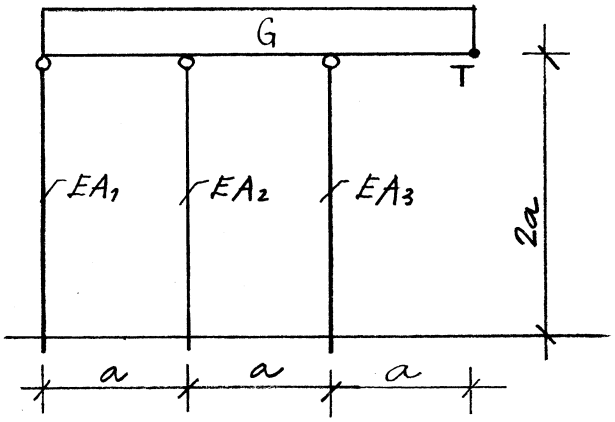
MTT IZPIT 23. 4. 1986

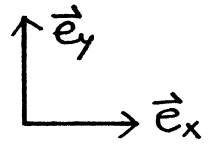
1.) Absolutno toga greda teže G je členkasto pritrjena na tri stebre. Določi vektor pomika točke T!



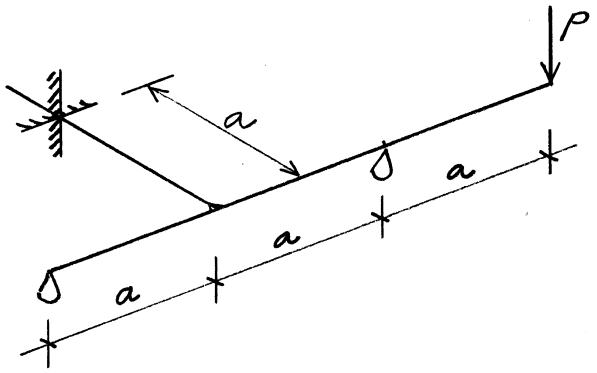
$A_1 = 3A_0$

$A_2 = 6A_0$

$A_3 = 2A_0$



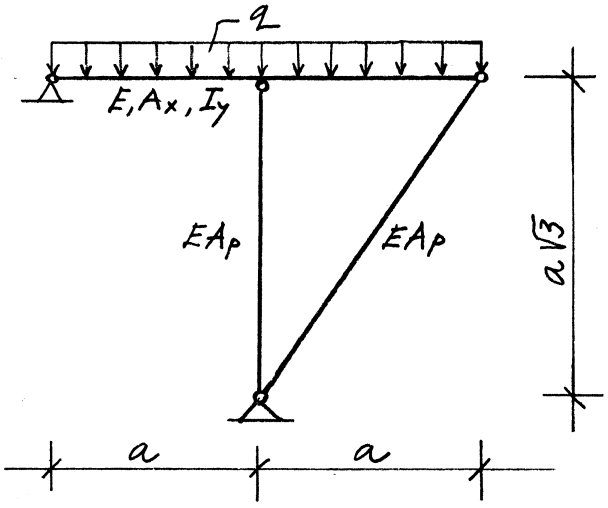
2.) Določi in skiciraj notranje sile!



Vsi elementi:

$EI_y = GI_x$

3.) Določi in skiciraj notranje sile!



