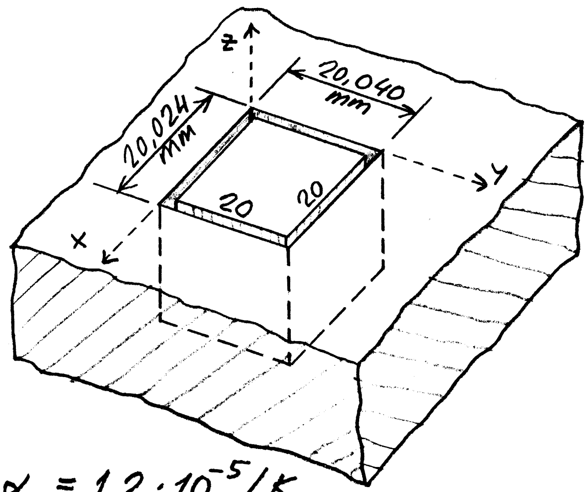
MTT IZPIT 14. 2. 1990

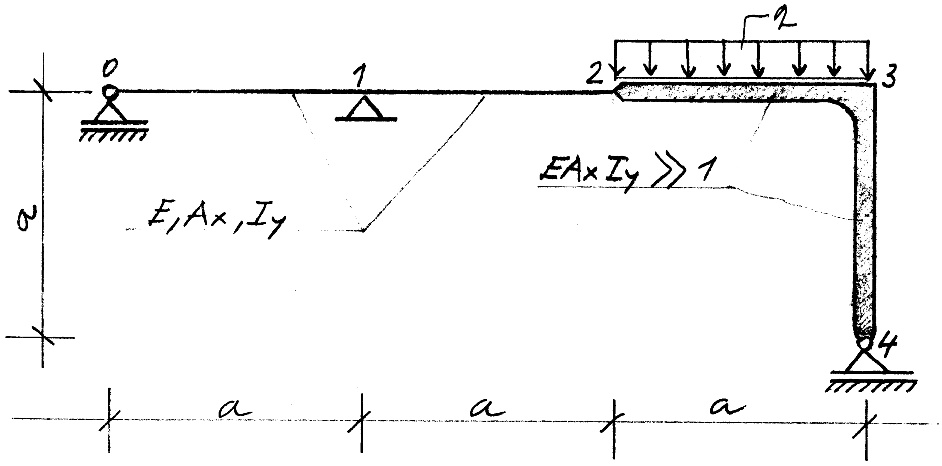
1. Jekleno kocko s stranico dolžine 20 mm centrično vstavimo v absolutno nepodajni utor dimenzij 20,024 x x 20,040 x 20 in jo začnemo segrevati. Določi:

a) Spremembo temperature, pri kateri se kocka prvič dotakne stene utora.
b) Spremembo temperature, pri kateri kocka v celoti zapolni utor.
c) Napetosti v kocki, če jo segrejemo za 160 K.



$E = 20\,000\ kN/cm^2, \quad \nu = 0{,}3, \quad \alpha_T = 1{,}2 \cdot 10^{-5}/K$

