu_j\, q = \frac{E_j}{E_b}\left[(q-p_h)\frac{D_1}{2\delta_b} + \nu_b\, q\right]$$

$$q\left[\frac{D_1}{\delta_b} + \frac{D_0}{2\delta_j} + 2\nu_b - \nu_j\right] = \frac{p D_0}{2\delta_j} + \frac{p_h D_1}{\delta_b}$$

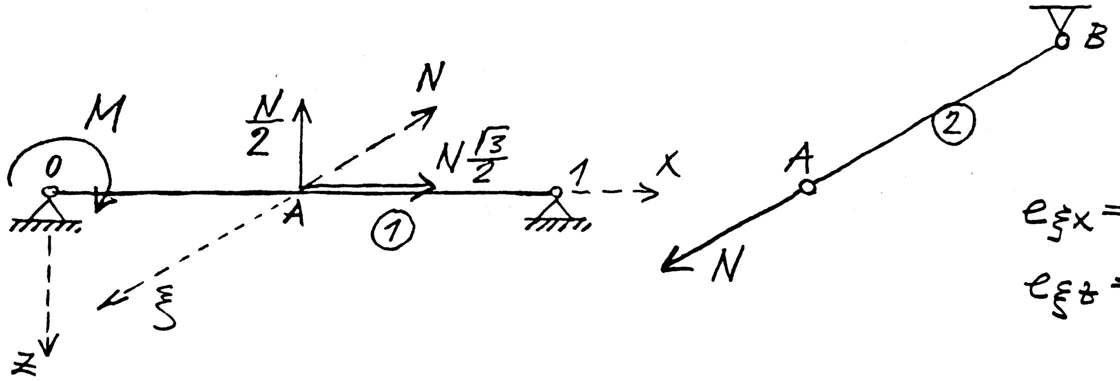
$85{,}047\, q = 5{,}483 \longrightarrow \boxed{q = 0{,}064\ kN/cm^2}$

$\sigma_{ss}^{j} = (0{,}15 - 0{,}064)\cdot\frac{50}{2\cdot 1} \longrightarrow \boxed{\sigma_{ss}^{j} = 2{,}138\ kN/cm^2}$

$\sigma_{ss}^{b} = (0{,}064 - 0{,}05)\cdot\frac{52}{2\cdot 1{,}5} \longrightarrow \boxed{\sigma_{ss}^{b} = 0{,}251\ kN/cm^2}$

Ad 2.)

$$l_{AB} = \frac{2a}{\sqrt{3}}$$



$$e_{\xi x} = -\frac{\sqrt{3}}{2}$$

$$e_{\xi z} = \frac{1}{2}$$

① $u_x^{(1)}(A) = \frac{1}{2} N \frac{\sqrt{3}}{2} \cdot \frac{a}{EA_n}$

$u_z^{(1)}(A) = M\frac{a^2}{4EI_y} - \frac{N}{2} \cdot \frac{a^3}{6EI_y}$

② $u_\xi^{(2)}(A) = N\frac{2a}{\sqrt{3}EA_p}$

$$u_\xi^{(2)} = u_x^{(1)} e_{\xi x} + u_z^{(1)} e_{\xi z}$$

$$N\frac{2a}{\sqrt{3}EA_p} = N\frac{a\sqrt{3}}{4EA_n} \cdot \left(-\frac{\sqrt{3}}{2}\right) + \left(M\frac{a^2}{4EI_y} - N\frac{a^3}{12EI_y}\right) \cdot \frac{1}{2}$$

$$\boxed{N\left(\frac{2}{\sqrt{3}A_p} + \frac{3}{8A_n} + \frac{a^2}{24I_y}\right) = M\frac{a}{8I_y}}$$

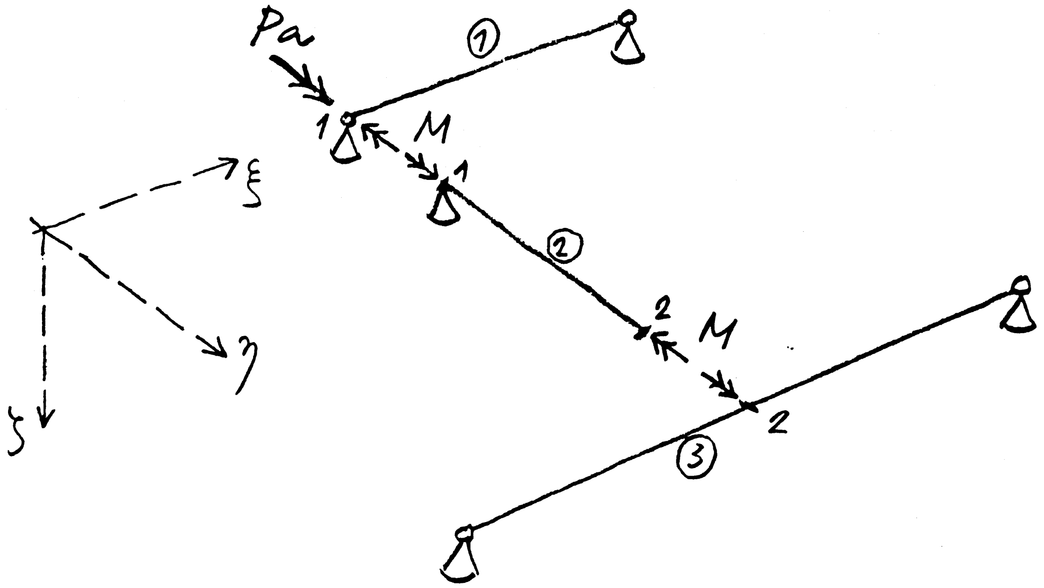
$$u_z^{(1)}(A) = M\frac{a^2}{4EI_y} - N\frac{a^3}{12EI_y} = 1 \text{ cm}$$

$$N = M\frac{3}{a} - 1 \cdot \frac{12EI_y}{a^3} \longrightarrow \boxed{N = 39{,}85 \text{ kN}}$$

$$39{,}85\left(\frac{1{,}155}{A_p} + 0{,}009 + 2{,}453\right) = 117{,}758$$

$$\boxed{A_p = 2{,}34 \text{ cm}^2}$$

Ad 3.)



① $\omega_\eta^{①}(1) = (Pa - M)\dfrac{a}{3EI_y}$

② $\omega_\eta^{②}(1) = \omega_\eta^{①}(1) = (Pa - M)\dfrac{a}{3EI_y}$

$\omega_\eta^{②}(2) = \omega_y^{①}(1) - M\dfrac{a}{GI_x} = (Pa - M)\dfrac{a}{3GI_x} - M\dfrac{a}{GI_x}$

$\omega_\eta^{②}(2) = P\dfrac{a^2}{3GI_x} - M\dfrac{4a}{3GI_x}$

③ $\omega_\eta^{③}(2) = M\dfrac{a}{6EI_y} = M\dfrac{a}{6GI_x}$

$\omega_\eta^{③}(2) = \omega_\eta^{②}(2) \longrightarrow M\dfrac{a}{6GI_x} = P\dfrac{a^2}{3GI_x} - M\dfrac{4a}{3GI_x}$

$$\boxed{M = P \cdot \frac{2a}{9}}$$

$\omega_\eta^{①}(1) = P\dfrac{a^2}{3EI_y} - P\dfrac{2a^2}{27EI_y} = P\dfrac{7a^2}{27EI_y}$

$w_A = P\dfrac{a^3}{3EI_y} + P\dfrac{7a^3}{27EI_y}$

$$\boxed{w_A = P\frac{16a^3}{27EI_y}}$$