$q = 40\ kN/m'$

$a = 3\ m$

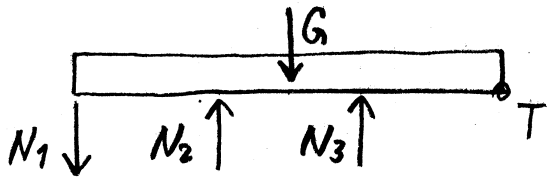
$E = 2 \cdot 10^5\ MPa$

$A_x = 0{,}02\ m^2$

$A_p = 0{,}002\ m^2$

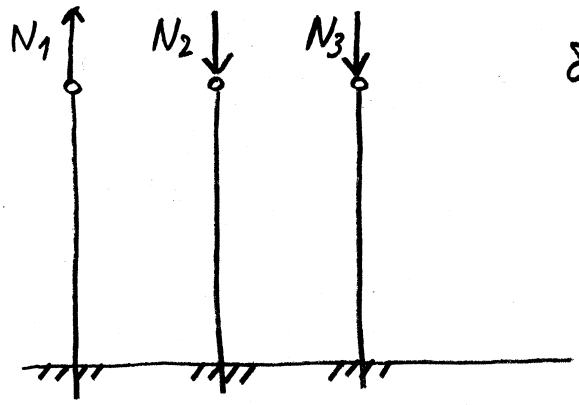
$I_y = 0{,}00008\ m^4$

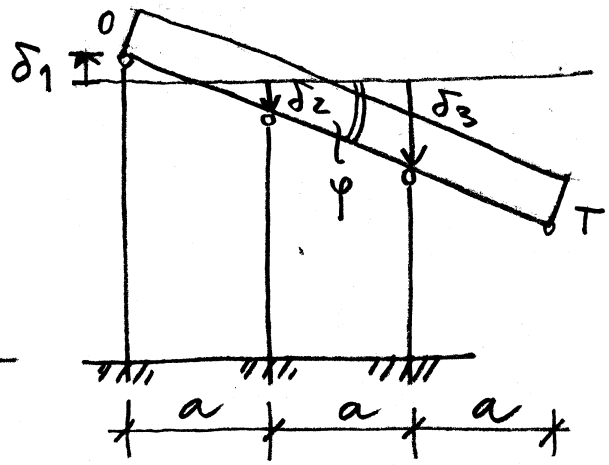
Ad 1)



$$-N_1 + N_2 + N_3 = G \quad \dots (1)$$

$$N_1 \cdot a + N_3 \cdot a = G \frac{a}{2} \quad \dots (2)$$





(2) $\boxed{N_3 = \frac{G}{2} - N_1}$

(1) $-N_1 + N_2 + \frac{G}{2} - N_1 = G \quad \dots \quad \boxed{N_2 = \frac{G}{2} + 2N_1}$

$\dots (3)$

$$\left.\begin{aligned} \delta_1 + \delta_2 &= a \cdot \varphi \\ \delta_1 + \delta_3 &= 2a\varphi \end{aligned}\right\} \quad 2\delta_1 + 2\delta_2 = \delta_1 + \delta_3$$

$$\boxed{\delta_1 + 2\delta_2 - \delta_3 = 0}$$

$\dots (4)$

(4) $N_1 \cdot \frac{2a}{3EA_0} + 2N_2 \cdot \frac{2a}{6EA_0} - N_3 \cdot \frac{2a}{2EA_0} = 0$

$4N_1 + 4N_2 - 6N_3 = 0$

$4N_1 + 4\left(\frac{G}{2} + 2N_1\right) - 6\left(\frac{G}{2} - N_1\right) = 0$

$14N_1 = G \rightarrow \boxed{N_1 = \frac{G}{14}}$

$\delta_1 = \frac{G}{14} \cdot \frac{2a}{3EA_0} \quad \dots \quad \boxed{\delta_1 = \frac{Ga}{21EA_0}}$

$$\delta_2 = \left(\frac{G}{2} + 2 \times \frac{G}{14}\right) \cdot \frac{2a}{6EA_0} \quad \ldots \quad \boxed{\delta_2 = \frac{4Ga}{21EA_0}}$$

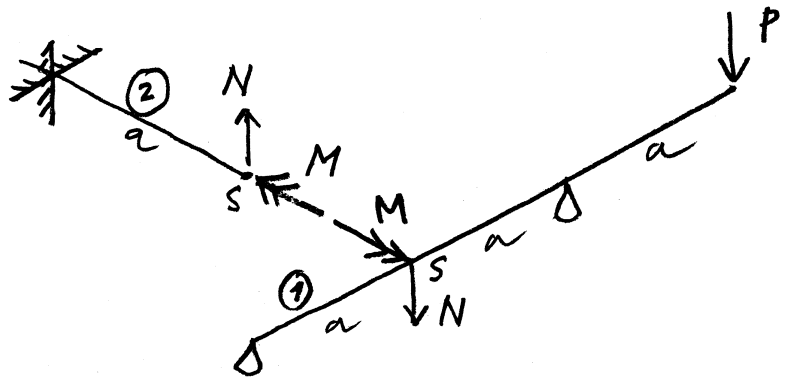
$$\varphi = \frac{1}{a}(\delta_1 + \delta_2) = \frac{G}{21EA_0}(1 + 4)$$

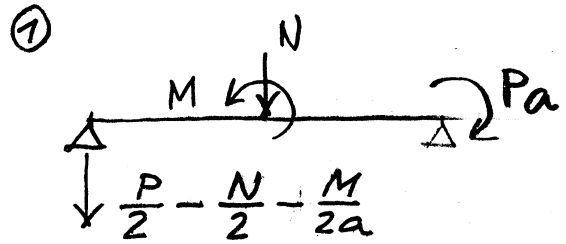
$$\boxed{\varphi = \frac{5G}{21EA_0}} \qquad \vec{u}_T = \vec{u}_0 + \vec{\omega}_0 \times \vec{r}_T$$

$$\vec{u}_0 = \delta_1 \vec{e}_y ; \qquad \vec{\omega}_0 = -\varphi \vec{e}_z ; \qquad \vec{r}_T = 3a \vec{e}_x$$

$$\vec{u}_0 = (\delta_1 - 3a\varphi)\vec{e}_y \rightarrow \boxed{\vec{u}_0 = -\frac{2Ga}{3EA_0}\vec{e}_y}$$