2. Določi in skiciraj notranje sile!



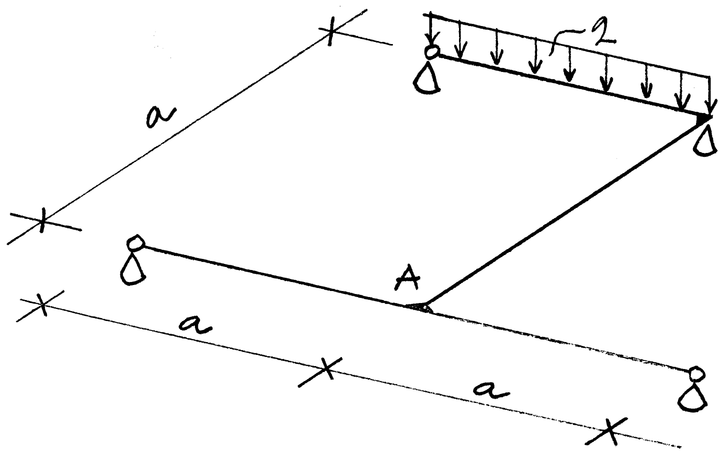
$E = 20\,000\ kN/cm^2$

$I_y = 6000\ cm^4$

$A_x = 160\ cm^2$

$q = 40\ kN/m'$

3. Določi poves točke A ter skiciraj notranje sile!



$EI_y = GI_x$

Ad 1

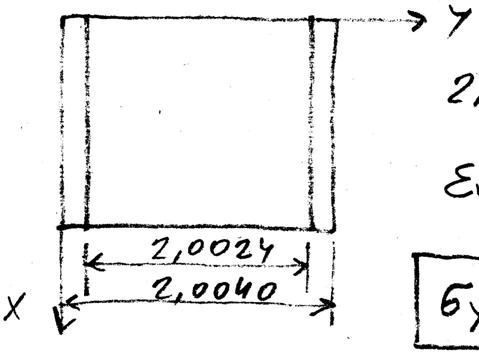
Enote: kN, cm, K

a) $2 \cdot (1 + \varepsilon_{xx}^{a}) = 2{,}0024$

$$\varepsilon_{xx}^{a} = \frac{2{,}0024 - 2}{2} = 0{,}0012$$

$$\varepsilon_{xx}^{a} = \alpha_T \Delta T_a \rightarrow \Delta T_a = \frac{\varepsilon_{xx}^{a}}{\alpha_T} \rightarrow \boxed{\Delta T_a = 100\ K}$$

b)



$$2{,}0024 \times (1 + \varepsilon_{yy}^{b}) = 2{,}0040$$

$$\varepsilon_{yy}^{b} = \frac{2{,}0040 - 2{,}0024}{2{,}0024} = 0{,}0008$$

$$\boxed{\sigma_{yy} = \sigma_{zz} = 0 \quad , \quad \varepsilon_{xx} = 0}$$

$$\varepsilon_{xx}^{b} = \frac{1}{E}\left[\sigma_{xx} - \nu(\sigma_{yy} + \sigma_{zz})\right] + \alpha_T \Delta T_b^{*} = 0$$

$$\varepsilon_{yy}^{b} = \frac{1}{E}\left[\sigma_{yy} - \nu(\sigma_{xx} + \sigma_{zz})\right] + \alpha_T \Delta T_b^{*} = 0{,}0008$$

$$-\frac{\nu \sigma_{xx}}{E} + \alpha_T \Delta T_b^{*} = \varepsilon_{yy}^{b}$$

$$\frac{\sigma_{xx}}{E} + \alpha_T \Delta T_b^{*} = 0 \longrightarrow \alpha_T \Delta T_b^{*} = -\frac{\sigma_{xx}}{E}$$

$$-(1+\nu)\frac{\sigma_{xx}}{E} = \varepsilon_{yy}^{b} \longrightarrow \sigma_{xx} = -\frac{20000 \cdot 0{,}0008}{1{,}3}$$

$$\boxed{\sigma_{xx}^{b} = -12{,}3\ kN/cm^2}$$

$$\Delta T_b^{*} = -\frac{\sigma_{xx}}{E \alpha_T} = \frac{12{,}3}{20000 \cdot 1{,}2 \cdot 10^{-5}}$$

$$\Delta T_b^{*} = 51{,}2\ K$$

$$\Delta T_b = \Delta T_a + \Delta T_b^{*} \rightarrow \boxed{\Delta T_b = 151{,}2\ K}$$

c) $\boxed{\sigma_{zz} = 0 \quad \varepsilon_{xx} = \varepsilon_{yy} = 0}$ $\quad \boxed{\Delta T_c^* = 8{\cdot}8\ K}$

$$\varepsilon_{xx}^c = \frac{1}{E}\left(\sigma_{xx}^c - \nu\sigma_{yy}^c\right) + \alpha_T\,\Delta T_c^* = 0$$

$$\varepsilon_{yy}^c = \frac{1}{E}\left(\sigma_{yy}^c - \nu\sigma_{xx}^c\right) + \alpha_T\,\Delta T_c^* = 0$$

