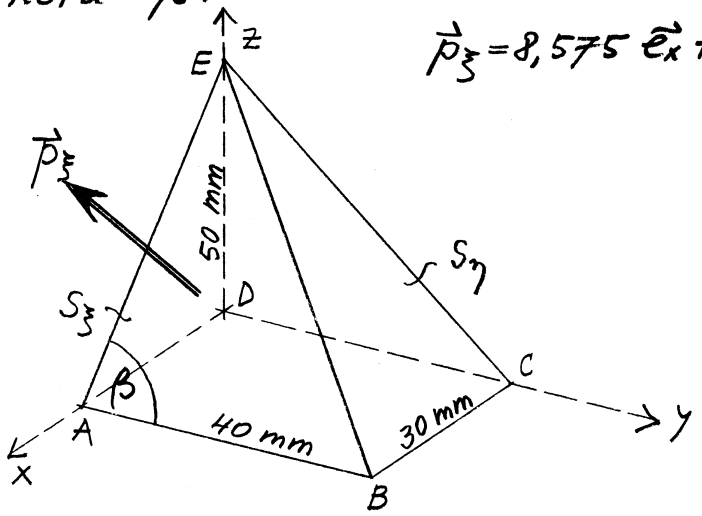
MTT IZPIT 10. 2. 1986

1.) Mejna ploskev S_η (BCE) prikazanega elementa telesa je neobtežena, na mejno ploskev S_ξ (ABE) pa deluje enakomerna zvezna obtežba $\vec{p}_\xi$. V ploskvi ADE ni normalnih napetosti. Določi spremembo pravega kota β!

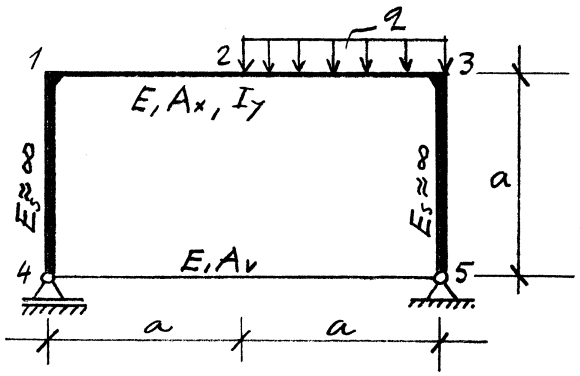


$\vec{p}_\xi = 8{,}575\,\vec{e}_x + 17{,}150\,\vec{e}_y - 21{,}437\,\vec{e}_z$

$E = 2 \cdot 10^5$ MPa

$\nu = 0{,}2$

2.) Določi in skiciraj notranje sile! ✶



$E = 2 \cdot 10^5$ MPa

$A_v = 0{,}004\ m^2$

$A_x = 0{,}040\ m^2$

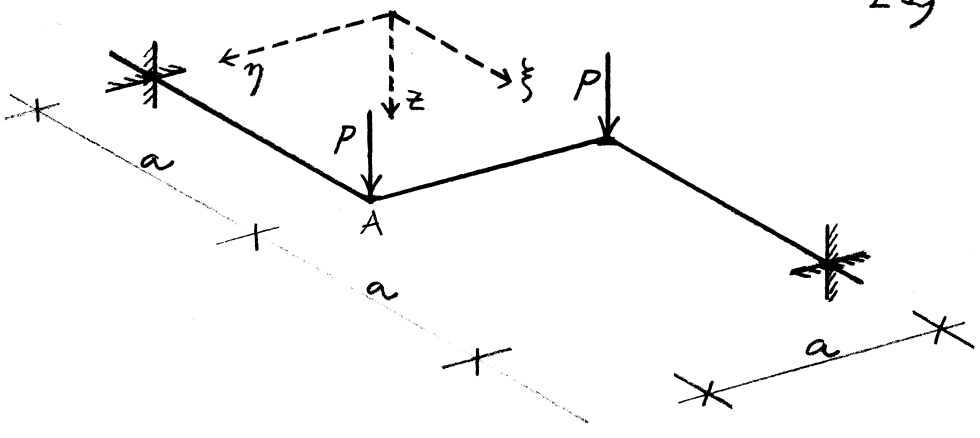
$I_y = 0{,}004\ m^4$

$a = 3m$

$q = 0{,}32$ MN/m'

3.) Določi pomik točke A!