Ad 2)





$$M_y = -\left(\frac{P}{2} - \frac{N}{2} - \frac{M}{2a}\right)x - N\langle x-a\rangle - M\langle x-a\rangle^0 = -EI_y w'$$

$$EI_y w'' = \frac{P}{2}x - \frac{N}{2}(x - 2\langle x-a\rangle) - \frac{M}{2a}(x - 2a\langle x-a\rangle^0)$$

$$EI_y w' = P\frac{x^2}{4} - \frac{N}{2}\left(\frac{x^2}{2} - \langle x-a\rangle^2\right) - \frac{M}{2a}\left(\frac{x^2}{2} - 2a\langle x-a\rangle\right) + C_1$$

$$EI_y w = P\frac{x^3}{12} - \frac{N}{2}\left(\frac{x^3}{6} - \frac{1}{3}\langle x-a\rangle^3\right) -$$

$$- \frac{M}{2a}\left(\frac{x^3}{6} - a\langle x-a\rangle^2\right) + C_1 x + C_2$$

$x=0 \ldots w=0 \rightarrow C_2 = 0$

$x=2a \ldots w=0$

$$P\cdot\frac{8a^3}{12} - \frac{N}{2}\left(\frac{8a^3}{6} - \frac{a^3}{3}\right) -$$

$$- \frac{M}{2a}\left(\frac{8a^3}{6} - a^3\right) + 2a C_1 = 0$$

$$\boxed{C_1 = -\frac{Pa^2}{3} + \frac{Na^2}{4} + \frac{Ma}{12}}$$

$$\boxed{w_S^{①} = -\frac{Pa^3}{4EI_y} + \frac{Na^3}{6EI_y}}$$

$$\boxed{\omega_S^{①} = \frac{Pa^2}{12EI_y} + \frac{Ma}{6EI_y}}$$

② $\boxed{w_S^{②} = -\frac{Na^3}{3EI_y}}$ $\qquad$ $\boxed{\omega_S^{②} = -\frac{Ma}{GI_x}}$

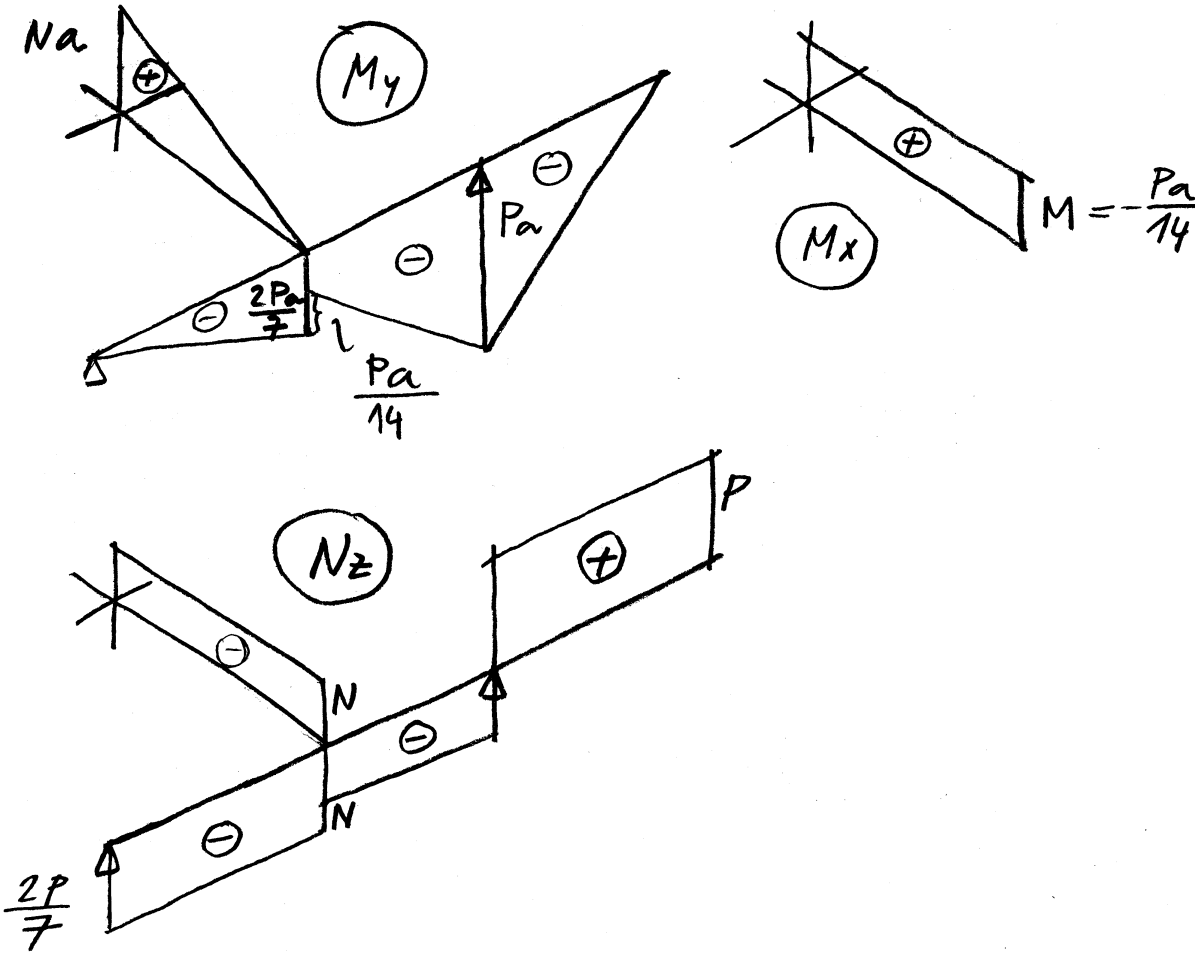
$$w_S^{①} = w_S^{②} \ldots \frac{a^3}{EI_y}\left(-\frac{P}{4} + \frac{N}{6}\right) = -\frac{Na^3}{3EI_y}$$