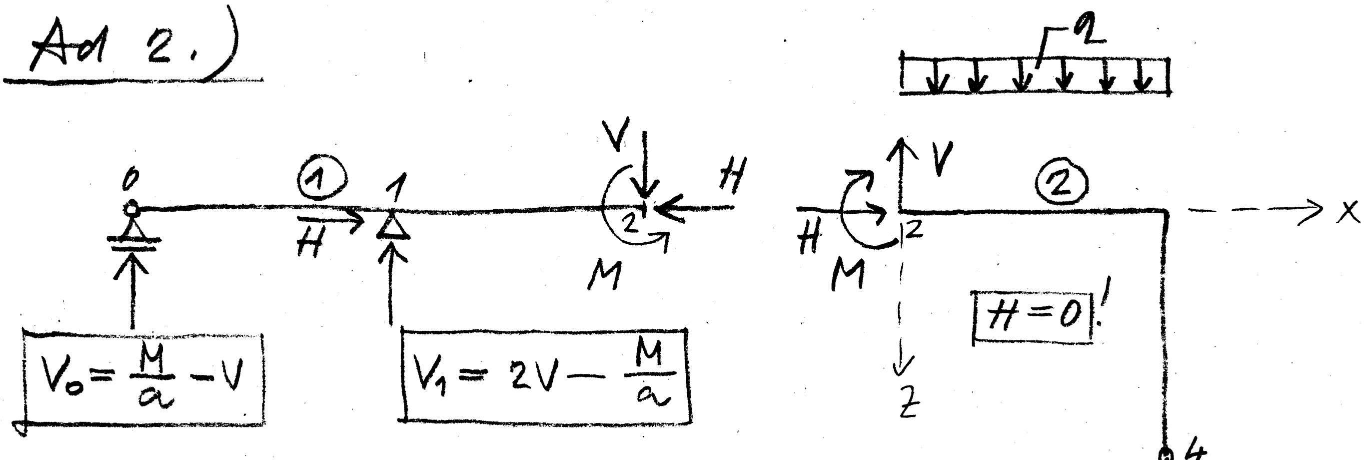
$$\boxed{\sigma_{xx}^c = \sigma_{yy}^c = -\frac{E\,\alpha_T\,\Delta T_c^*}{1-\nu}}$$

$$\sigma_{xx}^c = \sigma_{yy}^c = -\frac{20000\cdot 1{,}2\cdot 10^{-5}\cdot 8{\cdot}8}{0{,}7} = 3\ kN/cm^2$$

$$\boxed{\sigma_{xx}^c = \sigma_{yy}^c = -3{,}4\ kN/cm^2}$$

$$\sigma_{xx} = -12{,}3 - 3{,}4 \longrightarrow \boxed{\begin{aligned}\sigma_{xx} &= -15{,}3\ kN/cm^2\\ \sigma_{yy} &= -3{,}0\ kN/cm^2\\ \sigma_{zz} &= 0\end{aligned}}$$

Ad 2.)



$$M_y = V_0\,x + V_1\langle x-a\rangle =$$
$$= \left(\frac{M}{a} - V\right)x + \left(2V - \frac{M}{a}\right)\langle x-a\rangle$$

V_4

$$M_y = V\left(2\langle x-a\rangle - x\right) + \frac{M}{a}\left(x - \langle x-a\rangle\right) = -EI_y w''$$

$$EI_y\,w'' = V\left(x - 2\langle x-a\rangle\right) + \frac{M}{a}\left(\langle x-a\rangle - x\right)$$

$$EI_y w' = V\left(\frac{x^2}{2} - \langle x-a\rangle^2\right) + \frac{M}{2a}\left(\langle x-a\rangle^2 - x^2\right) + C_1$$

$$EI_y w = V\left(\frac{x^3}{6} - \frac{1}{3}\langle x-a\rangle^3\right) + \frac{M}{6a}\left(\langle x-a\rangle^3 - x^3\right) + C_1 x + C_2$$

$$x=0 \rightarrow w=0 \rightarrow C_2 = 0$$

$$x=a \rightarrow w=0 \rightarrow \frac{Va^3}{6} - \frac{Ma^2}{6} + C_1 a = 0$$

$$\boxed{C_1 = \frac{Ma}{6} - \frac{Va^2}{6}}$$

$x = 2a$:

$$EI_y w_2 = Va^3\left(\frac{8}{6} - \frac{1}{3} - \frac{1}{3}\right) + \frac{Ma^2}{6}(1 - 8 + 2)$$

$$\boxed{w_2^{(1)} = \frac{2Va^3}{3EI_y} - \frac{5Ma^2}{6EI_y}}$$

$$-EI_y \omega_2 = Va^2\left(\frac{4}{2} - 1 - \frac{1}{6}\right) + \frac{Ma}{2}\left(1 - 4 + \frac{1}{3}\right)$$

$$\boxed{\omega_2^{(1)} = -\frac{5Va^2}{6EI_y} + \frac{4Ma}{3EI_y}} \qquad \boxed{u_2^{(1)} = 0}$$