$EI_y = GI_x$



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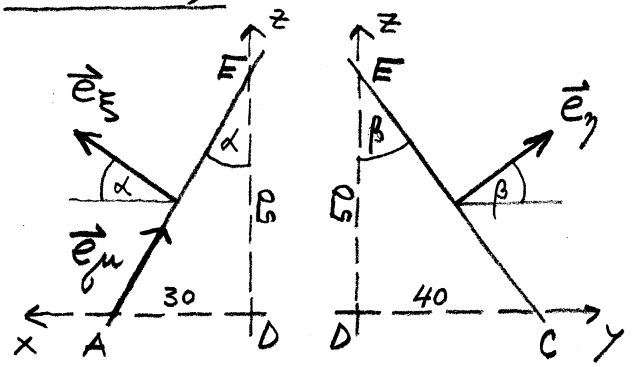
Ad 1.)



$\alpha = 30{,}96° \ldots \sin\alpha = 0{,}514$

$\cos\alpha = 0{,}857$

$\beta = 38{,}66° \ldots \sin\beta = 0{,}625$

$\cos\beta = 0{,}781$

$$\vec{e}_\xi = 0{,}857\,\vec{e}_x + 0{,}514\,\vec{e}_z$$
$$\vec{e}_\eta = 0{,}781\,\vec{e}_y + 0{,}625\,\vec{e}_z$$

$$\sigma_{yy} = 0$$

$$\begin{Bmatrix} p_{\eta x} \\ p_{\eta y} \\ p_{\eta z} \end{Bmatrix} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{xy} & 0 & \sigma_{yz} \\ \sigma_{xz} & \sigma_{yz} & \sigma_{zz} \end{bmatrix} \begin{Bmatrix} 0 \\ 0{,}781 \\ 0{,}625 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

$0{,}781\,\sigma_{xy} + 0{,}625\,\sigma_{xz} = 0 \rightarrow \sigma_{xz} = -1{,}25\,\sigma_{xy}$ ↓

$0{,}625\,\sigma_{yz} = 0 \rightarrow \boxed{\sigma_{yz} = 0}$

$0{,}781\,\sigma_{yz} + 0{,}625\,\sigma_{zz} = 0 \rightarrow \boxed{\sigma_{zz} = 0}$

$$\begin{Bmatrix} p_{\xi x} \\ p_{\xi y} \\ p_{\xi z} \end{Bmatrix} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{xy} & 0 & 0 \\ \sigma_{xz} & 0 & 0 \end{bmatrix} \begin{Bmatrix} 0{,}857 \\ 0 \\ 0{,}514 \end{Bmatrix} = \begin{Bmatrix} 8{,}575 \\ 17{,}150 \\ -21{,}437 \end{Bmatrix}$$

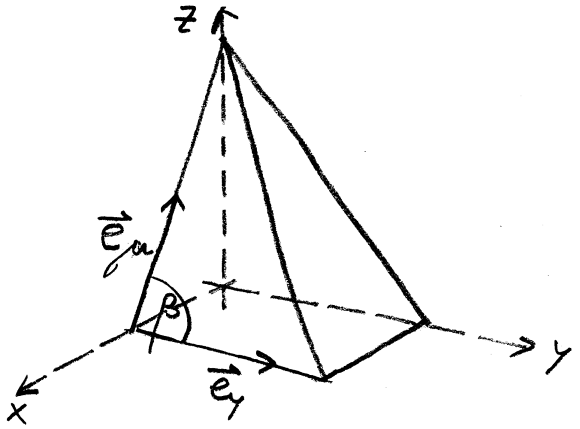
$0{,}857\,\sigma_{xx} + 0{,}514\,\sigma_{xz} = 8{,}575$

$0{,}857\,\sigma_{xy} = 17{,}150 \rightarrow \boxed{\sigma_{xy} = 20}$

$0{,}857\,\sigma_{xz} = -21{,}437 \rightarrow \boxed{\sigma_{xz} = -25}$

$\sigma_{xx} = \frac{1}{0{,}857}(8{,}575 - 0{,}514\,\sigma_{xz}) \rightarrow \boxed{\sigma_{xx} = 25}$

$$[\sigma_{ij}] = \begin{bmatrix} 25 & 20 & -25 \\ 20 & 0 & 0 \\ -25 & 0 & 0 \end{bmatrix}$$