$$\boxed{N = \frac{P}{2}}$$

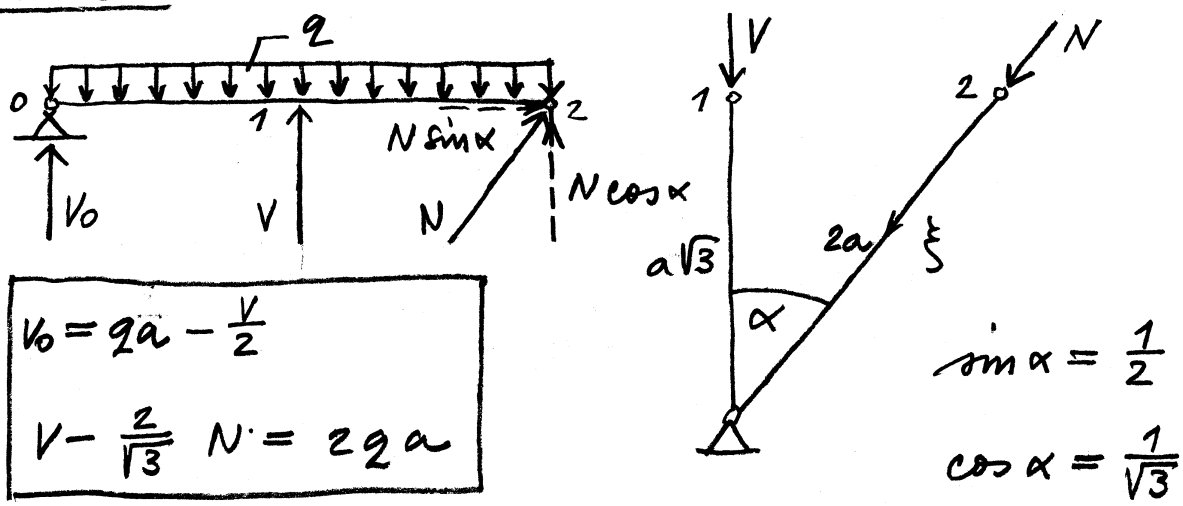
$$\omega_S^{①} = \omega_2^{②} \ldots \frac{a}{EI_y}\left(\frac{Pa}{12} + \frac{M}{6}\right) = -\frac{Ma}{EI_y}$$

$$\boxed{M = -\frac{Pa}{14}}$$

$$V_0 = \frac{P}{2} - \frac{P}{4} + \frac{P}{28} \rightarrow \boxed{V_0 = \frac{2P}{7}}$$



Ad 3)



$V_0 = qa - \frac{V}{2}$

$V - \frac{2}{\sqrt{3}} N = 2qa$

$\sin\alpha = \frac{1}{2}$

$\cos\alpha = \frac{1}{\sqrt{3}}$

$$M_y = \left(2a - \frac{V}{2}\right)x - \frac{qx^2}{2} + V\langle x-a\rangle = -EI_y w''$$

$$EI_y w'' = \frac{q}{2}(x^2 - 2ax) + \frac{V}{2}(x - 2\langle x-a\rangle)$$

$V_0 =$

$V =$

$N =$

$w_1 = 0,00188$ $\quad u_1 = 0,00002$

$w_2 = 0,00097$ $\quad u_2 = 0,00004$