(2) $\Sigma M^4 = 0 \longrightarrow M + Va - \frac{qa^2}{2} = 0$

$$\boxed{M = \frac{qa^2}{2} - Va}$$

$\vec{u}_4^{(2)} = u_4^{(2)} \vec{e}_x$	$\vec{\omega}_4^{(2)} = \omega_4^{(2)} \vec{e}_y$	$\vec{r}_2 = -a\vec{e}_x - a\vec{e}_z$

$$\vec{u}_2 = u_4^{(2)} \vec{e}_x + \omega_4^{(2)} \vec{e}_y \times (-a\vec{e}_x - a\vec{e}_z)$$

$$\vec{u}_2 = u_4^{(2)} \vec{e}_x + a\omega_4^{(2)} \vec{e}_z - a\omega_4^{(2)} \vec{e}_x$$

$$\boxed{\vec{u}_2 = \left(u_4^{(2)} - a\omega_4^{(2)}\right)\vec{e}_x + a\omega_4^{(2)} \vec{e}_z} \qquad \boxed{\vec{\omega}_2 = \omega_4^{(2)} \vec{e}_y}$$

$$\omega_4^{(2)} = \omega_2^{(1)} = -\frac{5Va^2}{6EI_y} + \frac{4a}{3EI_y}\left(\frac{qa^2}{2} - Va\right)$$

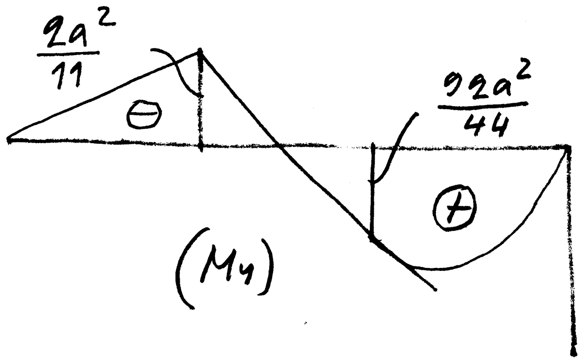
$$\boxed{\omega_4^{(2)} = -\frac{13Va^2}{6EI_y} + \frac{2qa^3}{3EI_y}}$$

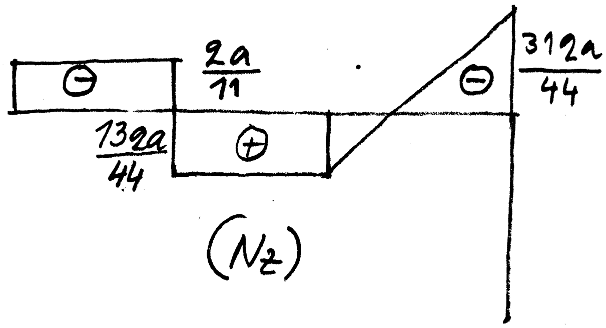
$$w_2^{(1)} = \frac{2Va^3}{3EI_y} - \frac{5a^2}{6EI_y}\left(\frac{qa^2}{2} - Va\right) = a\left(-\frac{13Va^2}{6EI_y} + \frac{2qa^3}{3EI_y}\right)$$

$$\frac{11Va^3}{3} = \frac{13qa^4}{12} \longrightarrow \boxed{V = \frac{13qa}{44}}$$

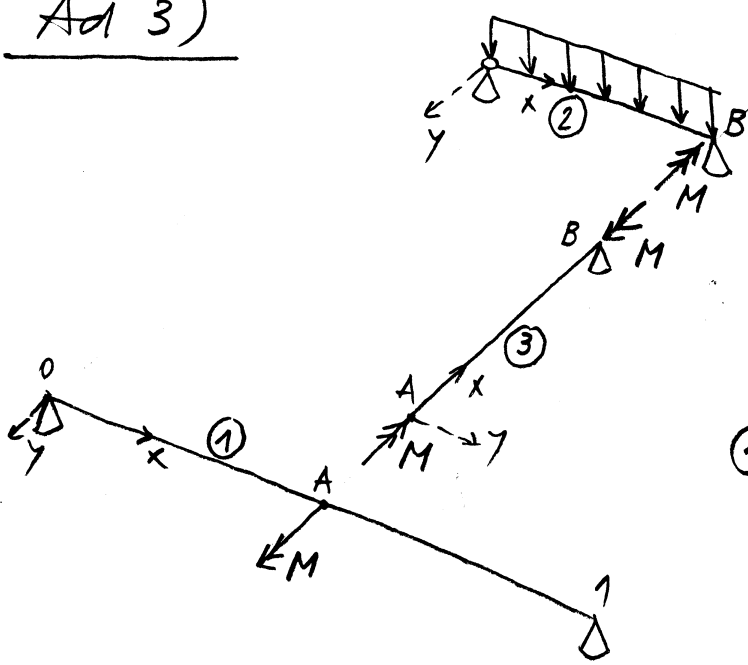
$$M = \frac{qa^2}{2} - \frac{13}{44}qa^2 \longrightarrow \boxed{M = \frac{9qa^2}{44}}$$