$$\vec{e}_\mu = -0{,}514\,\vec{e}_x + 0{,}857\,\vec{e}_z$$

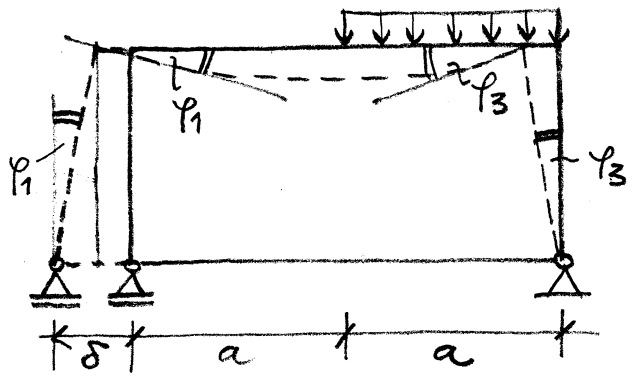
$$\sigma_{y\mu} = \sigma_{xx}\, e_{xy}\, e_{\mu x} + \sigma_{xy}(e_{yx}\, e_{\mu y} + e_{yy}\, e_{\mu x}) + \sigma_{xz}\, e_{yx}\, e_{\mu z}$$

$$\sigma_{y\mu} = 20 \times (-0{,}514) = -10{,}29$$

$$\varepsilon_{y\mu} = \frac{1+\nu}{E}\,\sigma_{y\mu} = -10{,}29 \times \frac{1{,}2}{2 \times 10^5} \rightarrow \boxed{\varepsilon_{y\mu} = -6{,}17 \times 10^{-5}}$$

$$\boxed{D_{y\mu} = 2\,\varepsilon_{y\mu} = -1{,}23 \times 10^{-4}}$$

Ad 2.)

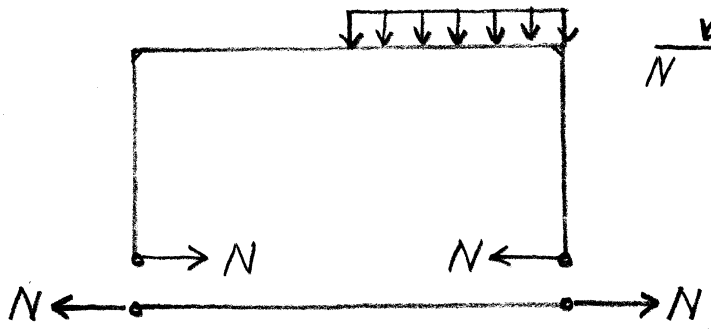


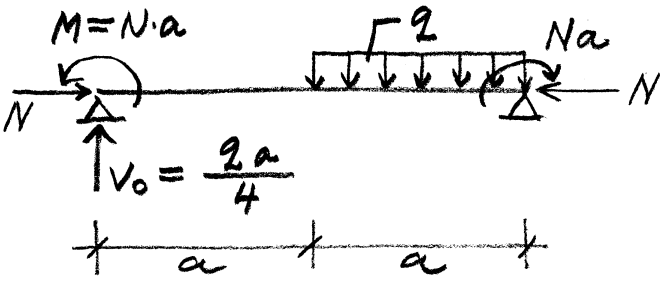
$$\varphi_1 = -\omega_y(1)$$

$$\varphi_3 = \omega_y(3)$$

$$\boxed{\delta = (\varphi_1 + \varphi_3)\cdot a - \frac{N}{EA_x}\cdot 2a}$$

$$\boxed{\delta = \frac{N}{EA_V}\cdot 2a}$$





$$M_y = \frac{qa}{4}x - Na - \frac{q}{2}\langle x-a\rangle^2 = \frac{q}{4}\left(ax - 2\langle x-a\rangle^2\right) - Na$$

$$EI_y\, w'' = Na - \frac{q}{4}\left(ax - 2\langle x-a\rangle^2\right)$$

$$EI_y\, w' = Nax - \frac{q}{4}\left(a\frac{x^2}{2} - \frac{2}{3}\langle x-a\rangle^3\right) + C_1$$

$$EI_y\, w = Na\frac{x^2}{2} - \frac{q}{4}\left(a\frac{x^3}{6} - \frac{1}{6}\langle x-a\rangle^4\right) + C_1 x + C_2$$

$$EI_y\, w = \frac{1}{2}Nax^2 - \frac{q}{24}\left(ax^3 - \langle x-a\rangle^4\right) + C_1 x + C_2$$

$$x=0 \ldots w=0 \rightarrow C_2 = 0$$

$$x=2a \ldots w=0 \rightarrow 2Na^3 - \frac{q}{24}\left(8a^4 - a^4\right) + 2C_1 a = 0$$

$$\boxed{C_1 = -Na^2 + \frac{7qa^3}{48}}$$

$$x=0 \ldots \boxed{w_y(1) = \frac{Na^2}{EI_y} - \frac{7qa^3}{48EI_y} = -\varphi_1}$$