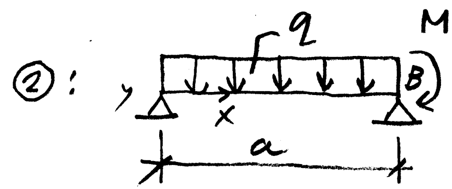
$V_0 = -\frac{qa}{11}$	$V_1 = \frac{17qa}{44}$	$V_4 = \frac{31qa}{44}$



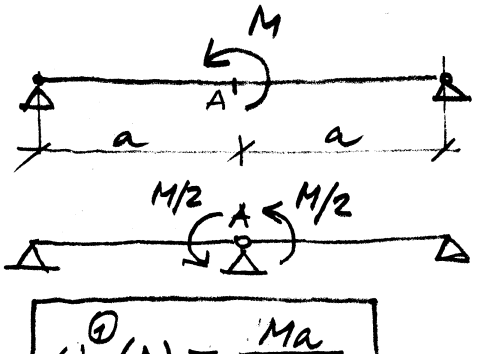


Ad 3)



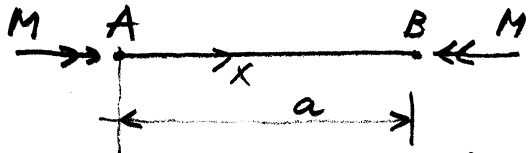


$$\boxed{\omega_y^{(2)}(B) = \frac{qa^3}{24EI_y} - \frac{Ma}{3EI_y}}$$



$$\boxed{\omega_y^{(1)}(A) = \frac{Ma}{6EI_y}}$$

3



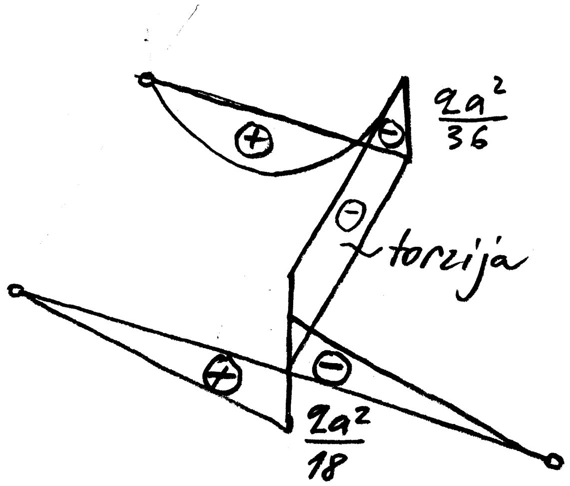
$$\boxed{M_x = -M}$$

$$\omega_x^{(3)}(B) = \omega_x^{(3)}(A) + a\alpha = \omega_x^{(3)}(A) - \frac{Ma}{GI_x}$$

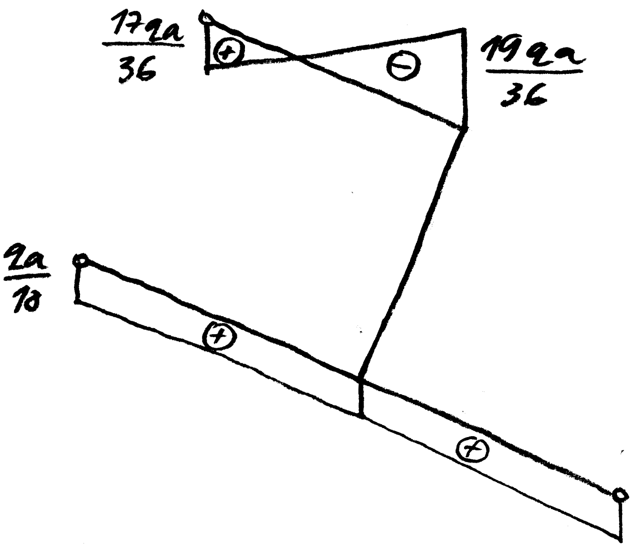
$$\boxed{\omega_x^{(3)}(A) = -\omega_y^{(1)}(A)} \qquad \boxed{\omega_x^{(3)}(B) = -\omega_y^{(2)}(B)}$$

$$-\frac{qa^3}{24EI_y} + \frac{Ma}{3EI_y} = -\frac{Ma}{6EI_y} - \frac{Ma}{GI_x}$$

$$EI_y = GI_x \longrightarrow \boxed{M = \frac{qa^2}{36}}$$



(M_x , M_y)



(N_t)