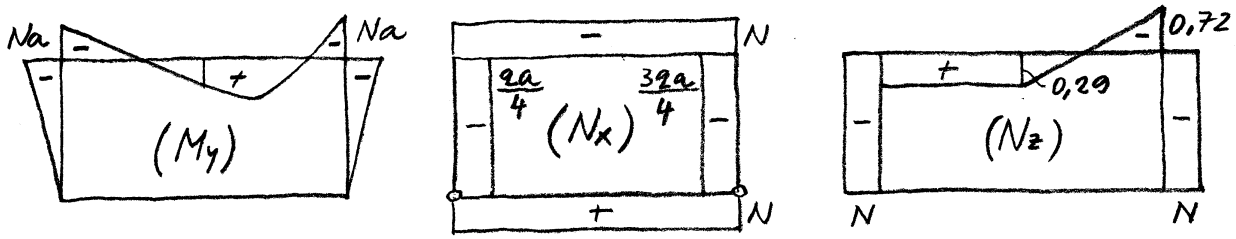
$$x=2a \ldots w_y(3) = \frac{1}{EI_y}\left[-2Na^2 + \frac{q}{4}\left(2a^3 - \frac{2a^3}{3}\right) + Na^2 - \frac{7qa^3}{48}\right]$$

$$\boxed{w_y(3) = -\frac{Na^2}{EI_y} + \frac{9qa^3}{48EI_y} = \varphi_3}$$

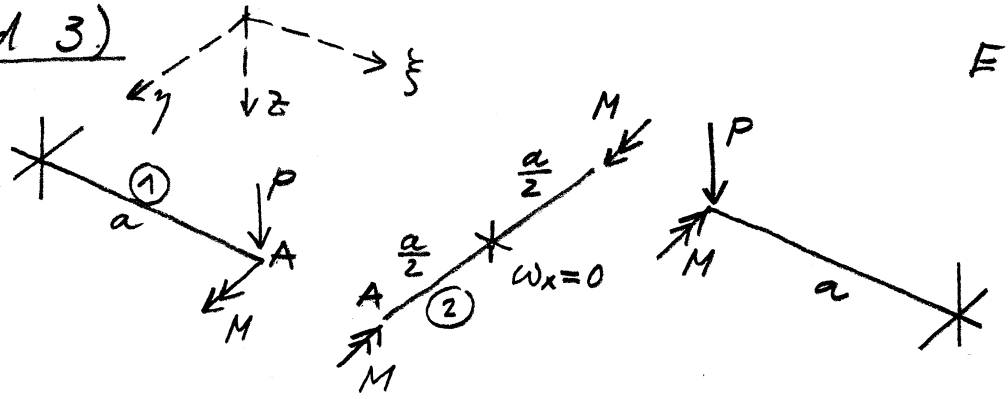
$$a\cdot\frac{a^2}{EI_y}\left(-N + \frac{7qa}{48} - N + \frac{9qa}{48}\right) - \frac{N}{EA_x}\cdot 2a = \frac{N}{EA_v}\cdot 2a$$

$$\boxed{N = \frac{qa^3}{6}\cdot\frac{A_v A_x}{I_y(A_v + A_x) + a^2 A_v A_x}}$$

$$\boxed{N = 0{,}1426\ MN}$$



Ad 3)



$EI_\eta = GI_x$

$$\omega_\eta^{①}(A) = M \cdot \frac{a}{EI_\eta} - P\frac{a^2}{2EI_\eta} \qquad \omega_\eta^{②}(A) = -M \cdot \frac{a}{2GI_x}$$

$$A: \quad \omega_\eta^{①} = \omega_\eta^{②} \longrightarrow M\left(\frac{a}{EI_\eta} + \frac{a}{2GI_x}\right) = P\frac{a^2}{2EI_\eta}$$

$$\boxed{M = \frac{Pa}{3}}$$

$$w_A = P\frac{a^3}{3EI_\eta} - M\frac{a^2}{2EI_\eta} \longrightarrow \boxed{w_A = \frac{Pa^3}{6EI_\eta}}$$